Assets Selection Criteria for Portfolio Diversification Based on Reward-to-Variability Ratio

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Key Words: *Assets selection; diversification; reward-to-variability ratio; risk measure; 'beta' coefficient; Value-at-Risk.*

Abstract*. Securities evaluation criteria are presented to select the assets for portfolio diversification. Modifications of the Sharpe coefficient are proposed, based on the new introduced risk measures. The effect of diversification is shown on the base of the introduced coefficients. The proposed selection criteria are evaluated according to the efficiency of a composed portfolio, measured by the Sharpe coefficient. Comparative analysis of the introduced securities evaluation criteria is held. Case study based on the Stock Exchange „Russian Trading System" is performed.*

1. Introduction

Investment portfolio analysis recently has become widely used due to the securities market development as is demonstrated by many publications on this subject (see e.g. [1-4]). One of the most discussed problems is the assets selection for portfolio diversification. There are different criteria developed to evaluate the securities to be included in a portfolio. From the portfolio diversification point the risk of a portfolio is reduced when non-correlated assets are added [6]. However in practice it can be difficult to select perfectly non-correlated assets [11]. The Sharpe coefficient (reward-to-variability ratio) and the Treynor coefficient (reward-to-volatility ratio) can be used to evaluate the securities to be included into a portfolio [11,12]. One of the discussed problems is the way to measure the risk or volatility of a security.

The Treynor coefficient uses stock betas from the CAPM to evaluate systematic risk, i.e. the return risk associated with market movements [7]. Even though being widely criticized, the beta-method is often used in financial analysis, and stood numerous empirical tests. When returns and factors are jointly normally distributed and independent over time, the classical method provides the most efficient unbiased estimator of factor risk premiums in linear models [14]. However many empirical studies report important beta variation over time. A standard approach to modeling and estimating time-varying betas has not yet emerged. Especially betas are biased, inconsistent, and inefficient in emerging markets, as has been shown in [13]. Given the fact that unstable betas might have serious consequences on the efficiency of beta based risk evaluations, there is a strong need for a better understanding of stock betas [10].

The Sharpe coefficient is a fundamental performance measure. Nevertheless, there have been some improvements of this ratio. The classical Sharpe ratio is based on normal distribution mean-variance analysis. When distributions are nonnormal or have fat tails, the performance rankings are not accurate. Two principal approaches to generalize the Sharpe ratio can be distinguished.

One strand of the literature is based on the use of utility functions. Hodges generalizes Sharpe ratio applying the exponential utility function with Arrow-Pratt risk-aversion index, which is constant for exponential utility independent of wealth [18]. Horges' reason for choosing exponential utility is the assumption of its equivalency to quadratic utility and mean-variance analysis.

Another generalized Sharpe measure is based on the family of negative power utility functions, also called constant relative risk aversion [18]. 'Gama'-generalized Sharpe ratio depends on the risk-aversion parameter and the initial wealth, so it does not have a unique value, as does the ordinary Sharpe ratio or Horges' generalized Sharpe ratio.

One of the recent approaches is the use of utility functions with hyperbolic absolute risk aversion (HARA). One of the strong points of this approach is that such utility functions allow the derivation of a generalized two-funds separation theorem thus leading to sample capital market evaluation formulas, and to the generalization of the Sharpe ratio and the Traynor ratio as well [9].

Nevertheless, utility function approaches, though important, are rather subjective. The degree of investor's risk-aversion and the selection of utility function remain discussed questions.

Another strand of literature aims at applying risk measures which are based on downside risk considerations. Ziemba and Schwartz propose to find the downside standard deviation, and the total variance is twice the downside variance [18]. Thus, a superior investor is not penalized for good performance. Ziemba calls the obtained performance ratio 'the symmetric downside risk Sharpe measure'.

Another downside risk measure is Value-at-Risk (VaR), a widely used concept for quantifying the risk of portfolios. VaR has received an official recognition after having been recommended by various regulating financial institutions as a portfolio risk-measurement tool. Thus, The Bank for International Settlements (BIS) recommends VaR method for defining the Market Risk Capital of a bank [17]. Moreover since the publication of the market risk measurement system $RiskMetrics^{TM}$ of J.P. Morgan in 1994 VaR has gained increasing acceptance and can now be considered as the industry's standard tool to measure market risks [8].

There are two main groups of models to calculate VaR. Parametric models such as delta-normal are based on statistical parameters such as the mean and the standard deviation of the risk factor distribution. Non-parametric models are simulations or historical models [8].

The aim of this paper is to propose new securities selection criteria based on the Sharpe coefficient and to evaluate their efficiency. We modify the Sharpe coefficient using the new introduced ($\overline{R} - VaR$)- and ($\overline{R} - R_{low}$)-risk measures. The new measures are based on *VaR*- and R_{low} -values, which refer to the downside measures. VaR is calculated using method of historical modeling considering inconsistency of the parametric VaR-models with the Russian stock market, as has been shown in [19].

The paper is organized as follows. The techniques of diversification as well as the Sharpe coefficient and the Treynor coefficient are discussed in the second section. Modifications of the Sharpe coefficient, based on the new introduced risk measures, are presented in the third section. Securities selection criteria are discussed in the forth section in case study, which is based on the Stock Exchange „Russian Trading System". The proposed securities evaluation criteria are compared from the point of view of the portfolio performance, measured by the Sharpe coefficient. The obtained results are discussed in the conclusion.

2. Theoretical Preliminaries

2.1. Portfolio Diversification Techniques

Portfolios with only a few assets may be subject to a high degree of risk represented by a relatively large variance of the return. As a general rule, the variance of the return of a portfolio can be reduced by including additional assets in the portfolio, a process referred to as diversification. The main techniques of diversification include blind diversification, the Markowitz diversification and the inclusion of a risk-free asset (see e.g. [2,6]).

Blind diversification means construction of a portfolio by taking equal portions of all *n* assets. That is, the weight w_i of the asset *i* is $w_i = 1/n,\ i = \overline{1,n}$. The overall expected return R_p of this portfolio is [6]:

$$(1)\quad R_p = \frac{1}{n}\sum_{i=1}^{n} R_i$$

where R_i is the mean return of asset *i*, $i = \overline{1,n}$.

The portfolio variance σ_p^2 is defined as:

$$(2)\quad \sigma_p^2 = \sum_{i=1}^{n} w_i^2 \cdot \sigma_i^2 = \frac{1}{n^2}\sum_{i=1}^{n} \sigma_i^2$$

where σ_i^2 is the variance of the asset *i*, $i = \overline{1,n}$.

In the definition (2) it is assumed that the individual returns are uncorrelated. The variance of a portfolio decreases rapidly as *n* increases. But in general, such diversification may reduce the overall expected return while the decrease of the variance is small. So, blind diversification without understanding of its influence on both the mean and the variance of return is not necessarily desirable.

The mean-variance approach developed by H. Markowitz makes the trade-offs between mean and variance explicit. The Markowitz Model is based on the theory of covariance between the assets.

The expected return R_p and the variance of the return σ_p^2 of a portfolio of *n* assets are obtained as:

$$(3)\quad R_p = \sum_{i=1}^{n} w_i \cdot R_i$$

and

$$(4)\quad \sigma_p^2 = \sum_{ij} w_i \cdot w_j \cdot Cov_{ij}$$

where Cov_{ij} is the covariance of the assets *i* and *j*.

