Adaptive Intelligence - Essential Aspects

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Key Words: *Adaptive Intelligence; Free Search; Differential Evolution; Swarm Intelligence; heuristics; No Free Lunch Theorem.*

Abstract:. *The article discusses essential aspects of Adaptive Intelligence. Experimental results on optimisation of global test functions by Free Search, Differential Evolution, and Particle Swarm Optimisation clarify how these methods can adapt to multi-modal landscape and space dominated by sub-optimal regions, without supervisors' control. In addition Free Search separately is evaluated on hard constraint global optimisation problem with unknown solution. It illustrates Free Search abilities for adaptation to unknown space. The achieved results are compared and analysed.*

Introduction

This article integrates and extends earlier conference publications. The comparison between different evolutionary methods and the attempts to assess their potential for adaptation and to cope with variety of problems are widely discussed in the literature [2,7,14,15,16,23]. A previous study [17] compares Free Search (FS) [18] Particle Swarm Optimisation (PSO) [8], and Differential Evolution (DE) [22] on several heterogeneous numerical problems. This article presents another investigation, which can be used for assessment of adaptation of these methods. The first part of the article presents specific properties, which enhance adaptation of intelligent systems. Free Search, relatively novel method, models behaviour of animals in nature, where they day by day explore surrounding environment in order to achieve their targets. FS model negotiates continuous space with discrete steps.

FS modifies all current solutions, which is similar to Evolutionary Programming (EP) [9], PSO and DE. The peculiarities, which differentiate FS from genetic [11,13], ant [6] and swarm algorithms, from Evolutionary Strategies (ES) [20,21], EP, DE, and from other methods, are black box search, independence from initial population, ability to diverge from one location across entire search space and capability to guide purposefully divergence and convergence during the process of global search avoiding stagnation in local sub-optima.

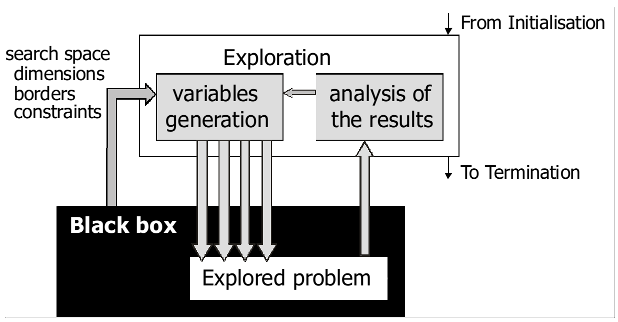

Figure 1. Black box optimisation

Black Box Concept

Free Search fully implements the concept for Black box exploration. From FS point of view any task is a black box. The algorithm does not require prior knowledge about the explored problems.

Figure 1 presents graphically the concept for Black box exploration implemented in Free Search.

During the event exploration the algorithm generates values of the variables and presents these values to the problem 'box'. The problem 'box' reacts and returns a value of the objective function, which corresponds to the variables values. By use of the value of the objective function, only, FS generates new variables and continues the optimisation process. The search space borders and existing constraints can be taken into account for generation of the new variables, as well.

Implementation of black box concept allows Free Search to adapt to any problem. It avoids preliminary settings of the optimisation parameters according to the particular optimisation problem.

Independence from Initial Population

Another advanced property, implemented in Free Search, is the idea for independence of optimisation process from initial populations. Free Search can operate on any initial population. This is a conceptual improvement in comparison to other real-value methods for optimisation of non-discrete problems. Analysis of Genetic Algorithm, Particle Swarm Optimisation and Differential Evolution suggests that these methods cannot operate when optimisation starts from one location. GA starts effective work after the first mutation and DE and PSO cannot start at all [18].

Start from Stochastic Initial Locations

Free Search can start from stochastically selected set of initial solutions where all the initial locations x_{0ji} are random values (*figure 2*):

$$x_{0ji} = Xmin_i + (Xmax_i - Xmin_i)*random_{ji}(0,1)$$

where $Xmin_i$ and $Xmax_i$ are the search space borders, $i = 1,..,n$, n is the number of dimensions, $j = 1,..,m$, m is the population size. $random(0,1)$ is a random value between 0 and 1. A start from random locations guarantees non-zero probability for access to any location from the search space. It guarantees, also, probabilistic transaction rules for initialisation.[11]

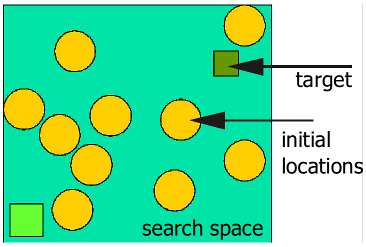

Figure 2. Stochastic initial population

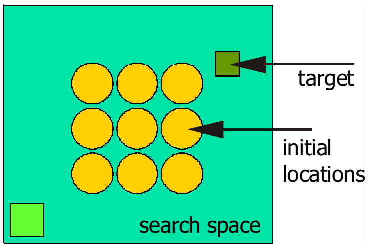

Figure 3. Certain initial population

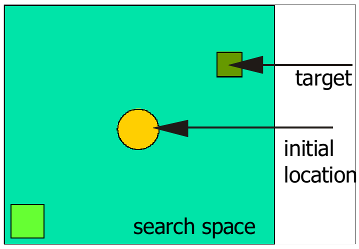

Figure 4. Initial population in one location

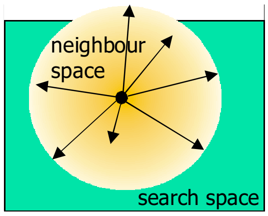

Figure 5. Exploration step generation

Start from Certain Initial Locations

Free Search can start from certain initial population where all the initial locations x_{0ji} are prior-defined values a_{ji} (*figure 3*):

$x_{0ji} = a_{ji}$, $a_{ji} \in [Xmin_i, Xmax_i]$

where $Xmin_i$ and $Xmax_i$ are the search space borders, $i = 1,..,n$, n is the number of dimensions, $j = 1,..,m$, m is the population size and a_{ji} are constants, which belong to the search space.

