e-Learning of Phase Change Processes under Vigorous Convection Heat Transfer

S. Kimura, K. Kanev

Key words: *Heat transfer; phase change; solidification; convection; e-learning; GIFES.*

Abstract: *In this work we discuss dynamic behaviours of solidification processes and formulate a one-dimensional model for their analysis. Based on it a fast and efficient code for numerical simulations and solidification layer thickness predictions has been implemented. A specialized experimental apparatus for observation and measurement of time varying ice layer thickness under different cooling conditions has been constructed and employed for gathering of experimental data. An extensive analysis and comparisons of model-based numerical results and experimentally measured layer thicknesses have been conducted. Following the very good agreement of simulated and measured results we are integrating the numerical simulation of solid layer dynamic responses to cooling temperature modulations and convective fluid flow deviations into traditional engineering courses. We are also focusing on e-learning where access to experimental facilities is much more limited and modelling and numerical simulations can play a central role. In the context of e-learning we discuss the registration and employment of the developed numerical code into the specialized highly configurable Graphical Interface Framework for Educational Support (GIFES.)*

Introduction

Solidification phenomena can be observed in a wide range of industrial and geophysical processes. For instance, techniques to control solidifying processes are crucial in casting industries and various materials development [1]. In geophysics, researching the formation of the Earth crust at mid-ocean ridges and the crystallization of magma in volcanic activities helps understanding interior Earth structures, plate tectonics and other geophysical processes [2,3]. In meteorology, monitoring and predicting of seasonal and inter-annual arctic sea ice fluctuations are very important because of the large thermal impact on atmospheric and ocean circulations [4].

Solidification of water, or ice formation, is one of the most commonly encountered phenomena around us. In this work we are considering the formation of an ice layer over a surface cooled to a subfreezing temperature. It is well recognized that convection flows adjacent to an ice front are extremely complex due to the presence of a density maximum near 4°C. Since various applications in industry and geophysics require evaluation of convective heat fluxes in the presence of a temperature inversion, these aspects of pure water freezing have been extensively researched in the past two decades. The aspect that we address in this paper is the solid layer response to transient cooling temperature variations.

It is a well-established fact that the metal-alloy morphology is very sensitive to the solidifying speed. In reality, adjusting the cooling temperature is the only practical way to control the solid layer growth and to ensure growing front speeds suitable for the formation of morphological structures of a certain quality. In this case, the cooling temperature is non-periodic and must decrease monotonically with the time. Periodic cooling temperature variations, however, are widely observed in thermal storage systems and natural environments as previously mentioned. Such periodic temperature conditions constitute one of the most fundamental cases for studying the transient response of a solid-liquid interface. In this paper, we consider the special case, where the thermal boundary condition is periodic in time at one end, and constant at the other end.

In fact, solidification and melting, subject to periodic surface temperature variations, appear not to have been studied in detail yet. To the present authors' knowledge, there are only a few reports on this problem, most of them dedicated to the thermal storage performance of phase-change materials (PCM). For example, Bransier [5] analyzed the conduction-dominated thermal behaviour of a PCM in contact with a fluid undergoing sinusoidal temperature variation in time. Kalhori and Ramadhyani [6] investigated the heat transfer in a vertical annular storage unit subject to a periodic steady state operation. Jariwana et al. [7] looked at a vertically-positioned cylindrical latent-heat storage container under a similar periodic operation. More recently, Ho and Chu [8] investigated periodic melting in a square box numerically, when the temperature at the hot vertical wall is varied in time. They reported the correlations of the mean heat transfer rate as well as the oscillating heat transfer amplitude. One of the current authors has already presented a one-dimensional model for downward solidification from the top boundary in a water layer with vigorous convection in Kimura and Vynnycky [9]. This time, we continue to investigate a possible one-dimensional model for predicting the response of the ice-layer front, when the cooling temperature oscillates at a steady period and amplitude. In the present paper, a horizontal water layer subject to downward solidification from the top is assumed. First, a one-dimensional model is presented and a coupled set of differential equations describing the energy conservation in the solid layer and the energy balance at the solid-liquid boundary is solved numerically. For verification of the proposed one-dimensional model we have gathered and employed experimental data for the ice-front movement in a square box as shown in the accompanying figures. In the final section of the paper our experimental results and the developed numerical model are considered in the context of e-learning.

