Configuring Robustness Properties in Control Systems

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Key Words: *Complex plane; time domain; pre-filter; robustness; sensitivity functions; Frequency domain.*

Abstract. *In this paper, an approach for achieving robustness, by proper configuration of a pre-filter in the control system, is proposed. A typical SISO, linear, continuous-time, regular, industrial class control systems is taken into consideration. The presence of delay, different order of astatism, as well as minimum phase properties of a larger class of models do not reduce the generality of the considerations and the conclusions made throughout the work. An assessment of the obtained performance in the time and frequency domains is presented also.*

1. Introduction

Guaranteeing robustness in control systems different in nature and function is a problem, which has found an effective solution in the combination of multiple approaches, based on different tools, ideas and alternatives, available in the complex plane and the frequency domain [1-7], [10-12]. Both domains, where robustness is searched for and guaranteed, have their, advantages and disadvantages.

The basic measure used to quantify control systems' robustness properties – the sensitivity function module has its useful and clear visualization and interpretation in the frequency domain. In the complex plane, interpreting the sensitivity function magnitude is a much harder task, though it is possible to have some insight depending on the n[th] - order zero poles. Complementary sensitivity function can be traced directly in the complex plane through root loci, allowing interpretation for all gains and frequencies covering the range from zero to infinity, but still its module is perceived in a much simpler and clearer way in the frequency domain.

The approach for guaranteeing robustness desired performance through a pre-filter [8] is a powerful tool, which requires rather good knowledge about the characteristics of the respective system. The pre-filter is a basic element in the QFT (Quantitative Feedback Theory) idea for robust control [2], [7], developed in the frequency domain, an approach, giving and using the relations between frequency-domain defined stability margins and time-domain specifications. Very feature, gives grounds to using complex-plane capabilities for specifying time-domain requirements [1], [5], [10-11], as an alternative for configuring a pre-filter. Most commonly, a pre-filter is needed and used when the capabilities of the system are insufficient to counteract adequately variations in the plant's dynamical parameters.

2. Goals and Problems

The goal of this paper is to present the application of an approach for „equipping“ a specific class of control systems, with robustness properties, by using correctly specified pre-filter. The study targets a class of control systems characterized by a standard controller (PI, PID) and a typical industrial plant model [9] in their structure.

This approach is reduced to an engineering procedure, solving design and analysis problems arising in the realization of the following steps:

a) performance analysis of the nominal uncompensated control system;

b) controller design, guaranteeing desired performance in nominal regime;

c) an assessment of system's performance under an a priori interval uncertainty in the dynamical parameters of the plant model;

d) pre-filter configuration in the complex plane;

e) verification and validation of the two-degrees of freedom control system's robustness properties in time and frequency domains.

An original aspect of the proposed approach is the configuration of a pre-filter, using performance specification tools, available in the complex-plane. It is shown that a motivated choice of the pre-filter's transfer function in the complex plane reflects in satisfying classic frequency-domain defined robustness criteria. Moreover, a quite clearer relation to time-domain performance is obtained since it can be explicitly interpreted in the complex plane.

3. Pre-filters' Application Domains

Figure 1 shows the signal flow graph of a linear SISO control system.

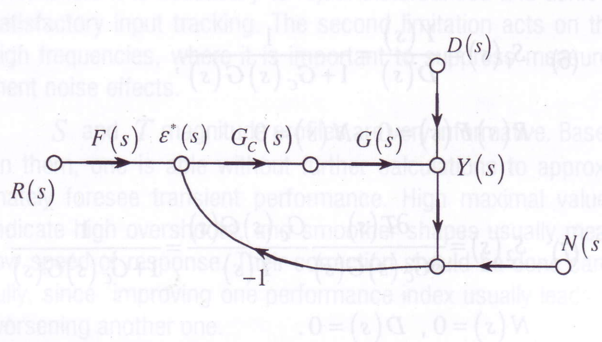


Figure 1

The basic equations relative to the output $Y(s)$ and error $\varepsilon(s)$ signals are given by the following expressions (1) and (2)

(1) $Y(s)=T(s)R(s)F(s)-T(s)N(s)+S(s)D(s)$,

(2) $\varepsilon(s)=R(s)F(s)-Y(s)= =S(s)R(s)F(s)+T(s)N(s)-S(s)D(s)$,

where:

$F(s)$ - pre-filter transfer function (TF), $R(s)$ - input, $\varepsilon(s)$ - error, $G_C(s)$ - controller TF, $G(s)$ - plant TF, $D(s)$ - disturbance, $Y(s)$ - output, $N(s)$ - measurement noise.

The error signal $\varepsilon(s)$ should be distinguished from the signal $\varepsilon^*(s)$, seen on the signal-flow graph, which itself is given by the following expression (3)

(3) $\varepsilon^*(s)=S(s)\left[R(s)F(s)-N(s)-D(s)\right]$.

It is namely $\varepsilon(s)$, the difference between reference input and system's output, the significant signal from control performance point of view.

$T(s)$ represents the complementary sensitivity TF and characterizes the closed-loop system's properties (4)

(4) $T(s)=F(s)G_C(s)G(s)(1+G_C(s)G(s))^{-1}$.

It is seen from (1) and (2), that the magnitude of $T(s)$, which is shaped by the pre-filter $F(s)$ determines the input tracking errors as well as the measurement noise influence. On the other hand, the sensitivity TF $S(s)$ has different interpretations (5) (6), (7), according to (1) and (2)

(5) $S(s)=\dfrac{\varepsilon(s)}{R(s)F(s)}=F(s)\dfrac{1}{1+G_C(s)G(s)}$,

$N(s)=0,\ D(s)=0$;

(6) $S_1(s)=\dfrac{Y(s)}{D(s)}=\dfrac{1}{1+G_C(s)G(s)}$,

$R(s)F(s)=0,\ N(s)=0$;

(7) $S_2(s)=\dfrac{\partial T(s)}{\partial G_C(s)G(s)}\dfrac{G_C(s)G(s)}{T(s)}=\dfrac{1}{1+G_C(s)G(s)}$,

$N(s)=0,\ D(s)=0$.