A start from certain locations is a valuable ability for multi-start optimisation. It is useful from a practical point of view, as well. A start from certain locations can be used when some values are already achieved and the algorithm can continue from these values instead of repeating starts from random locations.

Start from One Location

Free Search can start from an initial population where all the initial solutions x_{0ji} are in one location c_i:

$x_{0ji} = c_i$, $c_i \in [Xmin_i, Xmax_i]$

where $Xmin_i$ and $Xmax_i$ are the search space borders, $i = 1,..,n$, n is the number of dimensions, $j = 1,..,m$, m is the population size, c_i is constant. A start for one location is a difference from Genetic Algorithm, Differential Evolution and Particle Swarm Optimisation. It is similar to Ant Colony Optimisation modified for non-discrete search space [4].

The ability to start from one location supports escaping from trapping in local sub-optima.

The abilities to start from one location, from certain or from random initial locations support the independence of Free Search from the initial population. It illustrates that Free Search does not determinate or discriminate the search space at the initial stage of the search process. This is an example of successful harmonisation of different concepts from genetic, swarm and ant algorithms, which benefits exploration abilities and contributes to the better performance of the search process.

Free Movement within the Search Space

The individuals in Free Search have the ability for movement within the search space. They take exploration walks. During the walk they make exploration steps around the current start location. Generation of an exploration step in two-dimensional search space is illustrated in *figure 5*.

The exploration walk generates a step x_{tji}:

$$x_{tji} = x_{0ji} - \Delta x_{tji} + 2*\Delta x_{tji}*random_{tji}(0,1)$$

where x_{0ji} is the initial or previous location marked with pheromone. $random_{tji}(0,1)$ is a random value between 0 and 1. t is the current step $t = 1,..,T$, T is the step limit.

Δx_{tji} is the step. Modification strategy for step generation is:

$$\Delta x_{tji} = R_{ji} * (Xmax_i - Xmin_i) * random_{tji}(0,1)$$

where R_{ji} is a variable value of the neighbour space radius $R_{ji} = [Rmin, Rmax]$. $Xmin_i$ and $Xmax_i$ are the search space borders. $random_{tji}(0,1)$ is a random value between 0 and 1. The search space borders restrict the probability for access to any location within the search space, only. Variation of R_{ji} higher than one exceeds the search space borders and guarantees non-zero probability for access to any location within the search space. It guarantees, also, a probabilistic transaction rule for exploration of the whole space.

For a uni-dimensional step $i = l$, for a multi-dimensional step $i = 1,..,n$, n is the number of dimensions. The animals' number is j, $j = 1,..,m$, m is the population size.

This modification strategy avoids contradictions of modification strategies in GA, PSO and DE: (1) good convergence but trapping and (2) good divergence but inability to locate the optimum within an acceptable period. It is independent from a current or the best achievements. The strategy allows nonzero probability for access to any location of the search space and highly encourages escaping from trapping in local maxima. It can maintain a balance between convergence and divergence within the search process and supports adaptation to different problems without any settings of the optimisation parameters for control or regulation of the divergence, convergence or convergence speed. Experimental results fully confirm these capabiliies. [18]

Free Search Model for Adaptation

A specific original peculiarity of Free Search, which has no analogue in other Population-based Algorithms [9,8,19], is variable called sense. It can be described as a quantitative indicator of sensibility to particular search objectives. The sensibility varies between minimal and maximal values, which correspond to minimal and maximal values of the variable sense. The algorithm tunes sensibility during the process of search as function of explored problem. The same algorithm performs different regulations during exploration of different problems. This is considered to be a model of adaptation [18].

Presence of variable sense distinguishes individuals from solutions. Which is not the case in published by Congress on Evolutionary Computation FSDE [34].

Animals (or individuals) in Free Search are understood as search agents differentiated from the explored solutions and detached from the problems' search space. Solutions in FS are locations from continuous space marked with pheromone.

An individual in FS can be described as an entity, which can move and can evaluate (against particular criteria) locations from given search space and then it can indicate their quality.

The indicators can be interpreted, also, as a record from previous activities. A variable, which indicates locations' quality, is called pheromone by analogy with the indicators used by animals in nature. An individual can identify pheromone marks from previous activities and can use them to decide where to move.

The variable sense when considered in conjunction with the pheromone marks can be interpreted as personal knowledge, which the individual uses to decide where to move. From other point of view variable sense when related with variable pheromone plays a role of a tool for regulation of divergence and convergence within the search process and a tool for guiding the exploration [18].

Individual relation between sensibility and pheromone distributions affects decision-making policy of the whole population.

A short discussion on three idealised general states of sensibility distribution can clarify FS self-regulation and how chaotic on first view accidental events can lead to purposeful behaviour. These are – uniform, enhanced and reduced sensibility.

In *figure 6* (and in following *figures 7 and 8*) left side represents distribution of the variable sense within sensibility frame and across the animals from the population. Right side represents distribution of the pheromone marks within the pheromone frame and across the locations marked from previous generation.