Mathematical Formulation

One-Dimensional Model: We assume that a bulk of liquid is cooled from above at a uniform temperature below the solidi-

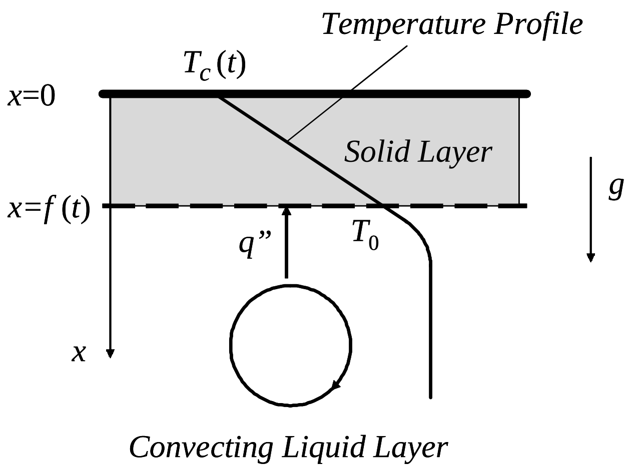


Figure 1. Schematic diagram of the one–dimensional model

fication point. If only the average thickness of the formed solid layer is considered, a one-dimensional model of the growth and the decay of the solid layer thickness can be formulated. The bulk of liquid below the solid layer is assumed to be dominated by vigorous convection due to the temperature difference between the solidification front and the bulk of liquid, which is large enough in volume and mass to maintain the same temperature, as time progresses and the solid layer grows.

Referring to *figure 1*, the energy equation within the solid layer and the energy balance on the solid-liquid boundary can be written as follows:

$$\frac{\partial T_s}{\partial t} = \alpha_s \frac{\partial^2 T_s}{\partial x^2}\,, \tag{1}$$

$$k_s \frac{\partial T_s}{\partial x}\bigg|_{x=f} - q'' = \rho_s L \frac{\partial f}{\partial t}\,. \tag{2}$$

The boundary conditions are

$$T_s = T_c \text{ on } x = 0 \text{ and } T_s = T_0 \text{ on } x = f, \tag{3}$$

where T, t, α, k, q'', ρ, L and f denote the temperature, time, thermal diffusivity, thermal conductivity, convective heat flux, density, latent heat of fusion, and the solidifying front respectively. The subscript s refers to the solid layer. We carry out the following non-dimensionalization

$$\left.\begin{array}{c} \theta_s = \dfrac{T_s - T_0}{T_0 - T_c},\ X = \dfrac{x}{l},\ F = \dfrac{f}{l},\ \tau = \dfrac{t\alpha_s}{l^2} \\[2ex] S = \dfrac{L}{c_p(T_0 - T_c)} \end{array}\right\} \tag{4}$$

Here l is the length scale defined by the solid layer thickness prevailing for a given cooling temperature and strength of convection heat flux:

$$l = \frac{k_s(T_0 - T_c)}{q''}\,. \tag{5}$$

One can notice that S is the inverse of the Stefan number of the system. To accommodate the time-varying position of the solid-liquid boundary the following coordinate transformation is introduced:

$$\xi = \frac{X}{F}\,. \tag{6}$$

The resulting dimensionless and coordinate transformed governing equations, corresponding to the equations (1) and (2) are as follows:

$$\frac{\partial \theta_s}{\partial \tau} = -\frac{\partial \xi}{\partial \tau}\frac{\partial \theta_s}{\partial \xi} + \left(\frac{\partial \xi}{\partial X}\right)^2 \frac{\partial^2 \theta_s}{\partial \xi^2} \tag{7}$$

and

$$\frac{\partial F}{\partial \tau} = \frac{1}{S}\left(\left(\frac{\partial \xi}{\partial X}\right)\frac{\partial \theta_s}{\partial \xi}\bigg|_{\xi = 1} - 1\right)\,. \tag{8}$$

The boundary conditions can be likewise transformed to

$$\theta_s = -1 \text{ on } X = 0 \text{ and } \theta_s = 0 \text{ on } X = 1. \tag{9}$$

This completes our formulation. The resulting set of equations (7), (8), and (9) is solved numerically. The initial conditions are derived from an arbitrary very thin solid layer and the Stefan solution for the temperature within the solid.