$S(s)$ can be defined as a disturbance $(D(s))$ to output $(Y(s))$ TF, noted as $S_1(s)$, or reference $(R(s)F(s))$ to error $\varepsilon(s)$ TF $S(s)$, or can be interpreted as a complex function describing the relative variation of the closed-loop to a relative variation of the open-loop system $-S_2(s)$.

The basic equations (1) and (2) suggest the zones where the pre-filter is able to affect the system performance. The pre-filter influences the sensitivity function, regarded as the reference to error TF, which means that it does not affect the disturbance rejection capabilities of the system.

When parameter uncertainty is considered, it can be claimed that the pre-filter has an influence, since the worst combination of parameter values leads to larger tracking and steady-state errors in the system. The noise influence on the control system performance is not affected by $F(s)$.

4. Basic Considerations for Pre-filter Configuration

The pre-filter $F(s)$ sets the desired dynamic behavior of the system by shaping its closed-loop transient response in order to meet certain time-domain specifications satisfying different criteria – integral, frequency or time domain.

The choice of pre-filter transfer function does not present a trivial problem at all. There is a common practice to assign pre-filter poles matching controller zeros [5]. Thus, a closed-loop system, with $n>m,\ m=0$ is obtained.

The use of a pre-filter is a delicate moment, representing a balance between performance specifications, since in order to work efficiently its dynamics must be significantly slower than the dynamics of the closed-loop control system (in and out presence of uncertainty). That is why, using a pre-filter aims and achieves a smaller overshoot or under-damped response sacrificing the speed of response of the system. In terms of frequency domain performance indices, the overshoot can be related to phase margin specification, which gives criteria for choosing $F(s)$.

Different direct and indirect frequency domain relations enable transforming these criteria for choosing desired values of sensitivity functions S and T.

Following the above mentioned considerations, taking formal expressions as (5), (6) and (7), the sensitivity function is used to characterize in a general way the control systems' performance. Nominal performance, robust stability and robust performance conditions are given (8), (9) и (10), [12]. A complex weighting function $w_S(j\omega)$ is used to specify desired performance of the control system. Following (8), the module of its inverse $\left|w_S^{-1}(j\omega)\right|$ can be viewed as a frequency-dependent bound on $|S|$. The weighting function $w_T(j\omega)$, taking part in the robust stability condition, accounts for the uncertainties in the model of the plant model, representing a multiplicative uncertainty model (11) or the relative change of the model at the „worst" combination of parameter values.

(8) $\max_{\omega}|w_S(j\omega)S(j\omega)|\le 1;$

(9) $\max_{\omega}|w_T(j\omega)T(j\omega)|\le 1;$

(10) $\sup_{\omega}(|w_S(j\omega)S(j\omega)|+|w_T(j\omega)T(j\omega)|)\le 1,$

$\forall\omega|\omega\in[0,\infty),$

where:

(11) $w_T(j\omega)=\dfrac{G_c(j\omega)G(j\omega,q_i^*)-G_c(j\omega)G(j\omega)}{G_c(j\omega)G(j\omega)},$

(12) $Q=\{q\,|\,q_i\in[q_i^-\ q_i^+],\ i=1,2...n\},$

Q - interval uncertainty range, n-dimensional rectangle [1], q_i - real, interval parameters, q_i^-, q_i^+ - a priori known upper and lower uncertainty bounds, respectively.

The pre-filter dynamics corrects the sensitivity functions' properties and from this point of view, the pre-filter' transfer function can be interpreted as a weighting function on *S*. Taking $F(s)$ into account, (10) is transformed into (13)

(13) $\sup_{\omega}(|F(j\omega)S(j\omega)|+|w_T(j\omega)T(j\omega)|)\le 1,$

$\forall\omega|\omega\in[0,\infty).$

The strictness of (13), the robust stability condition, is justified by clear geometric relations in the Nyquist plane [12]. It includes the sensitivity function, determining nominal performance and the complementary sensitivity function of the control system.

Requirements on control performance, in presence of uncertainty, can be translated into requirements on the sensitivity functions *S* and *T*, accounting for the pre-filter's properties, as in (14) and (15), [2]. The bounds w_{S_1}, w_{T_1} и w_{T_2} represent complex transfer functions, known as Horowitz-Sidi bounds [7], and can also be viewed as weighting functions. They determine the desired time-domain behavior of the control system, by shaping its frequency-domain response.

(14) $\max_{\omega}|F(j\omega)S(j\omega,q_i)|\le w_{S_1},$

(15) $w_{T_1}\le\max_{\omega}|F(j\omega)T(j\omega,q_i)|\le w_{T_2}$

The relations given by (13) and (14), (15) show that the specific properties of the pre-filter play significant part in determining the robustness of the control system.

5. Characteristic Properties of S and T

Sensitivity S and complementary sensitivity T functions represent robustness quantitative measures in the frequency domain. Achieving zero steady-state error (16) is an essential goal in all control approaches,

$$\varepsilon(\infty)=\lim_{t\to\infty}\varepsilon(t)=\lim_{s\to 0}sR(s)F(s)S(s)=$$

(16) $$=\lim_{s\to 0}sR(s)\left[F(s)(1-T(s))\right]=0.$$

Different functionals, including the ε signal are commonly minimized using the following criteria - IE, IAE, ISE, ITE, ITSE, ISTE, ITAE (17).