Figure 6. Uniform sensibility

In case of uniformly distributed sensibility and pheromone (*figure 6*), individuals with low level of sensibility can select for start position any location marked with pheromone.

Individuals with high sensibility can select for start position locations marked with high level of pheromone and will ignore locations marked with low level of pheromone. It is assumed that during a stochastic process within a stochastic environment any deviation could lead to non-uniform changes of the process.

The achieved results play role of deviator.

An enhancement of sensibility encourages animals to search around the area of the best-found solutions during previous exploration and marked with highest amount of pheromone. This situation appears naturally (within FS process) when pheromone marks are very different and stochastic generation of sensibility produces high values.

External adding of a constant or a variable to the sensibility of each animal could make forced sensibility enhancement. Individuals with enhanced sensibility will select and can differentiate more precisely locations marked with high level of pheromone and will ignore those indicated with lower level (*figure 7*).

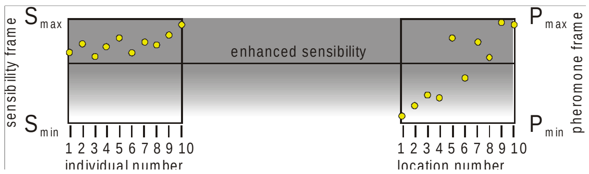

Figure 7. Enhanced sensibility

By sensibility reduction an individual can be allowed to explore around locations marked with low level of pheromone. This situation naturally appears (within FS process) when the pheromone marks are very similar and randomly generated sensibility is low. In this case the individuals can select locations marked with low level of pheromone with high probability, which indirectly will decrease the probability for selection of locations marked with high level of pheromone. Subtracting of a constant or a variable from sensibility of each animal could make a forced reduction of sensibility frame (*figure 8*).

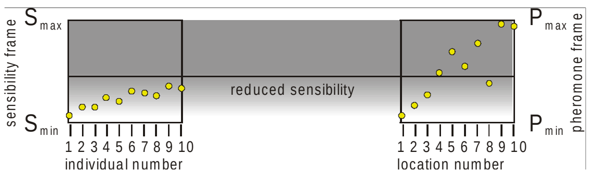

Figure 8. Reduced sensibility

Sensibility across all individuals varies. Different individuals can have different sensibility. It also varies during the optimisation process, and one individual can have different sensibility for different explorations.

Adaptive self-regulation of sense, action, and pheromone marks is organised as follows. An achievement of better solutions increases the knowledge of the population for best possible solution. This knowledge clarifies pheromone and sensibility frames, which can be interpreted as an abstract approach for learning and sensibility can be described as high-level abstract knowledge about the explored space. This knowledge is acquired from the achieved and assessed result only.

The individuals do not memorise any data or low-level information, which consumes computational resources. Sensibility can be interpreted as implicit knowledge about the quality of the search space and in the same time creates abilities to recognise further, higher or lower quality locations. Better achievements and higher level of distributed pheromone refines and enhances sensibility. A higher sensibility does not restrict or does not limit abilities for movement. It implicitly regulates the action of the individuals in terms of selection of a start location for exploration [18]. A learning new knowledge does not change individual abilities for movement. Animals continue to do small or large steps according to the modification strategy (By analogy with nature elephants will not do small ants size steps and vice versa). However, enhanced sensibility changes their behaviour. They give less attention to locations, which bring low quality results. They can be attracted with high probability to locations with better quality.

Test Problems

For all experiments the aim is to find the maximum therefore the test functions are transformed for in relevant manner.

Ackley test function [1] – *figure 9*

$$f(x) = a\exp\left[-b\left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right)\right]^{1/2} + \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(cx_i)\right) - a - \exp(1)$$

where a=20, b=0.2, c=2π. The maximum is $f(0)=0$. The search space borders are defined by $-32 < x_i < 32$.

Rastrigin test function [22] – *figure 10*

$$f(x) = nA + \sum_{i=1}^{n}(x_i^2 - A\cos(2\pi x_i))$$

where A=10 and $-5.12<x_i<5.12$. The maximum is $f(0) = 0$.

Norwegian test function [5,10] – *figure 11*

$$\prod_{i=1}^{n}\left(\cos(\pi x_i^3)\left(\frac{99+x_i}{100}\right)\right)$$

where search space borders are defined by $-1.1< x_i <1.1$. The maximum is $f(1.0)=1.0$.

Himmelblau test function [12] – *figure 12*

$$f(x, y0 = 200 - (x^2 + y - 11)^2 - (x + y^2 - 7)^2$$

It has four maxima equal height (200) at (3.584, -1.848), (3.0, 2.0), (2.805, 3.1313) and (-3.779, -3.283). The search space is restricted to $-10 < x, y < 10$.

Experimental Results

FS, PSO and DE are applied to the above-mentioned functions as follows – Each algorithm is evaluated four times per test function – (1) start from stochastic initial population with limit 100 iterations, (2) start from stochastic initial population with limit 2000 iterations, (3) start from one initial location with limit 100 iterations, (4) start from one initial location with limit 2000 iterations. The single initial location is defined as:

$x_0 = x_{min} + 0.9(x_{max} - x_{min})$.

Each evaluation is 320 experiments. Population size is 10 (ten) individuals for all algorithms for all experiments. For DE differential factor F varies from 0.5 to 1.5. For PSO iner tia W varies from 0.5 to 1.5. For FS neighbour space R varies from 0.5 to 1.5. As successful are accepted results: for Ackley test function higher than -0.1; for Himmelblau test function higher than 199.9; for Rastrigin test function higher than -0.1; for Norwegian test function higher than 0.99. The number of the successful results from all experiments is presented in *table 1*.