Convection Heat Flux. The convection heat flux q'' arriving on the solid-liquid boundary due to the vigorous convection in the liquid phase is a crucial parameter that determines the steady state thickness of the formed solid. Convection heat transfer coefficients for different convection processes, for example, natural and forced convection systems with various geometric conditions are readily available in heat transfer text books and handbooks, for instance [10].

Ice formation in water might be the most common solidification but due to the peculiar density variations near 4°C, convection heat flux from water tends to be difficult to correlate in a usual manner. Based on a series of two-dimensional numerical simulations, Blake et al. [11] proposed a heat transfer correlation for natural convection in water near 4°C. One of the present authors has developed and reported in [9] a one dimen-

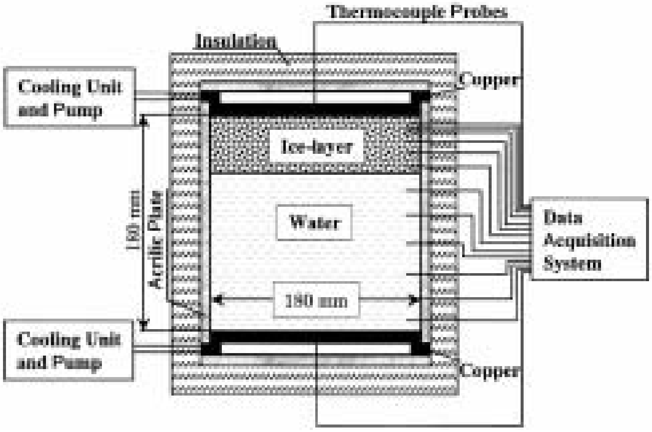


Figure 2. Schematic diagram of the experimental apparatus

sional model based on that correlation. Some discrepancies between the numerical predictions and the experimental results have been noted, however, indicating that the proposed heat transfer correlation might not be truly accurate.

In the present work, therefore, we employ a dedicated experimental apparatus for the direct measurement of the heat fluxes. Consequently we experimentally confirm that for every cooling temperature the prevailing ice layer thickness is an outcome of the heat balance between the conduction cooling in the ice and the convection heat flux on the solid-liquid boundary.

Experiments

Experimental Apparatus and Measurement. A specialized apparatus was constructed for monitoring the time-varying ice layer response to the cooling temperature modulation with time, while nearly constant convective heat flux from water to ice front is maintained during the experiment. *Figure 2* shows a sketch of the apparatus.

The core of the apparatus is a 180x180x180mm cubic box with acrylic resin sidewalls and copper plate top and bottom. The higher thermal conductivity of copper helps maintain uniform temperature distributions at the top and bottom boundaries of the box. Distilled water is filled in the box, and its top boundary is brought to a sub-zero temperature in order to initiate the formation of ice. The copper top plate of the box is chilled by running brine through it supplied from a cooling unit, while the bottom is kept at 8°C by another cooling unit. Such a setup ensures vigorous water convection in the tank even when the boundary temperature is varied with time. Temperatures in the respective copper plates and at certain vertical positions in the box are measured by thermocouples and recorded in a data acquisition system. The entire vessel is placed in another larger box made of polystyrene foam for minimizing the environmental temperature variations impact.