(17) $$J_j=\int_0^t f_j\{\varepsilon(t)\}dt\to\min.$$

In order to satisfy (16), it is required for S and T to have zero values.

The identity $S+T=1$ shows that there is certain contradiction, needing to be balanced by certain compromise, which is the goal of robust methods, [12]. This balance has its visualization in the frequency domain (18) and (19) and it is usually realized by special weighting functions.

(18) $M_S=\max_{\omega\in[0,\omega_1]}|S(j\omega)|<1,$

(19) $M_T=\max_{\omega\in[\omega_2,\infty)}|T(j\omega)|<1,\quad\forall\omega|\omega\in[0,\infty)$

and $\omega_2>\omega_1$

The first expression is relative to the low-end of frequencies, where it is necessary to reject disturbances and achieve satisfactory input tracking. The second limitation acts on the high frequencies, where it is important to suppress measurement noise effects.

S and T magnitude profiles are very informative. Based on them, one is able without further calculations to approximately foresee transient performance. High maximal values indicate high overshoots, and smoother shapes usually mean low speed of response. Their correction should be done carefully, since improving one performance index usually leads to worsening another one.

6. Numerical Example

The basic problem formulation can be subdivided into two parts by defining a global performance criterion, guaranteeing robustness, and a local one dealing with control system' nominal regime.

1. It is necessary to design a controller for the nominal plant (*figure 1*), able to achieve speed of response, twice as high as that of the uncompensated control system.

2. In presence of a priori bounded uncertainty in plant's dynamic parameters, it is required to guarantee transient closed-loop response, specified by an overshoot $\sigma = 20\% \pm 0.04$.

6.1. Time- and Frequency- Domain Analysis of the Nominal Uncompensated Control System

Typical plant transfer function [9] is given by the expression (20)

$$(20) \quad G(s) = \gamma\left(s(s+\alpha)(s+\beta)\right)^{-1}.$$

Proportional controller is considered $G_C(s) = k_p = 1$. According to *figure 1*, $F(s)=1$, $D(s)=0$ and $N(s)=0$ is assumed. The a priori uncertainty is obtained as a consequence of 20% variations of all three of the plant's parameters (21)

$$(21) \quad G(s,q) = q_1\left(s(s+q_2)(s+q_3)\right)^{-1},$$

where the upper and lower bounds of variations are expressed by (22)

$$(22) \quad q_1^{\pm} = \gamma \pm 20\%, \; q_2^{\pm} = \alpha \mp 20\%, \; q_3^{\pm} = \beta \mp 20\% .$$

The nominal dynamic parameters of the plant have the following values $\gamma = 24$, $\alpha=3$, $\beta=5$. These values determine indirect performance indices – phase margin PM, gain margin GM, bandwidth ω_{BW} and ω_{BW_S} , maximal sensitivity M_S , complementary sensitivity M_T , summarized in *the table, figure 2*.

These frequency domain defined indices guarantee transient performance with overshoot $\sigma = 20\%$ and settling-time $t_s \pm 2\% = 4.39s$ *figure 3*.

The maximal value of the sensitivity function M_S *figure 2*, is quite high, which means that small variations in plant parameters will lead to considerable changes in transient performance compared to the nominal regime of the system.

The P controller is able to improve the speed of response to its maximum at $G_C(s) = 0.344$. In this case, the control

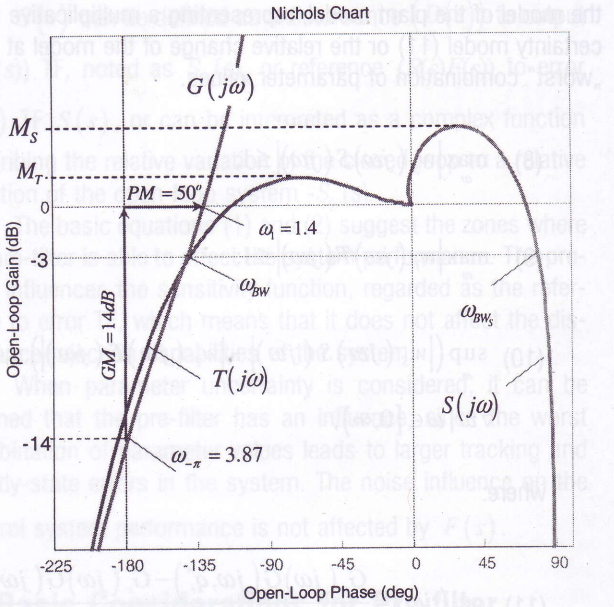


Figure 2

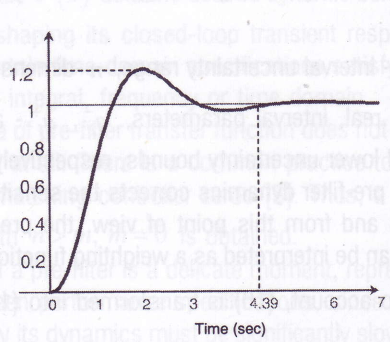


Figure 3

system will be characterized by a critically damped transient response, with settling-time still far from the required, *figure 4* (root locus for the uncompensated control system $G(s)$ in green).

The properly designed PD controller will ensure desired settling time, but there will be no reduction in sensitivity, since no poles are added at zero.

Reversely, a PI control will reduce sensitivity, but the settling-time requirements will not be met.

According to the preceding considerations and the speed-response requirements, PID controller design represents a convenient possible solution. A proper selection of its parameters will lead to relocating the nominal dominating closed-loop poles to the left, thus ensuring the effective functioning of the pre-filter in presence of a priori interval uncertainty. This is a necessary step since there is a theoretical stability loss possibility for the uncompensated control system for certain combination of parameter values in presence of uncertainty.