If the assets are uncorrelated, the variance of a portfolio can be made very small. If they are positively correlated, there is likely to be a lower limit to the variance that can be achieved [6].

The effect of a diversification may be seen in *figure 1*, which presents the set of efficient portfolios for different correlation values for a portfolio of two assets [4].

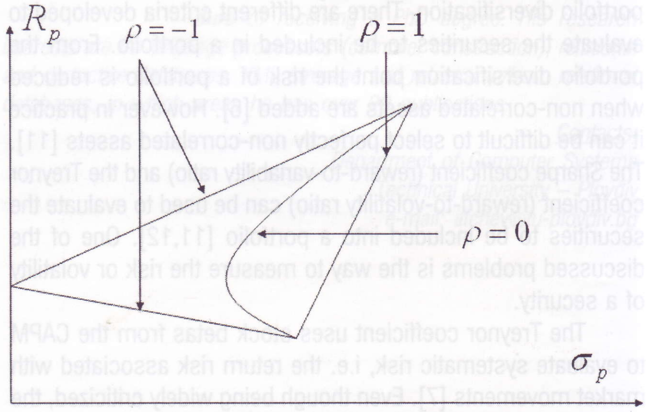

Figure 1. Standard-deviation – return diagram for different correlation values

The Markowitz model was further developed by the inclusion of a risk-free asset into a portfolio. According to the approach developed by J. Tobin, the portfolio is considered as a combination of a risky portfolio and a risk-free asset [7].

A risk-free asset has a deterministic return R_f (known with certainty) and therefore has a zero variance. In other words, a risk-free asset is a pure interest-bearing instrument. Its inclusion in a portfolio corresponds to lending or borrowing cash at the risk-free return R_f [6].

Let w_f denote the weight of a risk-free asset. Then the weight of the risky part is $(1 - w_f)$. Denote the variance of a risky part as σ^2 and the mean return as $\bar{R}$. Then the portfolio parameters are described as follows:

(5) $R_p = w_f R_f + (1 - w_f)\bar{R};$

(6) $\sigma_p^2 = (1 - w_f)^2 \sigma^2$.

This approach is used in The One-Fund Theorem, according to which „there is a single fund *F* of risky assets such that any efficient portfolio can be constructed as a combination of the fund *F* and the risk-free asset" [6]. So, the investor has to decide in which proportion his investments will be distributed between the risky part (fund *F*) and a risk-free asset.

The Capital Asset Pricing Model defines the Market portfolio to be such fund *F*. However, the Market portfolio still remains an object of numerous discussions [7]. As a benchmark portfolio the financial analysts often use market indices (for example, *S&P 500*, *DJIA*), which evaluate the mean-market return [3]. So the investor has to decide which assets will be included into the risky part of his portfolio.

2.2. Reward-to-Volatility and Reward-to-Variability Ratio

As has been shown, diversification is more efficient when non-correlated assets are added. However, for a given asset being at the same time positively or negatively, strongly or weakly correlated with other assets, it is difficult to select perfectly non-correlated assets to diversify the portfolio [11]. In such case the Sharpe and Treynor coefficients can be used to range the assets according to the reward-to-variability and reward-to-volatility ratio, respectively. The Sharpe coefficient, known as the 'reward-to-variability ratio', is defined to be [7]:

(7) $RVAR_p = \dfrac{R_p - R_f}{\sigma_p}$,

where R_p is the mean-return of the portfolio *p*, R_f is the risk-free asset return, and σ_p is the standard deviation of the portfolio *p*.

The Treynor coefficient (the 'reward-to-volatility ratio') is assumed to be:

(8) $RVOL_p = \dfrac{R_p - R_f}{\beta_p}$,

where β_p is the 'beta'-coefficient of the portfolio *p*, that is defined in the Market Model [7].

The Sharpe coefficient (7) and Treynor coefficient (8) can be equally used for assets evaluation [11]:

(9) $RVAR_i = \dfrac{\bar{R}_i - R_f}{\sigma_i}$;

(10) $RVOL_i = \dfrac{\bar{R}_i - R_f}{\beta_i}$

where $\bar{R}_i$ is the mean return of the asset *i*, $i = 1, n$, σ_i is the standard deviation and β_i is the coefficient of sensitivity of the asset *i* to market movement.

The choice of a security i^* to be added into a portfolio can be based on the maximization of the Sharpe coefficient (9) or the Treynor coefficient (10), i.e. $i^* = \arg\max_i RVAR_i$ or $i^* = \arg\max_i RVOL_i$, $i = 1, n$. It means a preference is given to the asset having the largest market prime per one risk-unit, measured by the standard deviation (the Sharpe coefficient) or by the 'beta'-value (the Treynor coefficient).

The choice of the coefficient depends on the set of the financial assets in the investor's portfolio. The risk for an investor, possessing other assets that are not included in the portfolio, should be measured by the 'beta'-coefficient since this coefficient evaluates risk relatively to the market [7]. When all instruments are included in the portfolio under consideration, the standard deviation can be seen as a suitable risk-measure, and the Sharpe coefficient can be used as an asset evaluation criterion.

Having selected the assets, the portfolio can be synthesized using the Markowitz Model [6]:

(11) $J_M = \sum_{ij} w_i \cdot w_j \cdot Cov_{ij} \to \min$

subject to

(12) $\sum w_i \cdot \bar{R}_i = R$

$\sum_i w_i = 1$

$w_i \geq 0$

where Cov_{ij} is the covariance of assets *i* and *j*, w_i and w_j are the weights of assets *i* and *j* in the portfolio, respectively, and *R* is a chosen value of the portfolio return.

3. Modifications of the Sharpe Coefficient

3.1. Alternative Risk-Measures

We introduce a new parameter, termed 'low-mean' return of the asset *i*, defined as:

(13) $R_{ilow} = \sum_{t \in Z} p_{it} \cdot R_{it}$, $R_{it} < \bar{R}_i$,

where Z is the set of indices *t* such that $R_{it} < \bar{R}_i$, and p_{it} is the probability of the return R_{it}.

$\overline{R}_{ilow}$ is the mean-return of a left ('bad') part of the return distribution of the asset *i*, i.e. the mean-value for the returns, which are less than the mean return of the asset $\overline{R}_i$.

In a similar way we obtain the mean-return of a right ('desirable') part of a return distribution of the asset *i*.

$$(14)\ \overline{R}_{iupper} = \sum_{t \in Z^+} p_{it} \cdot R_{it},\ R_{it} > \overline{R}_i,$$

where Z^+ is the set of indices *t* such that $R_{it} > \overline{R}_i$, and p_{it} is the probability of the return R_{it}.

In terms of 'low-mean' and 'upper-mean' returns the full variability of the return of the asset can be described by the difference of the 'upper-mean' and the 'low-mean' returns $(\overline{R}_{iupper} - \overline{R}_{ilow})$.

We define a new risk-measure, namely the difference between the asset mean return and the 'low-mean' return $(\overline{R}_i - \overline{R}_{ilow})$. This risk-measure is especially suitable for assets with asymmetric distributions as shown in *figure 2*.