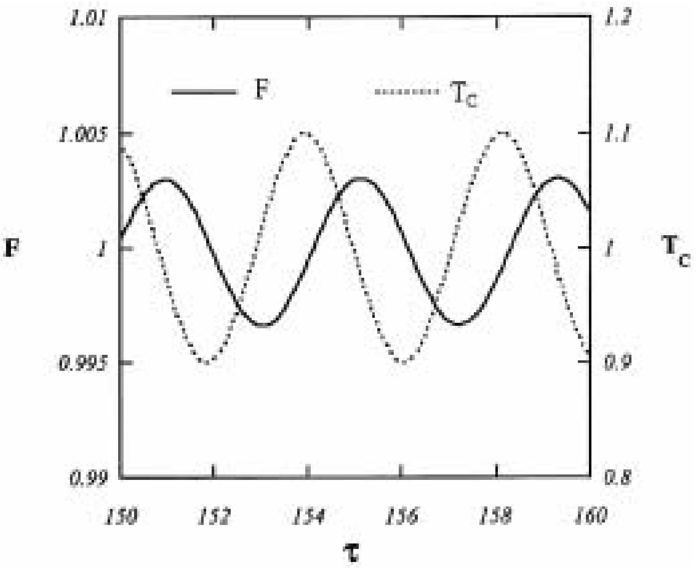


Figure 3. The nondimensional ice-water boundary response $F(t)$ for a sinusoidal cooling temperature modulation $T_c(t)$ with $S = 20$ and a nondimensional oscillation frequency of 20

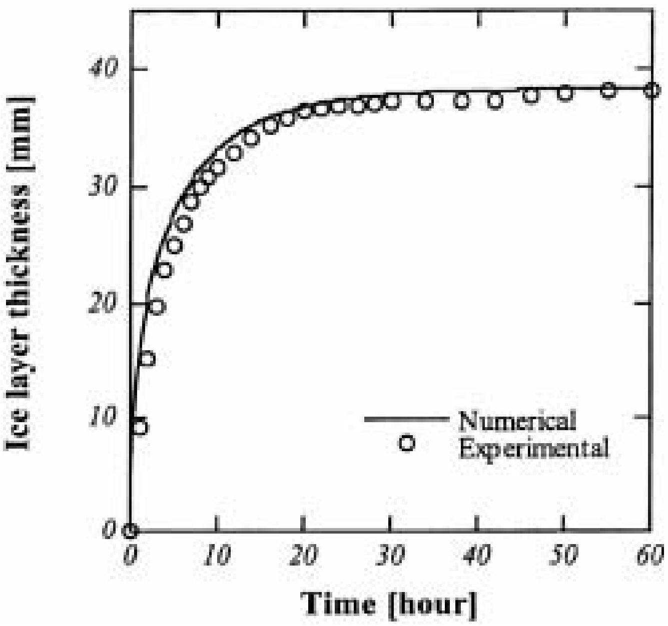


Figure 4. Ice-layer growth with time when the cooling temperature is fixed at -7°C

Experimental Procedure and Results. At the beginning of an experiment, ample time is allocated for reaching a thermal equilibrium of 8°C in the water, and only afterwards the top boundary temperature is lowered to an aimed sub-zero temperature. Water adjacent to the upper boundary shows a slight super cooling, and breaking of the meta-stable condition follows with a sudden formation of extremely thin dendrite ice layer and a release of latent heat, which is clearly detected by a spike in the upper boundary temperature. As we wait long enough, the ice layer growth terminates at a certain distance from the top boundary, which clearly depends on the magnitude of the cooling temperature T_c . When the cooling temperature is set to -10.5°C, an average ice thickness of about 6cm is reached after 30 hours. The ice-water interface is not flat; it exhibits a wavy surface with amplitude of a few millimetres and a wave length of a few centimetres, probably reflecting the small scale eddies present in the convecting water. Another interesting observation is the consistent inclination of the interface, leading to different local ice-layer thicknesses, in some cases reaching 15% of the average ice thickness.