Table 1. Experimental results

	F1	F2	F3	F4	Overall
FS R*-100	281	187	34	320	822
FS R-2000	313	267	253	320	1153
FS OL*-100	201	158	26	288	673
FS OL-2000	232	257	262	314	1065
DE R-100	283	202	4	294	783
DE R-2000	299	225	6	315	845
DE OL-100	0	0	0	0	0
DE OL-2000	0	0	0	0	0
PSO R-100	175	153	10	268	606
PSO R-2000	191	235	15	313	754
PSO OL-100	0	0	0	0	0
PSO OL-2000	0	0	0	0	0

F1-Ackley, F2- Rastrigin, F3- Norwegian, F4- Himmelblau;

* R indicates stochastic initial population; OL indicates start from one location. All the algorithms have found all four maxima of the Himmelblau test function for 100 iterations.

In *figures 13, 14, 15, and 16* are presented the achieved results per each test function.

Figure 18 illustrates the extent in which Free Search overcomes the dependence on the initial population. According to the achieved results the condition for a start from one location can be considered as hard. However an ability to find the optimum starting from one arbitrary location, which does not favourites any area of the search space, indicates how reliable could be a search algorithm on unknown problems. For 100 iterations from 1280 experiment on all tests for stochastic initial population FS produces 822 successful results (66%) versus 673 successful results (52%) for a start from one location.

This experiments and earlier published initial evaluation [17] and then extended evaluation [18] on classical test suggests good stable performance across heterogeneous problems. However classical tests usually have well known optimal solutions.