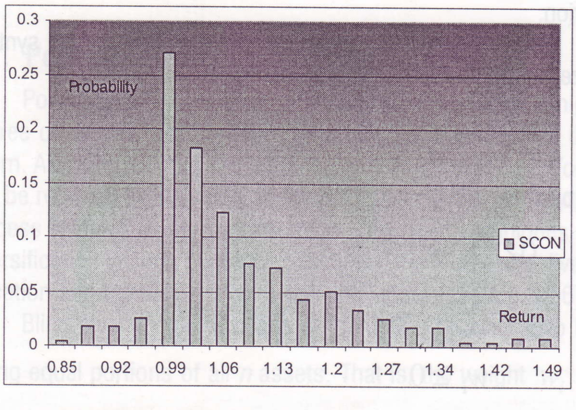

Figure 2. Return distribution for *SCON*-asset

For the case of a symmetric distribution the following equality holds:

$$(15)\ (\overline{R}_i - \overline{R}_{ilow}) = \left[\frac{\overline{R}_{iupper} - \overline{R}_{ilow}}{2}\right]$$

The value-at-risk (*VaR*) is a measure widely used in financial analysis. For a known asset return distribution, *VaR* defines the return that can be achieved with some probability level [2]:

$$(16)\ VaR_i = R_{iVaR} : [P\{R_{it} > R_{iVaR}\} = 1 - \alpha],$$

where α is the confidence level, which is usually set equal to 0.01, 0.05, or 0.1.

While the basic concept of VaR is simple, many complications can arise in practical use. A major drawback of VaR approach is that optimization problems, aiming at computing optimal portfolios with respect to VaR are typically hard to solve numerically. The reason is that VaR is in general not a convex function [8]. In this respect a related concept, conditional Value-at-Risk (CVaR) has recently been suggested as an alternative downside risk measure [15], which is determined as the expected mean loss after the VaR. This risk-measure is more consistent than VaR, due to some important properties such as subadditivity and convexity. CVaR is proved to be a coherent risk measure in the sense introduced by Artzner, Delbaen, Eber and Health, as shown in [16]. A more detailed study of CVaR and its application for assets selection can be a subject for further research.

There are several methods for computing VaR of nonlinear portfolios. Figure 3 presents the main approaches to VaR computation. Parametric models such as delta-normal are based on statistical parameters such as the mean and the standard deviation of the risk factor distribution. Non-parametric models are simulation approaches or historical models. An overview of frequently used VaR-models can be found in [8].

In the present paper we use method of historical modeling to calculate VaR considering inconsistency of the parametric VaR-models with the Russian stock market. This method is based on empirical distribution for a given period. VaR represents a quantile of an empirically estimated return distribution [19].

We propose another new risk measure, namely the difference between the asset mean return and the *VaR*-value for α-confidence level $(\overline{R}_i - VaR_i)$. The choice of the confidence level depends on the investor's attitude to risk. Risk preference allows setting high confidence-level, that increases *VaR*-value and decreases investor subjective evaluation of risk, measured by $(\overline{R}_i - VaR_i)$-value. And on the contrary, risk aversion implies low confidence-level.

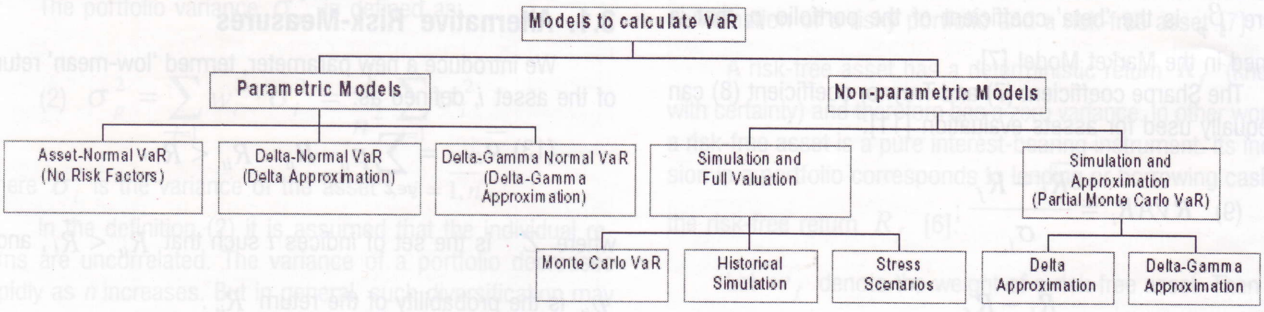

Figure 3. Approaches to VaR computation

Table 1. Securities parameters for the period June 2005 – May 2006

Asset	*SCON*	*KRNG*	*AVAZ*	*APTK*	*KLNA*	*AFLT*	*SNGSP*	*TATN*	*NLMK*	*NNSI*	*SIBN*	*SNGS*	*LKOH*	*RTKM*	*SBER*	*GMKN*	*EESR*	*MTSS*	*GASP*	*TRNFP*
Return	1.0781	1.062	1.105	1.082	1.05	1.045	1.0937	1.121	1.063	1.025	1.055	1.0874	1.1	1.075	1.0959	1.07	1.103	0.991	1.128	1.107
StDev	0.1136	0.234	0.187	0.108	0.085	0.087	0.0886	0.116	0.098	0.068	0.08	0.1017	0.09	0.124	0.1045	0.087	0.12	0.054	0.136	0.135
R Square	0.9882	0.949	0.975	0.991	0.99	0.991	0.9979	0.993	0.993	0.996	0.996	0.9969	0.998	0.987	0.9953	0.988	0.992	0.995	0.991	0.987
Beta	0.9818	0.963	1.002	0.993	0.968	0.956	0.9996	1.026	0.979	0.941	0.972	1.0002	1.006	0.975	1.01	0.976	1.005	0.912	1.039	1.015
Observations	236	236	236	236	236	236	236	236	236	236	236	236	236	236	236	236	236	154	70	224

3.2. Sharpe Coefficient for the ($\overline{R_i} - \overline{R_{ilow}}$)- and ($\overline{R_i} - VaR_i$)-Risk-Measures

On the base of the risk-measures introduced in 3.1, we propose the following modifications of the Sharpe coefficient for the asset *i* (7):

$$(17)\quad S_{ilow} = \frac{\overline{R_i} - R_f}{\overline{R_i} - \overline{R_{ilow}}};$$

$$(18)\quad S_{iVaR} = \frac{\overline{R_i} - R_f}{\overline{R_i} - VaR_i}.$$

The coefficient (17) describes the amount of excessive return (market prime) referred to one unit of risk, measured as a deviation of asset mean return from its 'low'- mean return. This coefficient may be recommended to evaluate especially the assets characterized by asymmetric distribution.

The coefficient (18) describes the amount of excessive return per one unit of risk, measured as a deviation of asset mean return from its *VaR*-value. *VaR*-value can be estimated for different α-confidence levels, which are set regarding the investor's risk preferences.

4. Case Study

4.1. $RVAR_i$ -Coefficient as Assets Selection Criterion for Portfolio Diversification

We consider the securities traded on the Russian Trading System Stock Exchange (RTS) [20]. The invested amounts are distributed between different branches of economics, represented by 20 companies. We have studied statistic data on selected securities for a one-year period, namely June 2005 – May 2006. We suppose the amounts are invested for a two-month period. The securities returns, standard deviations and 'beta'-coefficients have been calculated. The significance of the 'beta'-coefficients is confirmed by the high values of the coefficient of determination R^2 [5]. The results are presented in t*able 1*.