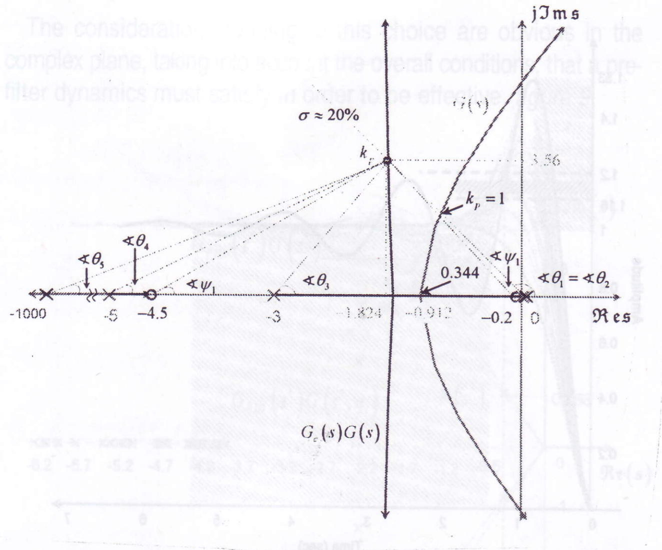


Figure 4

6.2. PID Controller with Pre-filter Standard Design

In this specific case it is appropriate to choose a PID controller with time-constants ratio of $T_I T_D^{-1} > 0.25$.

A delicate moment requiring certain intuition from the designer is the positioning of controller zeros, since there are many possible combinations. Purpose - wise it is suitable for the first zero to be located sufficiently close to the origin of the complex plane in order to eliminate the additional zero root influence on transient responses. The other zero is positioned from the right of the third pole of the uncompensated system, so that a valid second-order system approximation is obtained, *figure 4*.

In order for the PID controller to be physically implemented, a filter with transfer function (23)

(23) $G_{F_{PID}}(s) = (T_F s + 1)^{-1}$ $\quad T_F = 0.001s$

The pole of the filter is chosen so that it sits far left in the plane not influencing the properties of the designed control system. The PID controller design represents a pole-placement problem, sol-ved by satisfying modulus and argument conditions (24), (25), graphically represented in *figure 4*, ($\measuredangle\theta_1 = \measuredangle\theta_2 = 117.13^o$, $\measuredangle\theta_3 = 68.7^o$, $\measuredangle\theta_4 = 47.1^o$, $\measuredangle\theta_5 = 0.2^o$, $\measuredangle\psi_1 = 117.13^o$, $\measuredangle\psi_2 = 53.3^o$, $k = 0.5$).

(24) $\left|G_{PID}(s)G(s)\right| = \prod_{i=1}^{2}\left|s+z_i\right| / \prod_{i=1}^{5}\left|s+p_i\right| = \left|k\right|^{-1}$,

(25) $\angle G_{PID}(s)G(s) = \sum_{i=1}^{2}\measuredangle\psi_i - \sum_{i=1}^{5}\measuredangle\theta_i = \pm(2k+1)\pi$

The transfer function of the obtained controller is given by (26).

(26) $G_{PID}(s) = 0.5(s+0.02)(s+4.5)s^{-1}(0.001s+1)^{-1}$.

It is seen in *figure 5*, that the PID controller is able to the necessary phase margin $PM \approx 50^o$, which guarantees an overshoot $\sigma \approx 20\%$. The new crossover frequency $\omega_1 = 2.73r/s$ is twice as large as that of the uncompensated system $\omega_1 = 1.4r/s$, which leads to achieving the required settling – time.

Both PID controller real zeros introduce additional dynamics improving the speed of response and according to Bode relation, given by (27)

(27) $\angle G_{PID}(j\omega)G(j\omega) \approx n90^o$

where $n = -2$ is the slope of $\left|G_{PID}(j\omega)G(j\omega)\right|$, the compensated system has a an infinitesimal gain margin, $GM \to \infty$.

That means that the nominal system remains stable for each value of the generalized gain *figure 5*.

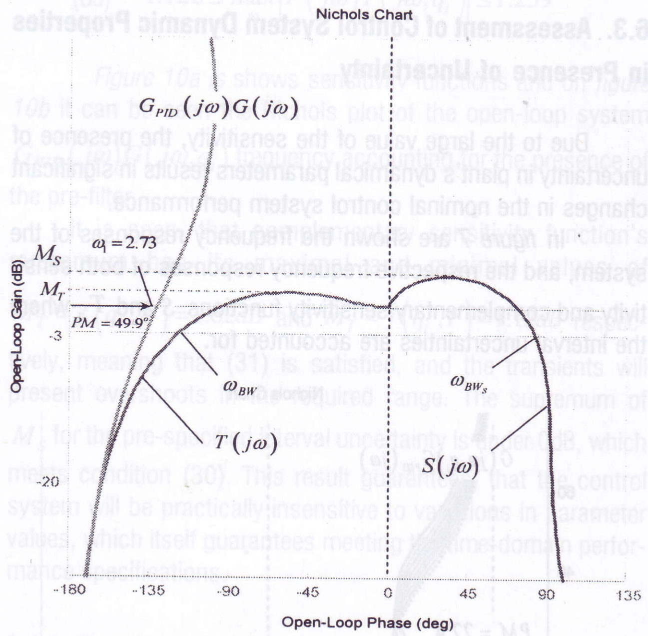


Figure 5

On the other hand, these appropriate values of gain and phase margins do not guarantee a low value of M_S of the system, though the PID controller additional zero pole reduces in certain degree the sensitivity. The compensated system is characterized by performance indices, summarized in the *table, figure 5*.