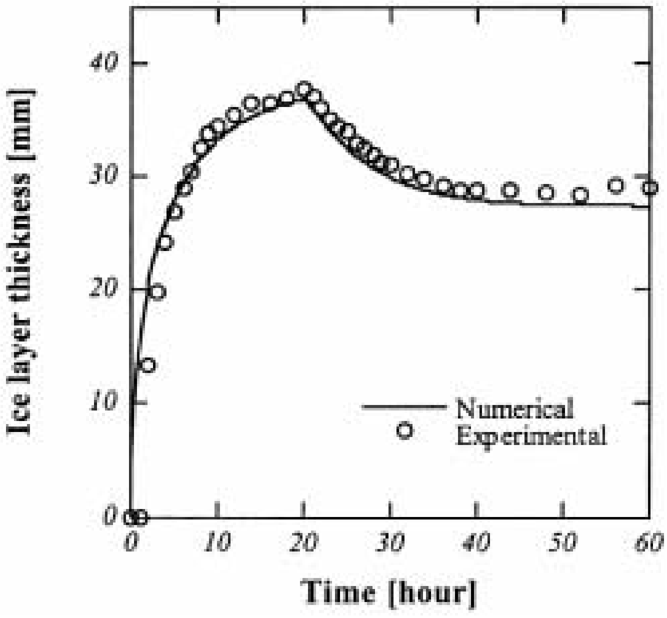


Figure 5. Ice layer thickness subject to a sudden change of cooling temperature. The cooling temperature rises from -7°C to -5°C.

Comparison between Numerical and Experimental Result

Numerical Results. Sample numerical results of the time-dependent ice-water interface movement are shown *in figure 3*.

When the cooling temperature oscillates periodically in a sinusoidal manner, the interface also oscillates sinusoidally with the same frequency. It becomes clear, however, that the interface response exhibits a significant phase delay relative to the temperature. One can also notice that for temperature oscillation amplitudes of about 10%, the interface oscillation amplitude remains within 1%.

Verification of the Numerical Code. Developed numerical code has been extensively tested against obtained experimental results.

Figure 4 shows the ice-layer growth with time, after the thermal equilibrium of 8°C has been obtained and the cooling temperature is quickly lowered to -7°C

Figure 5 shows a case when the cooling temperature is raised to -5°C after a 20-hour long cooling at -7°C. The three graphs in *Figure 6* show the interface oscillation when the cooling temperature is modulated with a rectangular wave with different periods and an average value of -5°C and amplitude of 2°C. It should be noted that the amplitude of interface boundary oscillation becomes smaller when the oscillation frequency increases. The numerical predictions in all cases agree well with the experimental results which concludes our code validation.