The annual risk-free return R_f is supposed to be 1.06. That is, the annual expected risk-free rate of return is 6% (the annual rate of return for governmental bonds in Russia (September 2005) [21]), or 1% for a two-month period in the considered example.

We have determined the values of the $RVAR_i$-coefficient (9). The assets have been ranged according to the $RVAR_i$-coefficient (9) as shown in *table 2*.

Table 2. Ranking of assets according to -coefficient

	Asset	$RVAR_i$		Asset	$RVAR_i$
1	*LKOH*	0.9940336	11	*SCON*	0.598925
2	*TANT*	0.95381 16	12	*SIBN*	0.55863
3	*SNGSP*	0.945251	13	*NLMK*	0.540745
4	*GASP*	0.8632978	14	*RTKM*	0.522085
5	*SBER*	0.8219275	15	*AVAZ*	0.509001
6	*EESR*	0.774932	16	*KLNA*	0.468108
7	*SNGS*	0.7612216	17	*AFLT*	0.399588
8	*TRNFP*	0.718538	18	*KRNG*	0.221683
9	*GMKN*	0.6879493	19	*NNSI*	0.221484
10	*APTK*	0.6632712	20	*MMTS*	-0.35475

We have composed the portfolio of 4 upper assets, selected from the *table 2* and having the highest Sharpe coefficient values, namely *LKOH-*, *TANT-*, *SNGSP-* and *GASP*-assets.

The efficient frontier for the portfolio of 4 assets has been determined. Remind that the efficient frontier is the upper portion of the minimum-variance set that lays upper than a minimum-variance point [6]. The points on the efficient frontiers have been determined by solving the optimization problem (11), (12): minimize the variance of the portfolio under the constraint of a fixed mean return R. The fixed values R in (12) have been chosen using a 0.1% step.

Then a portfolio of 5 assets has been composed by taking the 5 upper assets from the *table 2*, namely *LKOH-*, *TANT-*, *SNGSP-*, *GASP-* and *SBER*-assets. The same procedure was repeated for *n*, $n = \overline{4,11}$, assets. The efficient frontiers for portfolios, composed of *n*, $n = \overline{4,11}$, assets are presented in *figure 4*.

Figure 4 shows the effect of diversification. The portfolio, composed of 5 assets is more efficient than the one, composed of 4 assets, and less efficient than the portfolio that includes 6 assets, which is seen in the shifting of efficient frontiers leftward. *Table 3* demonstrates the effect of diversification for $R_p = 1.115$ (that is, the expected rate of return is 11.5%). The standard deviation in this example decreases from 0.08763 (*n*=4) to 0.08229 (*n*=9).

The results of *table 3* can be interpreted as follows: for an invested amount equal to 1000 euros the investor's gain is 115 euros with a standard deviation of 87.63 euros. That is, in the most pessimistic case, according to the rule of „3 σ " that holds for the majority of distributions, investor's possible loss will be 147.89 euros $(115-87.63\cdot 3)$ if he distributes his capital between 4 assets. If he invests in 5 assets, in the worst case he will loose 139.97 euros $(115-84.99\cdot 3)$ etc. After the sixth asset has been added, the standard deviation does not decrease considerably.

Table 3. Effect of diversification for $R_p = 11.5\%$

Number of assets	R_p	σ_p
4	1.115	0.087632091
5	1.115	0.084993942
6	1.115	0.082318053
7	1.115	0.082318053
8	1.115	0.082318053
9	1.115	0.082288547
10	1.115	0.082288547

Now we need to select portfolios on the efficient frontiers in *figure 4*. The portfolio performance may be evaluated using the Sharpe coefficient for a portfolio (7). The most efficient portfolio corresponds to the point having the highest value of the Sharpe coefficient, as shown in *figure 5*.

However, the maximum value of the Sharpe coefficient may correspond to a portfolio, having a low value of the standard deviation and of the portfolio return (a point on the low left part of the efficient frontier).

Low values of a portfolio expected return may be unacceptable for the investor. In such case the investor can determine a desirable zone on the efficient frontier, limited by an admissible level of the portfolio expected return in order to avoid portfolios with low values of the return.

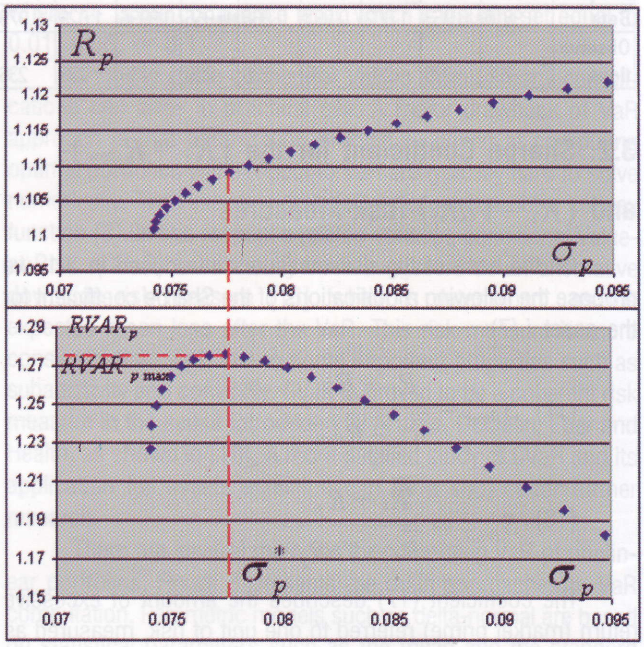

Figure 5. Selection of a portfolio on the efficient frontier

Thus, the investor will choose a portfolio having the maximum value of the Sharpe coefficient in the desirable zone. *Figure 6a and 6b* illustrate different admissible levels of the portfolio expected return ($R_{p\min}$) and the choice of the portfolio on the efficient frontier.