The M_T and PM values are the same as those in the uncompensated system, due to overshoot criterion, the characteristic frequencies are twice as large, due to speed of response requirements and the maximal sensitivity M_S is about $1dB$ lower.

Figure 6 shows comparative transient performance curves of both compensated and uncompensated control system. It can be concluded that the frequency domain design carried out is correct.

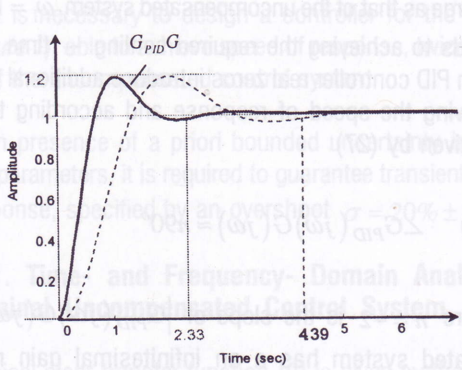


Figure 6

6.3. Assessment of Control System Dynamic Properties in Presence of Uncertainty

Due to the large value of the sensitivity, the presence of uncertainty in plant's dynamical parameters results in significant changes in the nominal control system performance.

In *figure 7* are shown the frequency responses of the system, and the respective frequency responses of both sensitivity and complementary sensitivity functions S and T, where the interval uncertainties are accounted for.

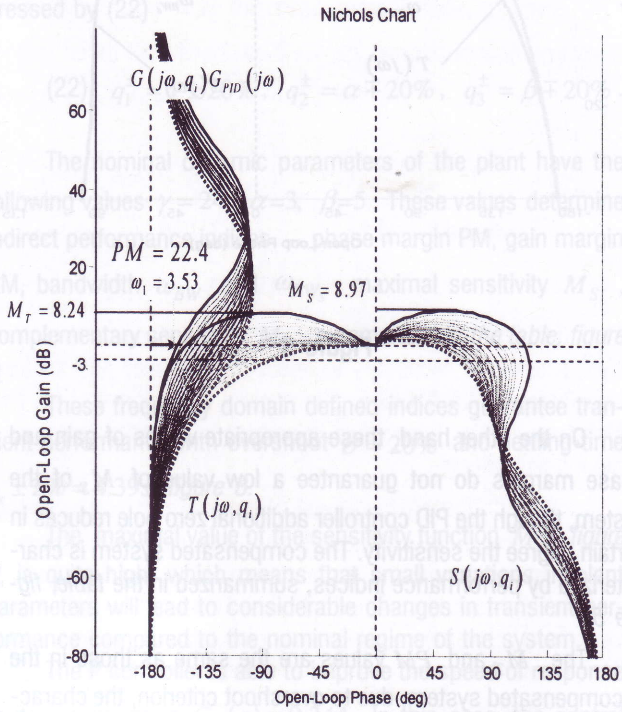


Figure 7

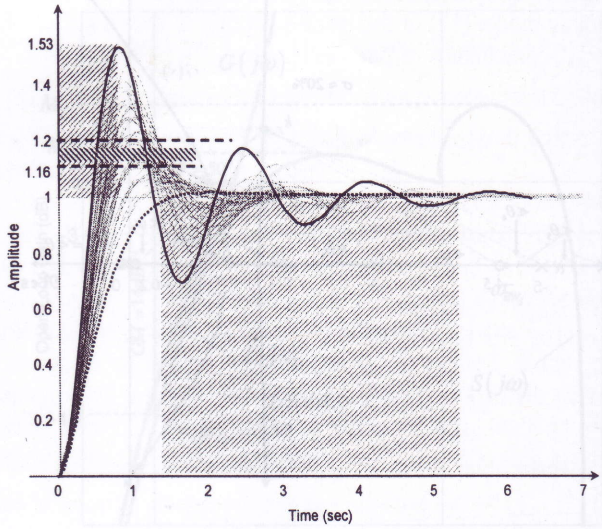


Figure 8

In dark the responses for the worst combination of parameters' values are visualized which leads to reduced values of stability margins and speed of response. The other boundary combination, given by a -20% variation in each parameter, also results in significant differences in comparison with the nominal system (represented with dashed line).

In the *table* are summarized all frequency domain indices for the cases of variations in plant parameters corresponding to the upper and lower bounds of their respective interval sets. It is seen that the control system does not exhibit robust properties, and that the PID controller is not able to achieve the desired performance in presence of uncertainty.

Some transient are shown in *figure 8* responses of the system with interval uncertainty. The transients for $G_{PID}(s)G(s,q_i^-)$ and $G_{PID}(s)G(s,q_i^+)$ are visualized with dark lines. The hatched areas enclose the upper and lower bounds of both direct performance indices - overshoot and settling-time respectively, It is seen that their variation ranges are significant. The zone corresponding to $\sigma = 20\% \pm 0.04$ is a visualization of the performance criterion and determines the robustness in this case, representing a time domain interpretation of the Horowitz boundary [2-4], [6-7].

6.4. Pre-filter Design and Performance Assessment

Introducing a second degree of freedom by a pre-filter in the control system aims at achieving robustness in a concrete sense. That is, maintaining the overshoot in the pre-specified limits $\sigma = 20\% \pm 0.04$ in presence of a priori uncertainties in plant parameters. A suitable choice for the task is a pre-filter with transfer function (28)

$$(28)\quad F(s) = \omega_n^2\left(s^2 + 2\xi\omega_n s + \omega_n^2\right)^{-1}$$

The considerations leading to this choice are obvious in the complex plane, taking into account the overall conditions, that a pre-filter dynamics must satisfy in order to be effective, *figure 9.*