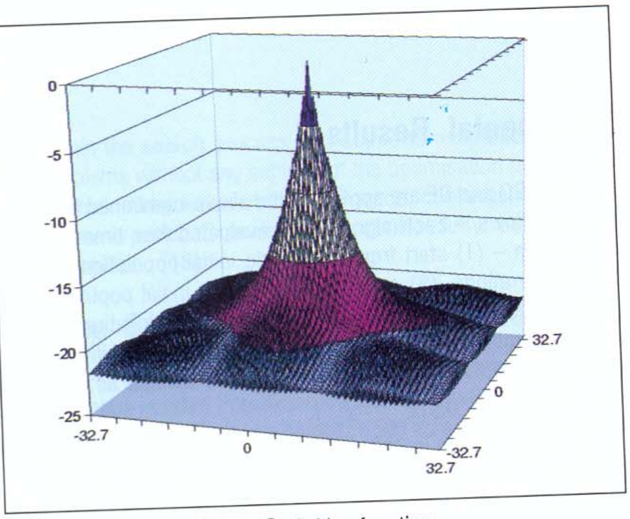

Figure 9. Ackley function

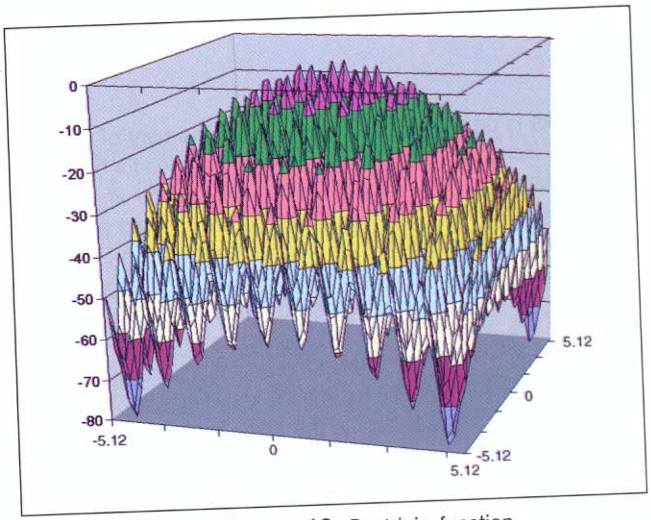

Figure 10. Rastrigin function

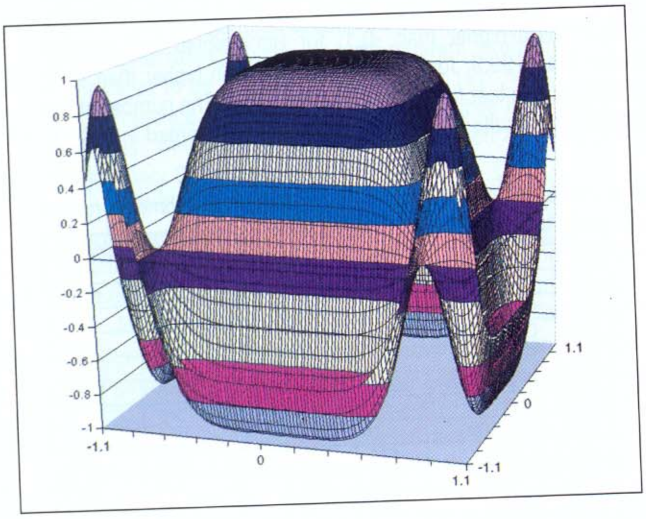

Figure 11. Norwegian function

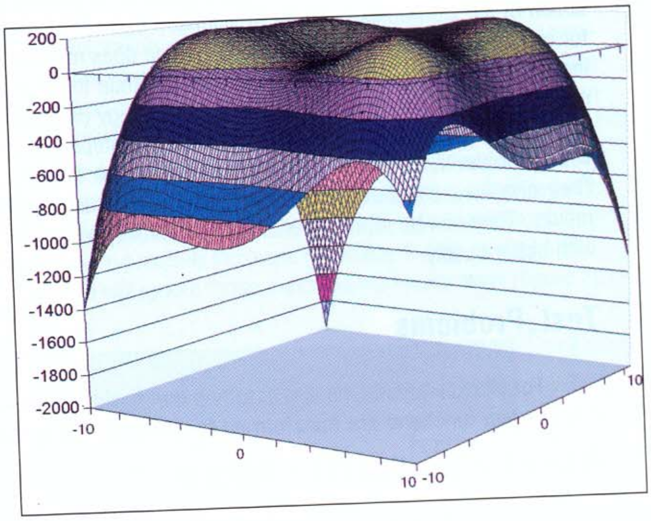

Figure 12. Himmelblau function

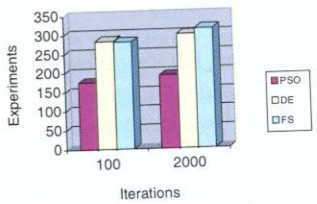

Figure 13. Ackley results

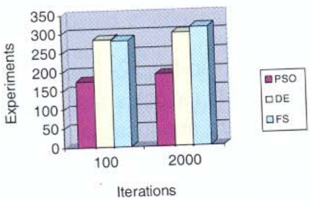

Figure 14. Rastrigin results

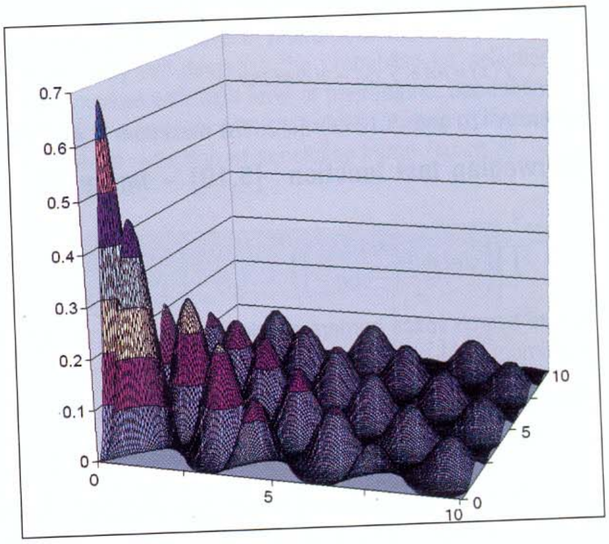

Figure 19. Keane test function two dimensional view

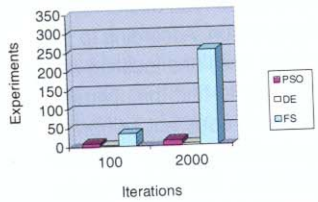

Figure 15. Norwegian results

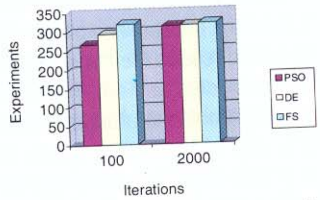

Figure 16. Himmelblau results

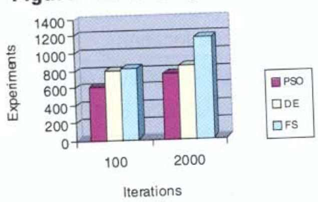

Figure 17. Overall results

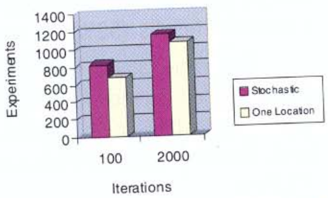

Figure 18. Dependence on initialisation

Adaptation to Unknown Space

In order to avoid any doubts whether Free Search can perform adaptation to unknown space and whether it requires prior knowledge a problem with unknown optimum was necessary. Therefore a hard, non-discrete, non-linear and constrained test problem published in the literature [24,26] is used.

The originally proposed in [24] test problem is:

Maximise:

$$\mathrm{f}(x_i) = \left| \sum_{i=1}^{n} \cos^4(x_i) - 2\prod_{i=1}^{n} \cos^2(x_i) \right| / \sqrt{\sum_{i=1}^{n} i x_i^2}$$

subject to: $\prod_{i=1}^{n} x_i > 0.75$ (1)

and $\sum_{i=1}^{n} x_i < 15n/2$ (2)

for: $0 < x_i < 10$, and $i=1,...,n$, start from $x_i = 5$, $i=1, ...,n$, where x_i are the variables (expressed in radians) and n is the number of dimensions [25].

It is generalised for multidimensional continuous search space. It has many peaks. It is very hard for most optimisation methods to cope with because its optimal value is defined by presence of constraint boundary formulated as a variables product (inequality 1) and because of initial condition – *start from* $x_i = 5$. Start from the middle of the search space excludes from initial population locations, which could be accidentally near to the unknown optimal value.

Maximal values *Fmax* and constraint boundaries defined as variables product (shortly noted as p) $p>0.75$ are located on a steep slope so that very small changes of variables lead to substantial change of the objective function. In the same time product constraint (1) defines very uncertain boundary. Various variables combinations satisfy the constraint (1) but produce different function values, which makes this test even harder. A given combination of variables produces highest function value when variables are in descending order.

The optimum is unknown. The best-achieved solutions are published in the literature. This test is a complex criterion for the ability of the optimisation method to cope with unknown, multi-modal, constrained task, similar to real-world optimisation problems.