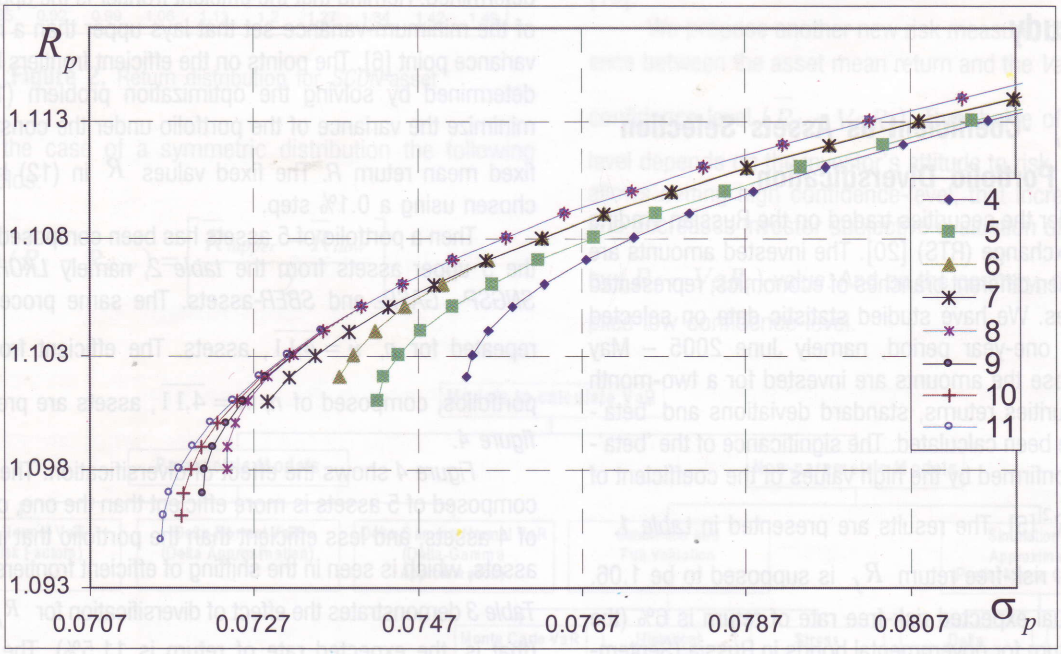

Figure 4. Efficient frontiers for diversified portfolios

We have evaluated the Sharpe coefficient for a set of portfolios, synthesized according to (11), (12). The values of R in (12) have been chosen sing 0.1% step. *Table 4* presents the weight distributions and portfolio parameters for portfolios composed of n, $n = 4{,}11$, assets for the portfolio return $R_p = 1.115$.

and by decreasing the weights of other assets. Thus, the diversification yields an increasing of the Sharpe coefficient (or not decreasing, at least).

The effect of increasing of the Sharpe coefficient can be illustrated as follows. If an investor distributes 1000 euros between 4 assets, he gains 115 euros (return is 1.115, the expected rate of return is 11.5%) with a standard deviation of

Table 4. Weight distribution for portfolios composed using $RVAR_i$**-coefficient** ($R_p = 1.115$)

Number of Assets	Weight of assets										Portfolio Parameters		
	LKOH	TATN	SNGSP	GASP	SBER	EESR	SNGS	TRNFP	GMKN	APTK	R_p	σ_p	$RVAR_p$
4	0.27386	0.32904	0.08137	0.3157							1.115	0.087632	1.198203
5	0.25153	0.32829	0.04923	0.3168	0.05417						1.115	0.084994	1.235394
6	0.22036	0.31516	0.01666	0.3056	0.03954	0.1027					1.115	0.082318	1.275552
7	0.22056	0.31527	0.0163	0.3054	0.03936	0.1031	0				1.115	0.082318	1.275552
8	0.19972	0.30053	0.00091	0.2925	0.03009	0.0957	0	0.0806			1.115	0.082318	1.275552
9	0.19981	0.30057	0	0.2924	0.03079	0.0959	0	0.0806	0		1.115	0.082289	1.275998
10	0.19961	0.30059	0.001	0.2925	0.03015	0.0957	0	0.0804	0	0	1.115	0.082289	1.275998

The results of the *table 4* show, that the more portfolio is diversified, the higher values of the Sharpe coefficient are obtained, because a more diversified portfolio cannot be less efficient than a portfolio composed of a less number of assets. If the inclusion of a new asset yields a new portfolio, which is less efficient then the previous one, then the optimization tool will indicate the inefficiency of this inclusion by setting the weight of this asset equal to zero. It follows that the attained previous parameters of a portfolio (the expected return and the standard deviation) and the Sharpe coefficient will not change. If the inclusion of a new asset yields a new portfolio, which is more efficient then the previous one, then the optimization tool will increase the portfolio performance by investing in this asset

87.63 euros. That is, he is awarded by 1.198 euro of excessive return (market prime) per 1 euro of risk taking $\left(\frac{0.115-0.01}{0.08763}\right)$. If he invests in 5 assets, these values are equal to 115, 84.99 and 1.235 euro respectively etc.

The experimental results have shown that the diversification allows achieving higher values of the Sharpe coefficient. However transaction costs should be taken into consideration while diversifying the portfolio.

Remind that each efficient frontier corresponds to a set of portfolios composed of different number of assets (see *figure 4*). Thus to evaluate the effect of diversification we compare port-

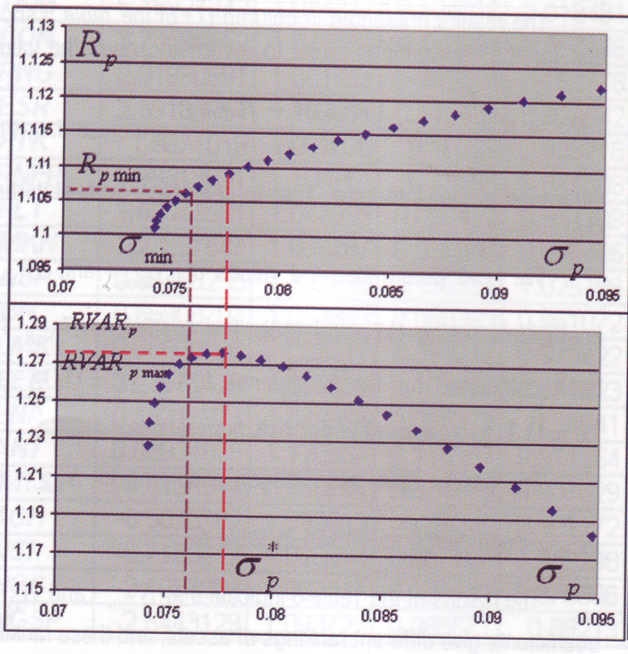

Figure 6a. Choice of the portfolio having the maximum value of the Sharpe coefficient

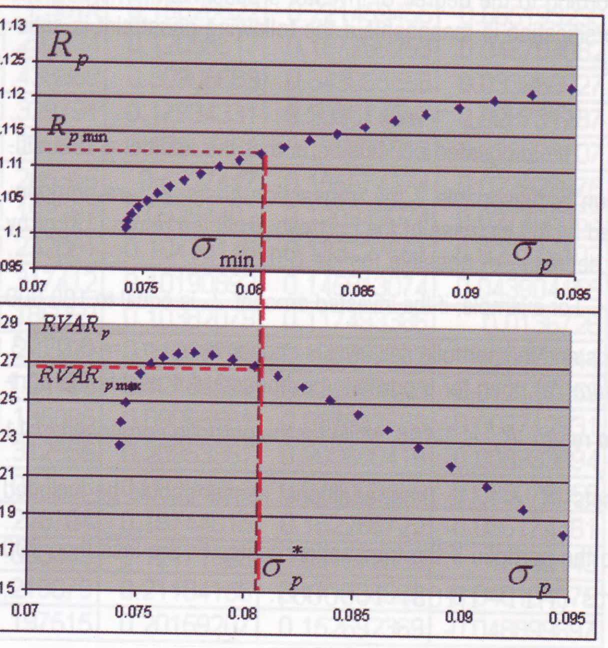

Figure 6b. Choice of the portfolio having the maximum value of the Sharpe coefficient in the desirable zone

folios taken from different efficient frontiers. We consider two cases: comparison of the portfolios for a fixed level of the portfolio risk or for a fixed level of the portfolio return (*figure 7*).

Figure 7a shows the increase of the portfolio return for a fixed level of the standard deviation σ' when a portfolio is diversified between 11 assets relatively to a portfolio of 4 assets. The diversification is reasonable if the following inequality holds:

(19) $(R_A - R_B)\cdot I > T$

where I is the invested amount.