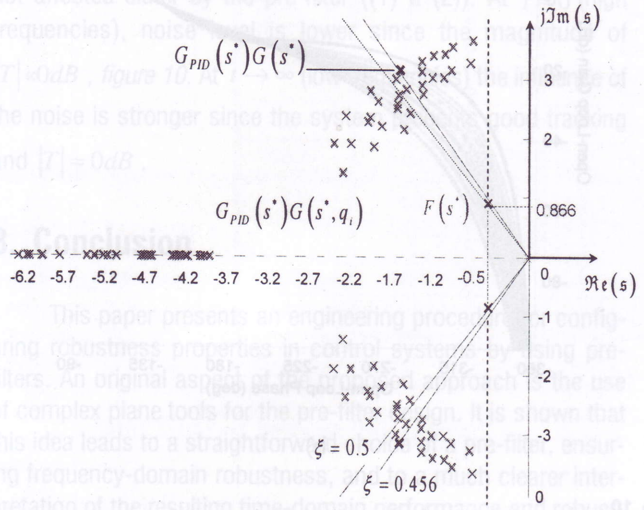


Figure 9

The repartition of characteristic equations' roots of (29) are shown in *figure 9*,

(29) $\Re\left(s,q_i\right)=1+k_e G_{PID}\left(s^*\right)G\left(s^*,q_i\right)=0$,

corresponding to variations of ±20% in all plant parameters. The "*" symbol indicates the roots specifying nominal performance. The roots determining the transient responses when plant parameters are set to their nominal values are marked in pink.

According to the "dominating poles" concept, both $F(s)$ poles must lie in specific regions to the right of the zone of uncertainty in conformity with the desired performance i.e. $\sigma = 20\%$. Since in all cases there has to be a compromise with the speed of response, a conditional speed of response is targeted.

A specific aspect that has to be taken into account comes from the fact that the overshoot determined by the pre-filter must be from 3 to 5% less than the desired, in order to achieve optimal results. The configuration of a pre-filter is an iterative procedure, requiring the engineering intuition of the designer. After a couple of steps into it the pre-filter transfer function $F(s)$ is specified by a damping factor $\xi = 0.5$ and an undamped natural frequency $\omega_n = 1$ which lead to transient performance characterized be an overshoot and settling-time (30), (31)

(30) $\sigma \approx \exp\left[-\pi\xi\left(1-\xi^2\right)^{-0.5}\right].100\% = 16.3\%$,

(31) $t_s \pm 2\% \approx 4\left(\xi\omega_n\right)^{-1} = 8s$.

The choice of a pre-filter by complex plane specifications is a possible alternative to the frequency-domain approach, since the complex plane gives an explicit interpretation of the time domain. That is why, if a model of the plant is available, it is convenient to configure the pre-filter in the complex plane. The design usually is reduced to specifying ξ и ω_n parameters, related to the most important characteristics of the transient response – overshoot and settling-time.

6.4.1. Frequency-Domain Analysis

The bounds on the sensitivity functions in (14) и (15), are chosen as shown in (30) and (31), a choice determined by the nature of the performance criterion $\sigma = 20\% \pm 0.04$. Negative values of $M_S(j\omega,q_i,F)$ over the entire frequency range are required to obtain a significant reduction of S. The bounds on $M_T(j\omega,q_i,F)$ are determined by the overshoot specifications.

(32) $\max\limits_{\omega}\left|F(j\omega)S\left(j\omega,q_i\right)\right| \le 1$,

(33) $1.122 \le \max\limits_{\omega}\left|F(j\omega)T\left(j\omega,q_i\right)\right| \le 1.259$

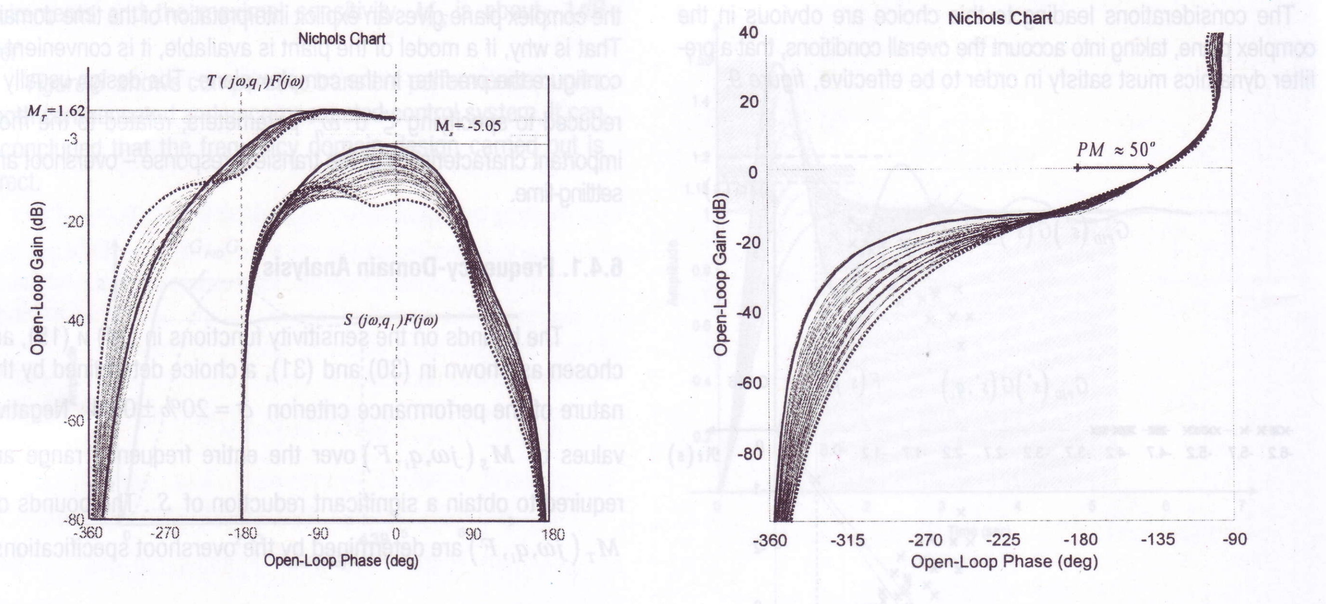
Figure 10a is shows sensitivity functions and on *figure 10b* it can be seen the Nichols plot of the open-loop system $G_{PID}(j\omega)G(j\omega,q_i)$ frequency accounting for the presence of the pre-filter.