Comparative Analysis

For earlier experiments reported in the literature the optimisation problem is implemented for 20 and 50 dimensions. Achieved by Genetic Algorithm results are 0.76 for both $n = 20$ limited to 20000 iterations and $n = 50$ limited to 50000 iterations; and 0.65 for 5000 and 12000 iterations respectively [24].

Other earlier publication, which introduces specific operators for search near boundaries and transforms original condition „start from centre“ to „start from random locations“ reports: - 0.80 for $n = 20$ limited to 4000 with population size 30. The best found is 0.803553. For $n = 50$ limited to 30000 iterations the best found value is 0.8331937 [31].

Perhaps the most comprehensive exploration of this test problem demonstrates how by use of Asynchronous Parallel Evolutionary Modelling Algorithm on distributed MIMD computational system can be discovered required knowledge for various numbers of dimensions from 2 to 50 and publishes the best-achieved results respectively [29]. Although no sufficient information about computational cost, population size and iterations number, these results can be used for comparison with other methods therefore they are entirely cited in *table 2*. In addition by using Runge-Kutta method a best value for 1000000 dimensions is predicted to be 0.849730 [29].

Table 2. Best values achieved by Asynchronous Parallel Evolutionary Algorithm – citation from [8]

n	f_{max}	n	f_{max}	n	f_{max}
2	0.36497975	19	0.79800887	36	0.82783593
3	0.51578550	20	0.80361910	37	0.82915387
4	0.62228103	21	0.80464587	38	0.82896840
5	0.63444869	22	0.80833226	39	0.83047389
6	0.69386488	23	0.81003656	40	0.82983459
7	0.70495107	24	0.81182640	41	0.83148885
8	0.72762616	25	0.81399253	42	0.83226201
9	0.74126604	26	0.81446495	43	0.83226624
10	0.7473103	27	0.81694692	44	0.83323002
11	0.7610556	28	0.81648731	45	0.83285734
12	0.76256413	29	0.81918437	46	0.83397823
13	0.77333853	30	0.82188436	47	0.83443462
14	0.77726156	31	0.82210164	48	0.83455114
15	0.78244496	32	0.82442369	49	0.8318462
16	0.78787044	33	0.82390233	50	0.83523753
17	0.79150564	34	0.82635733		
18	0.79717388	35	0.82743885		

All cited results above provide maximal values without corresponding variables and cannot be verified.

For comparison earlier achieved by FS results are:

$n = 20$, $fmax_{20} = 0.80361832569$ and

$n = 50$, $fmax_{50} = 0.83526232992710514$.

A confidence in FS ability to reach better results encourages publication of the corresponding variables. They can be found in [28].

Widely recommended by the CEC 2006 set of tests for real parameter optimisation transforms this test for minimisation and states:

„The global minimum x* = (3:16246061572185; 3:12833142812967; 3:09479212988791; 3:06145059523469; 3:02792915885555; 2:99382606701730; 2:95866871765285; 2:92184227312450; 0:49482511456933; 0:48835711005490; 0:48231642711865; 0:47664475092742; 0:47129550835493; 0:46623099264167; 0:46142004984199; 0:45683664767217; 0:45245876903267; 0:44826762241853; 0:44424700958760; 0:44038285956317), the best we found is $f(x^*) = -0.80361910412559$ (which, to the best of our knowledge, is better than any reported value)“ [33].

However, verification for maximisation of the cited above vector produces:

$fmax_{20} = 0.80361910412558735$. Better results are published in the literature [27,18] and presented below.

By modified Differential Evolution has been achieved the following results:

$n = 20$, $fmax_{20} = 0.80361910412558857$ and
$n = 50$, $fmax_{50} = 0.83526234835804869$, corresponding variables are published in [27]. They are followed by conclusion: „I verified these solutions by a gradient method (MATLAB). Starting from these points the local searcher could not find a better solution. In fact, 64bit encoding provides numerical validity for tolerance more than or equal to 1.0e-16. So, my solutions can be considered as the global optima.“ [27 page 142]

These results give Free Search a chance to confirm its ability for start from a single location and achieved results are:

$n = 20$, $fmax_{20} = 0.80361910412558879$
constraint is $p_{20} = 0.75000000000000076 > 0.75$
$n = 50$, $fmax_{50} = 0.83526234835811175$
constraint is $p_{50} = 0.75000000000000144 > 0.75$.

Together with corresponding variables they are published in [18] and can be verified. These results exceed published best solutions achieved by improved version of DE and require reconsiderations of the assumptions and conclusions about the global maxima of this function, cited above.

Rigorous investigation on various dimensions from $n = 2$ to $n = 50$ confirms the results from $n = 2$ to $n = 27$ in *table 2*. However from $n = 28$ to $n = 50$ Free Search reaches different results.

For example after 320 runs with population size 10 individuals and start from $x_i = 5$, limited to 50000 iterations FS achieves for $n = 28$ $f_{28} = 0.817228$, constraint is $p_{28} = 0.75010348812072747$. In *table 3* corresponding variables are presented for verification.

On single core single thread Intel processor at 4.3 GHz this experiment takes about 20 minutes. Optimisation process in FS in organised in explorations with 5 steps per individual, so that 50000 iterations correspond to 10000 explorations. Therefore evaluations and assessments of these locations are much less than in conventional iterations [18].