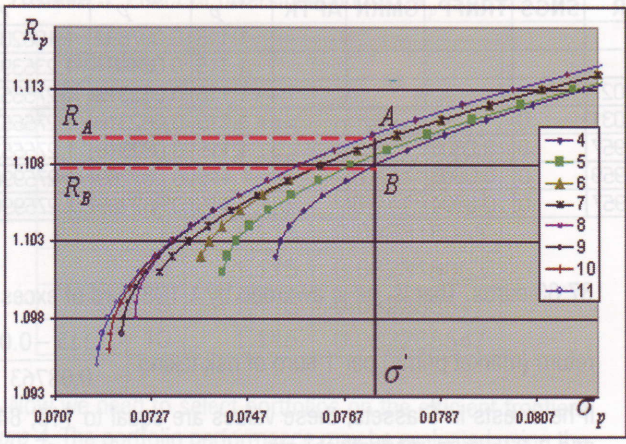

Figure 7a. Increase of the portfolio return due to the portfolio diversification

For a fixed value of the portfolio return R' (*figure 7b*) the transaction costs T should be compared with the decrease of possible losses, the last being evaluated as $k\sigma$, $k \in [1,3]$, according to the degree of investor's pessimism. The portfolio diversification is reasonable if the following constraint is satisfied:

(20) $(k\sigma_A - k\sigma_B)\cdot I > T$.

The suggested conclusion is that the additional diversification is reasonable if the transaction costs T are compensated by the increase of the portfolio expected return (19) or by the decrease of possible losses (20).

For example, if the invested amount I is equal to 100 000 euros and the portfolio risk level is chosen to be $\sigma' = 0.0773$ (*figure 7a*), then for a portfolio composed of 4 assets the portfolio return R_p is 1.108, while for the portfolio composed of 11 assets R_p = 1.11. Thus, additional assets should be included into the portfolio if the transaction costs T are less then 200 euros $(1.11 - 1.108)\cdot 100000$).

4.2. S_{ilow} - and S_{iVaR} -Coefficients as Asset Selection Criteria for Portfolio Diversification

The empirical distributions have been obtained for all the assets under consideration. Figure 8 shows some of the symmetric and asymmetric distributions.

We have determined the values of $(\overline{R_i} - \overline{R}_{ilow})$ and $\dfrac{(\overline{R}_{iupper} - \overline{R}_{ilow})}{2}$, using (13) and (14). The coefficients of

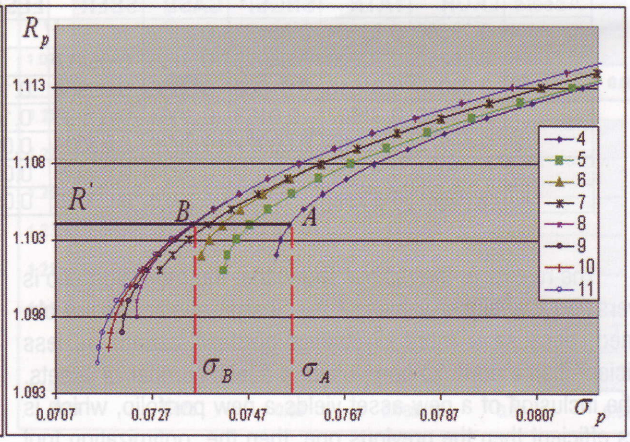

Figure 7b. Standard deviation decrease due to the portfolio diversification

skewness for the asset distributions have been obtained, also (see e.g. [5]). The results are shown in *table 5*. Positive values of the coefficients of skewness indicate a heavier left ('bad') part of the distribution while negative values indicate a heavier right ('desirable') part of the return distribution (see *figure 8*, as well).

The results presented in column (7) of the *table 5* show, that for the distributions close to symmetric ones the values $(\overline{R_i} - \overline{R}_{ilow})$ and $\dfrac{(\overline{R}_{iupper} - \overline{R}_{ilow})}{2}$ are close, i.e., their difference is close to zero. That is consistent with statement (15).

We have determined the values of S_{ilow} - and S_{iVaR} -coefficients, using (17) and (18). The coefficient S_{iVaR} has been calculated for the confidence levels $\alpha = 0.05$ and $\alpha = 0.1$ to assess different risk preferences.

The assets have been ranged according to S_{ilow} - and S_{iVaR} -coefficients as shown in *table 6*.

The results of the Table 6 indicate that S_{ilow} - and S_{iVaR} -coefficients give different rankings of assets, and these rankings are different from the results presented in *table 2*.