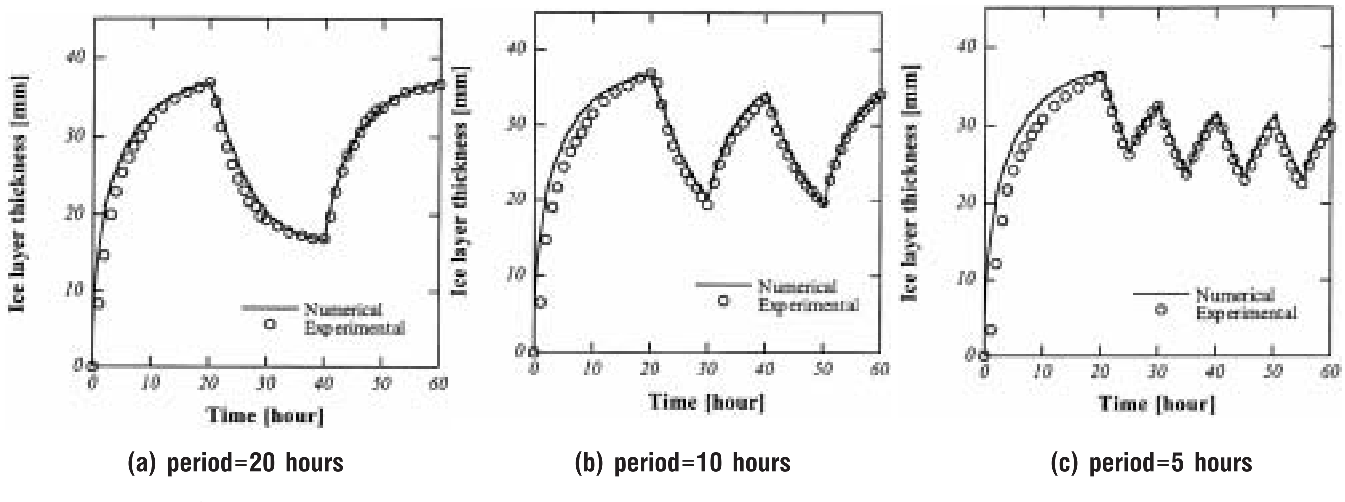
Heat and Mass Transfer and e-Learning

Heat and mass transfer plays an important role in many engineering disciplines. Because of the invisible nature of heat flows and temperature fields, it is usually difficult for engineering students to understand them without proper visualization support and experimentation. We often use, for example, experimental apparatuses such as the one shown in *figure 2* to introduce students to phase change processes under vigorous convection heat transfer. In such experiments the ice layer formation, however, is a relatively lengthy process (*figure 4*) which makes direct student observation of ice layer thickness rather impractical (*figure 5* and *figure 6*). On the other hand, the one dimensional numerical model that we have developed calculates the ice layer thickness quite rapidly. With it, illustrative educational content is easily generated and could be assembled into video sequences corresponding to different physical experimental environments and conditions.

Such illustrative content may be good for lectures and as supportive material in traditional education but when it comes to e-learning and direct numerical simulations, students need to be supplied with a suitable interface. Nowadays window and widget based Graphical User Interfaces (GUI) constitute the standard for Human Computer Interactions (HCI). They come, however, in many different styles and flavours that sometimes hamper the smooth user adaptation. In e-learning, in particular, this may pose problems, since students should be focusing on studying the course content rather than on learning interfaces coming with new packages and courses. We address this problem by adopting the specialized highly configurable Graphical Interface Framework for Educational Support (GIFES) that allows employment of unified interface structures across a multiplicity of courses and software packages.

Numerical simulations are applicable to a great diversity of disciplines, so corresponding models and numerical codes are naturally developed by different specialist using different, often incompatible languages and platforms. By all means, those specialists are best capable of developing a „black box" numerical code that takes available input values, applies a physical model, and outputs corresponding numerical results. But such numerical codes, although fully functional, are hard to use for educational purposes without an appropriate interface.

The developed code for numerical simulation of solid layer dynamic responses to cooling temperature modulations and convective fluid flow deviations has also been initially con-



(a) period=20 hours **(b) period=10 hours** **(c) period=5 hours**

Figure 6. Oscillation of the ice-layer thickness in response to different frequencies of cooling temperature modulation

(The cooling temperature is a rectangular wave with an average of -5°C and amplitude of 2°C)

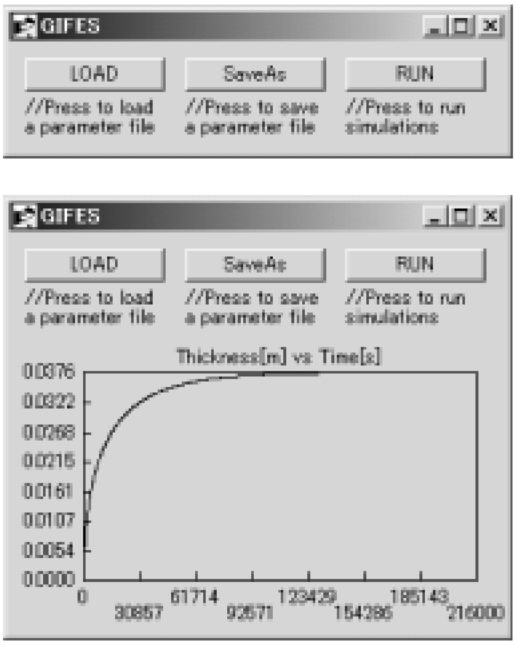


Figure 7. The generic GIFES (top) and its parameterized version with graphically represented simulation results (bottom)

structed as a „black-box" with purely numerical input and output. We have employed then GIFES to generate graphical user interfaces suitable for it as shown in *figure 7*. Interface content and structure are controlled through a dedicated parameter description file while interface appearance is automatically adapted to the visualization screen parameters and the window system of the host OS.