Detailed analysis of computational cost could be a subject of further publications. In order to refine initial results to better precision, additional multi-start experiments from best-achieved locations have been made. Achieved results different from *table 2* are:

$n = 28$, $fmax_{28} = 0.81852140091357606$
constraint is $p_{28} = 0.750003706880362 > 0.75$
$n = 29$, $fmax_{29} = 0.81973931385618481$
constraint is $p_{29} = 0.75000009598193296 > 0.75$
$n = 33$, $fmax_{33} = 0.82519820563520652$
constraint is $p_{33} = 0.75001891510962826 > 0.75$
$n = 40$, $fmax_{40} = 0.83101104092886091$
constraint is $p_{40} = 0.75000002707051705 > 0.75$
$n = 43$, $fmax_{43} = 0.83226645247409203$
constraint is $p_{43} = 0.75000000097094244 > 0.75$
$n = 48$, $fmax_{48} = 0.83455172285394108$
constraint is $p_{48} = 0.75000000020088442 > 0.75$
$n = 49$, $fmax_{49} = 0.83485653417902916$
constraint is $p_{49} = 0.75000009159705938 > 0.75$

For $n = 50$ FS also has different solution shown above. Let us clarify that these are currently best solutions but they are not the best possible solutions.

As far as solution for $n = 49$ highly differs from *table 2* corresponding variables are presented in *table 4* and can be verified.

In descending order these variables produce:

$fmax_{49} = 0.8351028653158$ and after another run FS adds more precision: $fmax_{49} = 0.8351568574438$.

These results suggest that Free Search outperforms Asynchronous Parallel Evolutionary Algorithm [29]. In order to examine computational abilities of modern hardware tests above 100 dimensions can be used [18].

$F_{max100} = 0.84568545609618284$ is the best achieved by FS result for 100 dimensions, constraint is $p_{100} = 0.75000000000001477$.

A result achieved by FS for 200 dimensions $F_{max200} = 0.850136$ and $p_{200} = 0.750000585348$ requires reconsideration of the assumptions and predictions about maximum for 1 000 000 dimensions cited above [29]. Very likely these current results will be improved, even so, will be a challenge to see variables, which generate better solutions.

An earlier investigation for convergence speed measurement made two series of 640 experiments each for $n = 50$. [18] The termination criterion is complex, satisfaction of an optimal value or expiration of a certain number of iterations.

For the first series the criterion is reaching optimal value $f_{crtr} = 0.8$ or expiration of 50000 iterations. The minimal number of iterations g_{min} and the average number of iterations g_{av}, used to reach $f_{crtr} = 0.8$ are measured.

The average value is calculated from the sum of iterations used to reach optimisation criterion, divided by number of experiments. In case of expiration of iteration limit, the limit value is used for calculations. For the second series the criterion is satisfaction of a higher optimal value $f_{crtr} = 0.83$ or expiration of 50000 iterations. The minimal number of iterations g_{min} used

Table 3. Variables for $Fmax_{28}$ = 0.817228

x0	6.2531702561222771	x14	0.44681770745280147
x1	3.1395951938332418	x15	0.44790346171332279
x2	3.1591029163830173	x16	0.46589591188197471
x3	3.0951442686509703	x17	0.44907968614227989
x4	3.0894286765848036	x18	0.45954159038857201
x5	3.021224964495548	x19	0.43544094760156993
x6	3.0278444599429202	x20	0.48880559202627838
x7	3.0411909726874309	x21	0.43400319816736921
x8	2.9278202125513797	x22	0.45329262605710513
x9	2.9346673711895144	x23	0.44583286216813689
x10	2.9258715958074002	x24	0.44240282562524524
x11	0.51047982594421493	x25	0.43046375194518827
x12	0.56576531902196214	x26	0.43159621964160683
x13	0.48211851961377578	x27	0.43988961518963099

Table 4. Variables for $Fmax_{49}$ = 0.83485653417902916

x0	6.2792861323024578	x25	0.48148669422147178
x1	3.1637278169038185	x26	0.47360423616335467
x2	3.1499159855039895	x27	0.46935293479313345
x3	3.1407828766805119	x28	0.47088228065895599
x4	3.1224592557204516	x29	0.45860019925104595
x5	3.1099831636225046	x30	0.44735588040258434
x6	3.0946396293094467	x31	0.44860532142340692
x7	3.083081445582879	x32	0.44593857815209992
x8	3.0677291965449376	x33	0.43907381895990599
x9	3.0548401056665453	x34	0.45232777376532651
x10	3.040612103252581	x35	0.41731946646250828
x11	3.0273934320125049	x36	0.46877508274045099
x12	3.012842782491592	x37	0.43389198331683859
x13	2.9996619453917561	x38	0.44355729597864224
x14	2.9874015596539265	x39	0.42430322063529485
x15	2.9722089890878403	x40	0.44422719158626583
x16	2.9584503859956279	x41	0.4153021935173587
x17	2.9417205474505268	x42	0.42390449585240647
x18	2.9275505838361253	x43	0.44529679982447551
x19	2.9126901025010241	x44	0.43937682282922974
x20	0.4618330010351604	x45	0.41611766386400928
x21	0.44551463869304436	x46	0.45247621216839901
x22	0.46542657823328093	x47	0.43385424668902811
x23	0.48029088059211045	x48	0.43205299747850506
x24	0.49187419690441819		

Table 5. Convergence speed 50 dimensions start from $x_i = 5$, $i = 1,..,n$ – citation from [15]

f_{crtr}	f_{max}	constraint (1)	g_{min}	g_{av}
0.8	0.8001062254	0.8027977569	5335	36650
0.83	0.8300132503	0.7503894274	44610	-

to reach f_{crtr} = 0.83 is measured. For these experiments the population is 10 individuals, starting from $x_i = 5$, $i = 1, ...n$, neighbouring space varies from 0.1 to 2.1 with step 0.1. [15]

How does the new concept reflect on performance? Free Search outperforms other methods discussed in the literature [27,29,24,25,26,30,31,33].