It is seen, that complementary sensitivity function's supremum has its maximal and minimal values of $M_T = f\left(q_i^+,F\right)=1.62dB$ and $M_T = f\left(q_i^-,F\right)=1.12dB$ respectively, meaning that (31) is satisfied, and the transients will present overshoots in the required range. The supremum of M_S for the pre-specified interval uncertainty is under 0dB, which meets condition (30). This result guarantees, that the control system will be practically insensitive to variations in parameter values, which itself guarantees meeting the time-domain performance specifications.

6.4.2. Time-Domain Analysis

In *figure 11* the transient responses of the two degrees of liberty control system in the case of variations in plant parameters are shown. The uncertainty in plant parameters results in an overshoot variation of about 1% when the pre-filter is added, which is a lot less than the case without it - from 0% to 51.4%. The local performance criterion $\sigma = 20\% \pm 0.04$ is met, since all transient curves are found in the specified zone. When speed of response is considered, the control system is robust, since the settling-time varies in a very narrow range.

Figure 12 visualizes the evolution of the error in the time domain. It is seen that the introduction of the pre-filter reduces the error, respectively the system's sensitivity.



a) b)

Figure 10

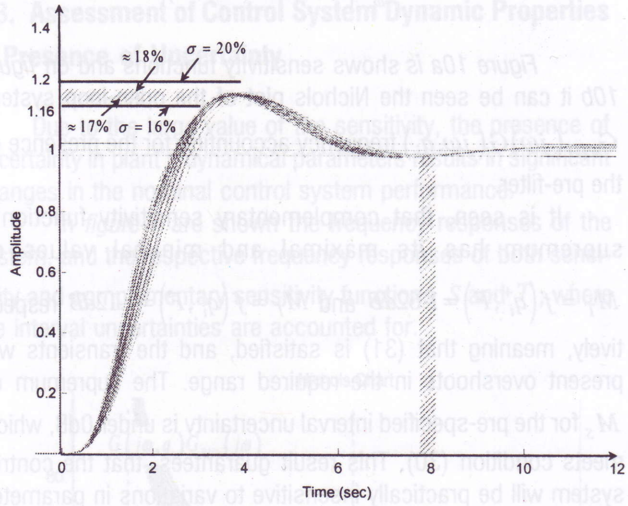


Figure 11

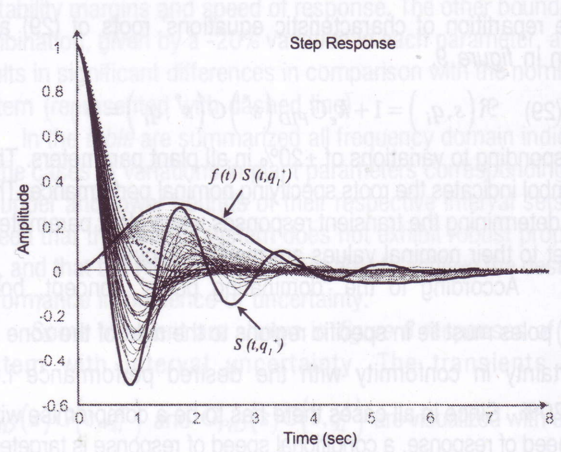


Figure 12

6.4.3. Robust Stability and Performance Analysis

Figure 13 presents a graphical expression of robust stability and robust performance conditions (9) and (13).

The robust stability condition, given by (9) is satisfied (*figure 13a*). Condition (13), concerning robust performance, obviously is not strictly met.

On the other hand, *figures 10* and *11*, and *the table* indicate that the control system possesses robust properties, since it is practically insensible to variations in plant parameters. It is possible to satisfy the requirement, given by (13) by introducing additional weighting functions. In this case, the time domain requirement $\sigma = 20\% \pm 0.04$ will be violated since the magnitude $\left|T(j\omega, q_i)\right|_{dB} < 0$, which means an under-damped (aperodic) transient.

7. Analysis of Disturbance and Measurement Noise Effects

Let $D(s) = -0.3s^{-1}$, as in *figure1*. When disturbance rejection is considered, the pre-filter $F(s)$ does not affect control system performance, *figure 14*.

The output reaction of the system to disturbance signals at the worst combination of parameter values is not altered by the presence of the pre-filter in the control system. In both cases it takes the same amount of time to reduce the disturbance related deviation of the output.

Output reactions to noise in feedback measurements are not affected either by the pre-filter ((1) и (2)). At $t \approx 0$ (high frequencies), noise level is lower since the magnitude of $|T| \ll 0dB$, *figure 10*. At $t \to \infty$ (low frequencies) the influence of the noise is stronger since the system presents good tracking and $|T| \approx 0dB$.

8. Conclusion

This paper presents an engineering procedure for configuring robustness properties in control systems by using pre-filters. An original aspect of the proposed approach is the use of complex plane tools for the pre-filter design. It is shown that this idea leads to a straightforward choice of a pre-filter, ensuring frequency-domain robustness, and to a much clearer interpretation of the resulting time-domain performance and robustness.