The GIFES enhanced numerical simulation tool is employed as a component of an e-learning Digital Laboratory where students are first requested to study various preset cases. Each case is defined by its parameter file that can be accessed through the LOAD function of GIFES (*figure 7*). Loading a new case is a two-fold process that incurs:

- redefinition of calculation parameters and conditions for consequent numerical simulations, and
- automatic GIFES GUI content and structure parameterization.

GIFES serves as a frontend to numerical codes that are executed in real time so that many different cases can be explored in the course of e-learning.

After mastering the preset cases students can advance to the second stage of e-learning at the Digital Laboratory. Here they formulate and attempt to solve practical problems by employing available numerical models and codes. GIFES accommodates numerical codes with various types of parameters that are presented to students for modification as shown in *figure 8*. Note that along with the single value parameters more complex ones such as functional representations are also supported. The last parameter of the interface shown in *figure 8*, for example, defines a time dependent cooling temperature.

Phase change is a complex phenomenon involving both conduction and convection, which is hard to predict, especially when the boundary conditions, such as the cooling temperature, are altered in time. Since it has, however, important application in energy technology, material processing, and other fields, both engineering students and practicing thermal design engineers need to deal with it in an efficient manner. The graphical approach that we employ can assist the quick comprehension of

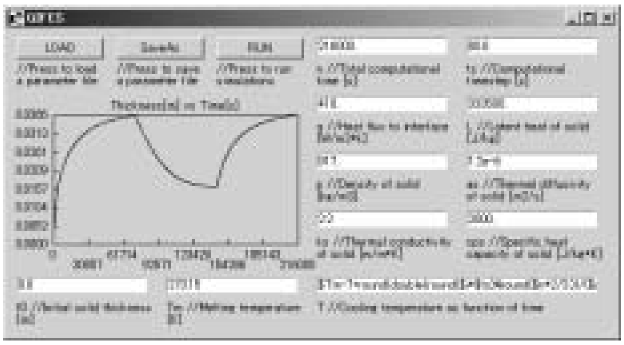


Figure 8. GIFES interface with editable parameter entry fields employed for solving practical problems

the basics of phase change processes for a wide range of parameters, such as cooling temperatures, thermal properties, and strengths of convection heat flux. We believe that present software has a great potential as an e-learning and design tool in the many areas where solidification takes place.

Developed numerical simulation code and corresponding visualization components are considered in the scope of a broader heat and mass transfer e-learning environment conceptually close to the design reported by Hung et al. [12]. Within its extent, solidification models for other materials, such as steel [13], etc., along with some previously designed and reported fluid flow e-learning components [14], could be considered for integration.

Conclusion

A numerical code for dynamic solidification processes, based on a one-dimensional model has been developed, and extensively tested with various experimental measurements. The good agreement between the numerical and the experimental results for a wide range of parameters proves that the code is capable of accurate solidification calculations. The present code can be used for simulations and visualization of dynamic solidification processes for engineering and educational purposes and especially in e-learning environments.