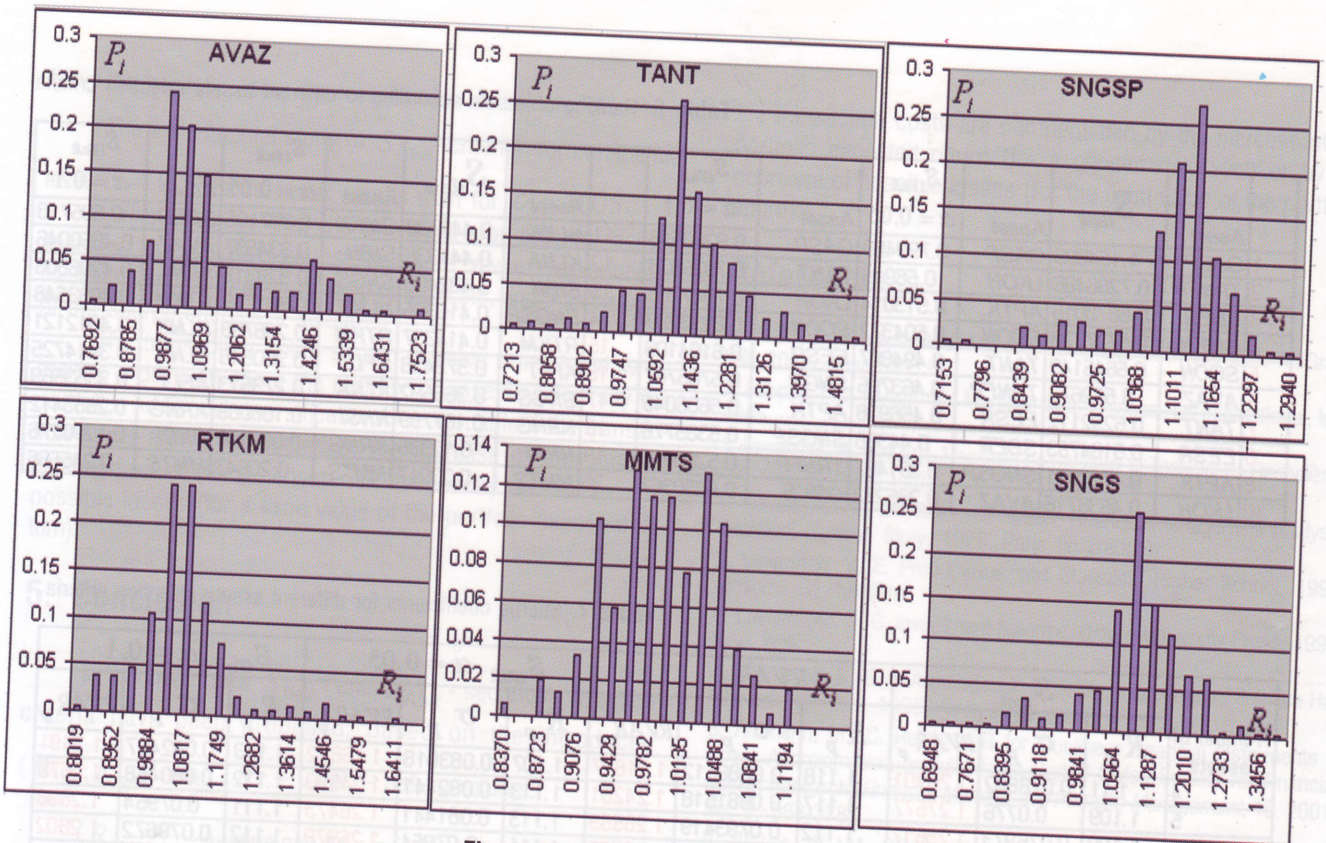

Figure 8. Return distributions for the assets

Table 5. The assets parameters

Asset	(1) Skewness	(2) $\overline{R}_i$	(3) σ_i	(4) $\overline{R}_{ilow}$	(5) $\overline{R}_{iupper}$	(6) $\overline{R}_i - \overline{R}_{ilow}$	(7) $\frac{(\overline{R}_{iupper} - \overline{R}_{ilow})}{2}$	(8) (7)-(6)
AVAZ	10.2717502	1.105033	0.186704	0.933082	1.451943	0.17195092	0.259430665	0.087479743
RTKM	5.16921242	1.074747	0.124016	0.918481	1.3614	0.15626622	0.221459455	0.065193238
SCON	3.73595652	1.078061	0.113638	0.957496	1.292034	0.12056517	0.167269253	0.046704082
KRNG	2.91990691	1.061864	0.233958	0.752778	1.438889	0.30908663	0.343055556	0.033968927
EESR	2.59108057	1.102985	0.119991	0.923642	1.329791	0.17934331	0.203074597	0.023731287
APTK	1.9871016	1.081826	0.10829	0.929718	1.280981	0.15210761	0.175631313	0.023523707
TRNFP	1.96319631	1.106751	0.13465	0.974006	1.285355	0.1327447	0.155674374	0.022929675
AFLT	1.53073581	1.044933	0.087422	0.951389	1.193287	0.09354402	0.120949074	0.027405053
GMKN	1.23837547	1.069787	0.086906	0.965405	1.232861	0.10438146	0.133727561	0.029346102
GASP	0.86468255	1.127695	0.136331	1.025786	1.317412	0.10190892	0.145813074	0.043904156
SIBN	0.66908707	1.054893	0.080363	0.951072	1.186059	0.10382079	0.117493386	0.0136726
NLMK	0.36888808	1.06311	0.098217	0.943422	1.202221	0.11968792	0.129399362	0.00971144
SBER	0.21575428	1.095869	0.104473	0.971023	1.24495	0.12484589	0.136963148	0.012117259
KLNA	0.18313974	1.049965	0.085377	0.95941	1.135095	0.09055561	0.087842683	-0.002712931
TANT	0.08934975	1.120887	0.116257	0.911354	1.312555	0.20953351	0.200600418	-0.008933094
MMTS	-0.0136944	0.990711	0.054373	0.907629	1.057635	0.08308251	0.075003455	-0.008079058
LKOH	-0.5075362	1.099613	0.09015	0.902172	1.226704	0.19744018	0.162265722	-0.035174461
NNSI	-0.7183617	1.025014	0.067787	0.903298	1.086443	0.12171563	0.091572635	-0.030143
SNGS	-2.1089895	1.087447	0.101741	0.875606	1.219079	0.21184156	0.171736688	-0.040104875
SNGSP	-2.6443129	1.093722	0.088571	0.89213	1.197515	0.20159207	0.152692369	-0.048899697

Table 6. Ranking of assets according to different assets selection criteria

	Asset	S_{ilow}	Asset	S_{iVaR} $\alpha = 0.05$	Asset	S_{iVaR} $\alpha = 0.1$		Asset	S_{ilow}	Asset	S_{iVaR} $\alpha = 0.05$	Asset	S_{iVaR} $\alpha = 0.1$
1	GASP	1.154899	GASP	0.799469	GASP	0.9260516	11	NLMK	0.443738	GMKN	0.367185	SBER	0.505718
2	TRNFP	0.7288486	LKOH	0.56019	TANT	0.7329823	12	KLNA	0.441336	SIBN	0.334537	AVAZ	0.4850046
3	SBER	0.6878016	APTK	0.513014	LKOH	0.7047283	13	SIBN	0.432408	SNGS	0.329319	SIBN	0.4296808
4	GMKN	0.5727728	SCON	0.504372	SCON	0.6233289	14	SNGSP	0.415305	NLMK	0.301739	RTKM	0.4033548
5	SCON	0.5645143	TANT	0.494807	EESR	0.6104108	15	RTKM	0.414337	RTKM	0.296799	NLMK	0.4012121
6	AVAZ	0.552673	TRNFP	0.463705	SNGS	0.5765787	16	AFLT	0.373438	AFLT	0.280546	KLNA	0.3414725
7	TANT	0.5292112	EESR	0.456928	APTK	0.5606045	17	SNGS	0.365592	KLNA	0.273513	AFLT	0.3353569
8	EESR	0.5184753	SBER	0.44358	SNGSP	0.5385776	18	KRNG	0.167799	NNSI	0.106089	KRNG	0.2533412
9	APTK	0.4722041	SNGSP	0.391467	TRNFP	0.5280216	19	NNSI	0.12335	KRNG	0.092308	NNSI	0.1290276
10	LKOH	0.4538718	AVAZ	0.385272	GMKN	0.5230357	20	MMTS	-0.23217	MMTS	-0.20643	MMTS	-0.245566

Table 7. Sharpe coefficients for different assets selection criteria

Number of assets	S_{ilow}			$RVAR_i$			$S_{iVaR}, \alpha = 0.05$			$S_{iVaR}, \alpha = 0.1$		
	R_p	σ_p	$RVAR_p$	R_p	σ_p	$RVAR_p$	R_p	σ_p	$RVAR_p$	R_p	σ_p	$RVAR_p$
4	1.11	0.078802	1.26901	1.118	0.0886512	1.21827	1.107	0.083018	1.16842	1.113	0.082457	1.2491
5	1.109	0.0776	1.27577	1.117	0.0861518	1.24201	1.113	0.082441	1.24939	1.112	0.080458	1.2678
6	1.109	0.076973	1.28617	1.112	0.0793419	1.28559	1.113	0.081441	1.26473	1.111	0.07954	1.2698
7	1.109	0.076973	1.28617	1.112	0.0793411	1.28559	1.111	0.07954	1.26979	1.112	0.079672	1.2802
8	1.109	0.076495	1.29421	1.112	0.0793419	1.28559	1.111	0.078464	1.28721	1.109	0.077261	1.2814
9	1.109	0.076495	1.29421	1.111	0.0781951	1.29165	1.109	0.076495	1.29421	1.11	0.077508	1.2902
10	1.109	0.076495	1.29421	1.111	0.0781951	1.29165	1.109	0.076495	1.2942	1.11	0.077508	1.2902

The portfolios have been diversified using S_{ilow} - and S_{iVaR} - coefficients as assets selection criteria. The portfolio performance has been evaluated using the Sharpe coefficient. The portfolios having the maximum values of the Sharpe coefficient for different number of assets have been determined. *Table 7* shows the results of diversification for different securities selection criteria: $RVAR_i$ -, S_{ilow} - and S_{iVaR} - coefficients.