A short systematization of the properties and the capabilities of the pre-filter, as a specific functional element in control systems is done. This enables assessing and properly reflecting the impact of its presence in robustness related conditions and their frequency-domain graphical interpretations. It is shown how pre-filter's characteristics shape sensitivity and complementary sensitivity functions in the frequency domain, as well as system's frequency response in order to obtain desired performance in the presence of an a priori uncertainty. The relations between time and frequency domains and the complex plane are efficiently exploited through the respective performance specifications.

Control system's dynamic properties are compared in configurations with and without a pre-filter in the structure, for all combinations of parameter values in the a priori uncertainty range.

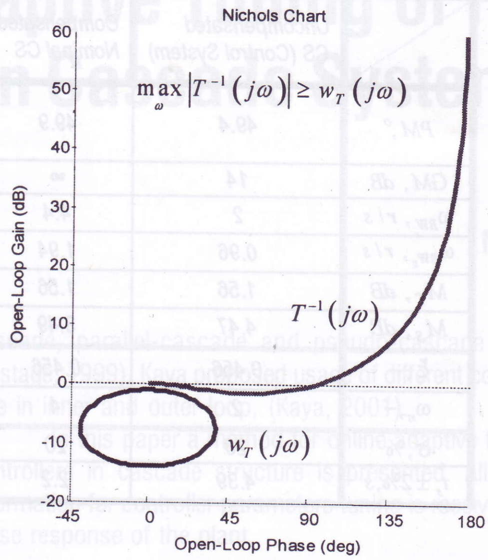
The following conclusions can be drawn from the study presented in the paper:

1. The specification of a pre-filter presents a not trivial, iterative problem, though unambiguous and clear. Introducing a pre-filter in the control system does not affect disturbance and measurement noise influence on the output.

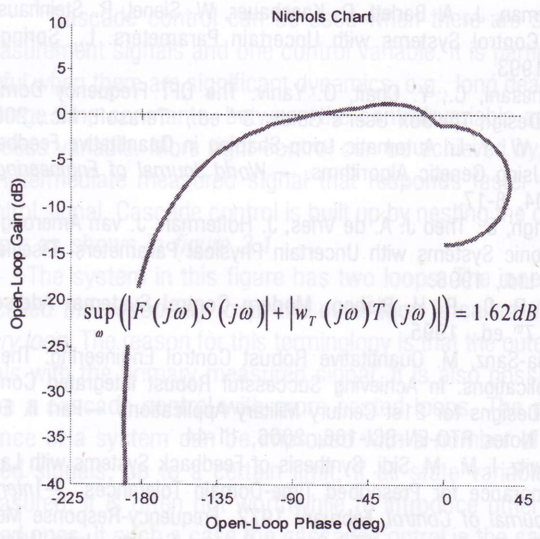
2. The use of a pre-filter in presence of parametric uncertainty requires minimal relative stability margin of the closed-loop system, which can be ensured by a proper choice of control law.

3. The study, presented in the paper, shows that meeting simultaneously time-domain and robust frequency-domain specified criteria is contradictory and ambiguous. That is why usually robust criteria are formally guaranteed in the frequency domain, while a less severe inter-pretation of robust properties is acceptable in the time domain and its direct relation with the complex plane.

4. The results drawn from this typical control system study can be generalized and applied to any SISO control system, whose sensitivity function presents no particularities from interpretation point of view.



a)



b)

Figure 13

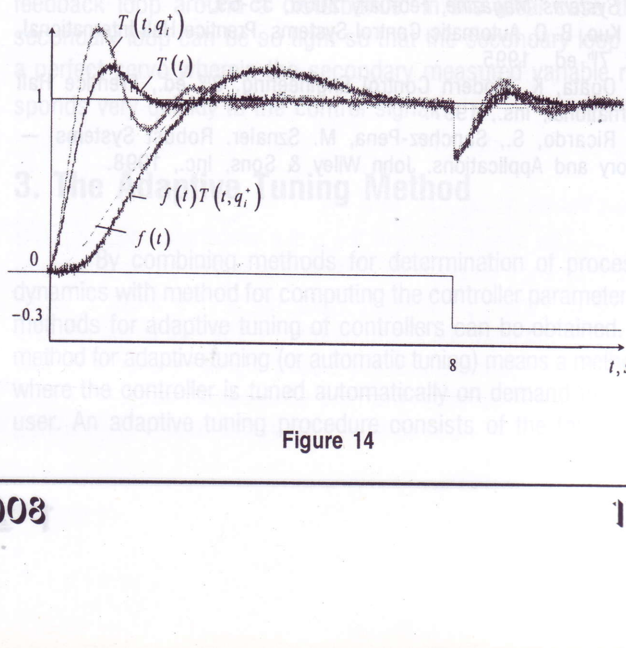


Figure 14

	Uncompensated CS (Control System)	Compensated Nominal CS	CS in presence of Uncertainty without F		CS in presence of Uncertainty with F	
			$\bar{q}_i^-$	$\bar{q}_i^+$	$\bar{q}_i^-$	$\bar{q}_i^+$
$PM,^o$	49.4	49.9	71.6	22.4	49.1	52.4
$GM,\ dB$	14	∞	∞	∞	∞	∞
$\omega_{BW},\ r/s$	2	4.4	2.58	5.72	1.4	1.4
$\omega_{BW_s},\ r/s$	0.96	1.94	1.56	2.35	-	-
$M_T,\ dB$	1.56	1.56	0.08	8.24	1.12	1.62
$M_S,\ dB$	4.47	3.49	1.47	8.97	-13.4	-5.05
$\xi,-$	0.456	0.456	1	0.2	0.5	0.5
$\omega_n,-$	2	4	2.9	3.74	1	1
$\sigma,\%$	20	20	0	51.4	16.8	18.8
$t_s \pm 2\%,s$	4.39	2.2	1.37	5.34	7.5	8.07

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Manuscript received on 14.02.2008

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