References

1. Flemings, M. C. Solidification Processing. McGraw-Hill, New York, 1974.
2. Macdonald, K. C. Mid-Ocean Ridges: Fine Scale Tectonic, Volcanic and Hydrothermal Processes within the Plate Boundary Zone. – *Ann. Rev. Earth Planet. Sci.*, 10, 1982, 155-190.
3. Huppert, H. E., R. S. J. Sparks. Double-Diffusive Convection due to Crystallization in Magmas. – *Ann. Rev. Earth Planet. Sci.*, 12, 1984, 11-37.
4. Hibler III, D., J. E.Walsh. On Modelling Seasonal and Interannual Fluctuations of Arctic Sea Ice. – *J. Physical Oceanography*, 12, 1982, 1514-1523.
5. Bransier, J. Stockage Periodique par Chaleur Latente Aspects Fondamentaux Lies a la Cinetique des Transferts. – *Int. J. Heat Mass Transfer*, 22, 1979, 875-883.
6. Kalhori, B., S. Ramadhyani. Studies on Heat Transfer from a Vertical Cylinder, with and without Fins, Embedded in a Solid Phase Change Medium. – *J. Heat Transfer*, 107, 1985, 44-51.
7. Jariwala, V.G., A. S. Mujumdar, M.E.Weber. The Periodic Steady State for Cyclic Energy Storage in Paraffin Wax. – *Can. J. Chem. Engng*, 65, 1987, 899-906.
8. Ho, C. J., C. H. Chu. Periodic Melting within a Square Enclosure with an Oscillatory Surface Temperature. – *Int. J. Heat Mass Transfer*, 36, 1993, 725-733.
9. Kimura, S., M. Vynnycky. Time History of Ice-layer Formation at the Cooled Top Boundary and its Dynamic Response to the Time-varying Cooling Temperature. Moving Boundaries V. Edited by Sarler, B., C. A. Brebbia, H. Power, WIT press, 1999, 47-56.
10. Bejan, A., A. D. Kraus (editors). Heat Transfer Handbook. Wiley, 2003.
11. Blake, K. R., D. Poulikakos, A.Bejan. Natural Convection Near 4°C in a Horizontal Water Layer Heated From Below. – *Phys. Fluids*, 27, 1984, 2608-2616.
12. Hung, T. C., S. K. Wang, S. W. Tai, C. T .Hung. An Innovative Improvement of Engineering Learning System Using Computational Fluid Dynamics Concept. – *Computers & Education*, 48, 2007, 44-58.
13. Babich, A., D. Senk, K. Mavrommatis. Combination of Lab Experiments and Online Simulations in Master Course. Proceedings of iCEER2005-iNEER Conference for Engineering Education and Research, Tainan, Taiwan, March 1-5, 2005.
14. Kanev, K., S. Kimura, M. Yoneya. e-Learning Fluid Flows: Tank Discharge through an Orifice. Inter Academia 2008, 15-18 September 2008, Pecs, Hungary, 144-151.

Manuscript received on 12.11.2010

Shigeo Kimura *received the M.Sc. and the Ph.D. degrees in Mechanical Engineering, both from University of Colorado, Boulder, Colorado, USA in 1980 and 1983, respectively. He is a Professor with Institute of Nature and Environmental Technology, and the School of Mechanical Engineering, Kanazawa University, Japan, where he teaches fluid mechanics, thermodynamics and heat transfer. His main research interests are in transport processes due to environmental fluid motions. On this and related topics he has authored and coauthored more than 100 scientific journal and conference papers and patents. Dr. Kimura is a member of the Japan Society of Mechanical Engineers (JSME) and the American Society of Mechanical Engineers (ASME).*

Contacts:
Institute of Nature and Environmental Technology, Kanazawa University
Kanazawa 920-1192
Japan
e-mail: skimura@t.kanazawa-u.ac.jp

Kamen Kanev *received the M.Sc. degree in mathematics and the Ph.D. degree in computer science from Sofia University in 1984 and 1989, respectively. He is a Professor with the Research Institute of Electronics, the Graduate School of Informatics, and the Graduate School of Science and Technology, Shizuoka University, Japan, where he teaches and supervises students majoring in computer and information science. His main research interests are in interactive computer graphics, user interfaces and surface based interactions, and in nanovision image information processing. On this and related topics he has authored and coauthored more than 100 scientific journal and conference papers and patents. Dr. Kanev is a member of the IEEE, the Association of Computing Machinery (ACM), and the Asia-Pacific Society for Computers in Education (APSCE).*

Contacts:
Research Institute of Electronics, Shizuoka University
Hamamatsu 432-8011
Japan
e-mail: kanev@rie.shizuoka.ac.jp