It can be seen, that the Sharpe coefficient achieves its greatest values when S_{ilow} is applied as assets selection criterion.

Figure 9 summarizes the Sharpe coefficients for different assets selection criteria.

Table 7 and *figure 9* show, that the diversification is most effective when S_{ilow} -criterion is applied. It enables to achieve the highest values of the Sharpe coefficient and allows diversi-

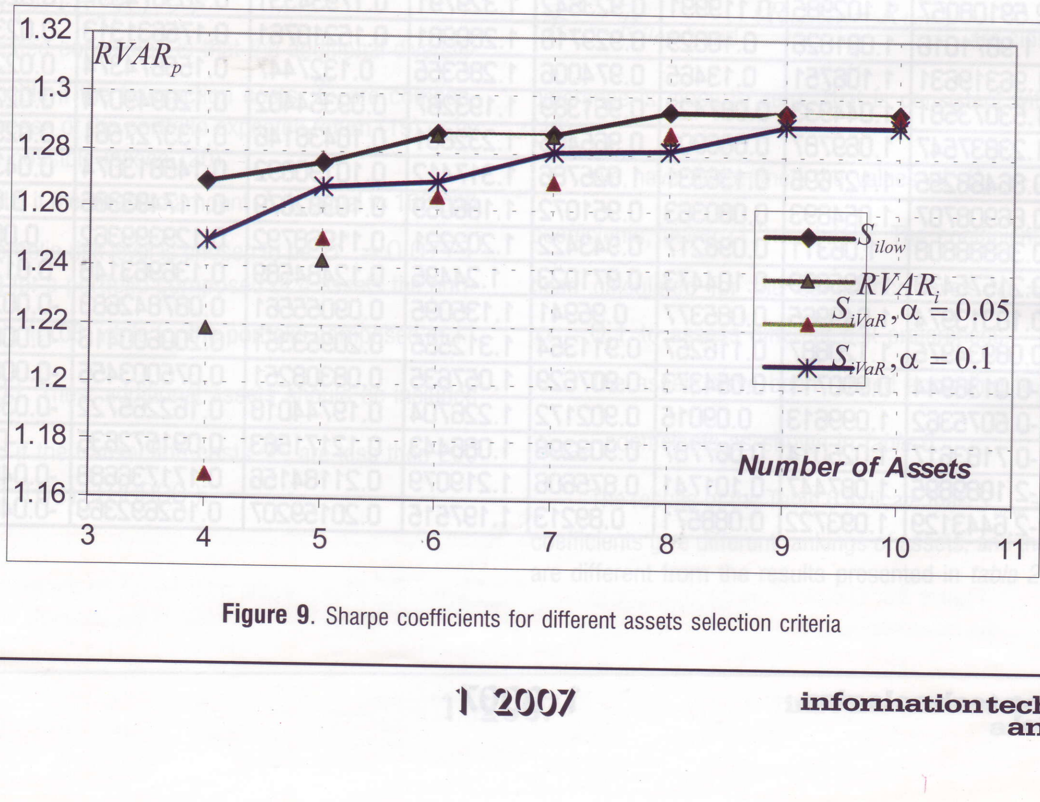

Figure 9. Sharpe coefficients for different assets selection criteria

fying the portfolio using the less number of selected assets.

The experimental study of S_{iVaR} -criterion for confidence level $\alpha = 0.1$ yields better results than for $\alpha = 0.05$.

Table 7 shows that the application of different assets selection criteria yields almost the same 'reward-to-variability' value when portfolio is composed of 9 assets, and that the value of the Sharpe coefficient stabilizes.

The transaction costs should be also considered while making a decision about the number of assets to diversify the portfolio. Additional diversification is reasonable if the transaction costs are compensated by the increase of the portfolio expected return (for a chosen risk level) or by the decrease of possible losses (for a fixed value of the portfolio expected return).

5. Conclusion

In this paper new S_{ilow}- and S_{iVaR} - securities selection criteria have been proposed, based on the introduced $(R_i - R_{ilow})$- and $(R_i - VaR_i)$- risk-measures.

$(R_i - R_{ilow})$-value can be a suitable risk-measure especially for the asymmetric distribution of the return of an asset. $(\overline{R_i} - VaR_i)$-value allows the investor to set acceptable deviation of the return from the *VaR*-value for different confidence levels, considering his risk preferences. For further research measures based on conditional VaR can be considered, since CVaR possesses such important properties as subadditivity and convexity.

The efficiency of proposed selection criteria has been analyzed using the Sharpe coefficient. S_{ilow} -criterion can be recommended as the most efficient for assets selection from the point of view of the portfolio performance. The experimental study of S_{iVaR} -criterion for confidence level $\alpha = 0.1$ has yields better results than for $\alpha = 0.05$. It can be recommended to apply other methods for VaR computation. Considering the inconsistency of parametric VaR-methods with the Russian stock market, simulation methods are preferable. In the present paper we have used the method of historical modeling. We suppose that other simulation methods could increase the efficiency of the VaR-approach. For example, Monte Carlo simulation, which is widely used in practice.

The efficient frontier can be limited by the admissible level of the portfolio expected return in order to avoid portfolios with low values of the return. The investor will choose the portfolio having the maximum value of the Sharpe coefficient in the desirable zone.

The experimental results have shown that the diversification implies increase of the Sharpe coefficient. The number of assets to compose a portfolio may be determined on the base of the (quazi)stabilization of the Sharpe coefficient and depends on transaction costs. Additional diversification is reasonable if the transaction costs are compensated by the increase of the portfolio expected return (for a chosen risk level) or by the decrease of possible losses (for the fixed value of the portfolio return).

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for the case studies of the N queens problem and Sam Loyd's puzzle we can conclude that the efficiency of parallel combinatorial search on multicomputer platform is about 30% and the speedup increases slowly versus the problem size not exceeding 1.8 for 5 computers.

In order to make a prognostication for the efficiency of parallel combinatorial search on multicomputer platform we have to estimate the isoefficiency that is a metric to characterize system scalability. The efficiency of the parallel system E depends on the workload W, the number of processors n and the system overhead T_o i.e. $E=f(W,n,T_o)$. In order to keep up 30% efficiency for parallel combinatorial search scaling the machine size requires the adequate scaling of the parallel application i.e. the board size should be enlarged – above 15x15 for the N Queens' problem and 7x7 for Sam Loyd's puzzle. Nevertheless, we have to take into consideration that scaling up the workload will result in increasing the system overhead and eventually the efficiency might drop below 30%.

The future work should encompass investigation of the efficiency of parallel combinatorial search on computer cluster and the utilization of more efficient mechanisms for dynamic load balancing.

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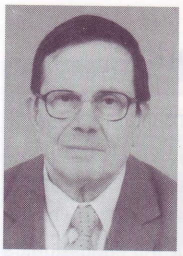

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