How does the new concept affect the search process and individuals within the population? The animals own sensory perception creates an element of individualism. This individualism supports personal and then social creativity within the population. One individual in FS can escape easily, from unessential stereotypes, expired acceptances and hypotheses, than the whole population in other methods [18].

In particular, the experiments with a start from best local solution, illustrate how animals can escape easily from trapping in local optima. The results suggest that modelling of individual adaptive intelligence contributes to a better search performance. The individuals in FS can adapt their own behaviour flexibly during the optimisation process for global exploration, local search or search near to the constraints edge. Presented results demonstrate, also, that individual abilities for adaptation enhance effectiveness.

The success of this approach is based on the appropriate regulation of probability for access to any location within the search space. It is based on the assumptions: Uncertainty another uncertainty can cope with. Infinity another infinity can cope with. Difference between discrete finite space and continuous space can be classified as qualitative rather than quantitative. Therefore continuous infinite interpretation of the search space allows Free Search to operate with arbitrary precision (in other words with arbitrary granularity). This is accepted as compulsory condition for successful adaptation. Any limits including current knowledge (for example about best solution) if restrict behaviour and decision making then could constraint adaptation.

Discussion

The results from the experiments with a start from stochastic initial population demonstrate that all the algorithms can adapt to the explored tests without external adjustments for any concrete test. FS outperforms DE and PSO on Norwegian test problem, which confirms that DE and PSO have some difficulties in search near to the search space borders, published earlier [18]. For the experiments with start from one location DE and PSO have some difficulties to reach the optimum in accepted limited number of iterations. It is a consequence from the requirements for non-equal individuals for adaptive settings of the optimisation parameters from DE and PSO. Prior adjustment of the optimisation parameters for each particular problem does not seem to be an acceptable approach for adaptive algorithms. An improvement of the adaptivity of these algorithms can be a subject of further research.

One question which deserves attention is: What is the benefit from differentiation of the search agent from the solutions in FS? The benefit is: The agent independently can explore (can abstract cognition), learn (can indicate with pheromone) and use the learned cognition (can decide what to do and can do it). Why this is considered to be a benefit? For successful exploration of unknown problems FS not need prior settings and adjustments of the search parameters to the explored problem. The decision-making policy in FS is explicitly implemented in an idealised space by a relation between the frames of sensibility and pheromone independently from the problem [18]. Then during the process of search any particular problem is normalised to the frame of pheromone. So that FS can adjust the problem to itself or FS can adjust itself to the problem. It means FS has a potential to perform well across heterogeneous problems. Whether this is in contradiction to the statement „no algorithm can perform the best across all possible problems" [23] [14]? FS is not in contradiction with this statement and those theorems. FS is not preliminary set to any task and good overall performance requires time. This time is a slight delay, which FS needs for an adjustment to the explored problem. It is confirmed form published experimental results that some algorithms on some problems, for some appropriate initial populations can perform faster [18]. The approach, which FS uses, suggests that: The algorithm, which performs the best across all possible problems, will be the algorithm, which takes less time to adjust itself to the explored problem. This conclusion is inline with the „No Free Lunch Theorems for Optimisation", which clearly declare that „measures of performance based on factors other than (e.g., wall clock time) are outside the scope of our results" [23, page 5], and leaves behind the assumptions on which they are proven. The inductive proof assumes for example: „The new y value, $(m+1)$, will depend on the new x value, f and nothing else. So we expand over these possible x values" [23 page 23] .

The measurement of the performance, which respects „wall clock time", besides the dependence of the new y value on the x value and f, establishes dependence of the new y value on time $(mT+1)$. Where T is a finite period of time and mT is a time dependent number of iterations.

A proof or a disproof of this conclusion can be a subject of further research. Presented experiments are made with respect and according to the defined time constraints, namely the criterion for termination is expiration of an iterations limit.

Conclusion

The article presents Free Search properties, which enhance adaptivity. Investigation of the global landscape of multimodal test functions illustrates achieved level of intelligence and self-organisation. Explored algorithms demonstrate good capabilities for adaptation to different problems without supervisor's control and without additional adjustment to the concrete problem. Presented results confirm, also, Free Search capability to negotiate hard constrained task with unknown optimum and illustrate adaptation to unknown space.

FS which harmonises valuable ideas from other evolutionary algorithms such as probabilistic transaction rules, learning from experience, probabilistic access to whole search space with the concepts for uncertain individual behaviour and creative interpretation of the achieved current results has higher overall performance on explored test. FS overcomes common disadvantages of existing evolutionary population-based algorithms such as dependence from the initial population and inability for orientation within the search space, which is supported with the experimental results. Capability for adaptation within multidimensional search space can contribute to the multidimensional space study and to a better understanding of real space.

Presented results can be valuable, also, for comparison with and assessment of other methods. The question which level of adaptation – to unknown or – to changeable space is harder could be a subject of further investigation by evaluation of other unknown and changeable test problems.

Free Search can advance a wide range of disciplines in the efforts to cope with complex, uncertain problems, such as engineering, physics, chemistry, economics, business, finance, and operations research.

Further investigations can focus on enhancement of convergence speed, evaluation with dynamic and time dependent search space, including implementation in autonomous systems. For further work and study, Free Search could be applied to hard, unknown or time dependent real-world applications where data, space, information, conditions and objectives vary over time such as e-business and e-market [32].

A pragmatic area for further research is application to industrial and scientific tasks.

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