A New Approach for Adaptive Tuning of PI Controllers. Application in Cascade Systems

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Key Words: *Adaptive tuning controllers; online tuning; cascade system.*

Abstract. *In this paper, a new method for online adaptive tuning of PI controllers is proposed. Additional plant information is not necessary for this method. All necessary information for calculation of the controller parameters is received directly through data derived from the pulse response of the plant. In order to demonstrate the performance of the adaptive tuning approach, which has been designed for linear systems, an example with control of a three tank system is used. Simulation results using MATLAB/Simulink and real experiments using WinCon to create and execute real time code from a Simulink model are given in this paper.*

1. Introduction

Some authors (Äström, Hang, Zhuang, Kaya and etc.) suggest to use rules presented in (Ziegler and Nichols, 1942), (Äström, 1984), (Rotach, 1984), (Rotach, 1985), (Hang, 1991), (Hang, 2002), (Äström, 2004), or rules presented in (Zhuang, 1993) for tuning controllers in cascade systems.

The Ziegler-Nichols (ZN) rules were originally designed to give systems with good responses to load disturbances. They were obtained by extensive simulations of many different systems. The design criterion leads to a damping ratio $\xi = 0.22$, which is often too small. For this reason the Ziegler-Nichols rules (method) often requires retuning. In (Äström, 1984) authors suggested using of phase and amplitude margins as design criterion. Hang presented refined ZN tuning for PI control (Eq. 1) in (Hang, 1991) and for PID control in (Hang, 2002).

$$K_C = \frac{5}{6}\left(\frac{12+K}{15+14K}\right)K_u; \qquad (1)$$

$$T_i = \frac{1}{5}\left(\frac{4}{15}K+1\right)T_u;$$

where: $K = K_P K_u$, K_P is the process gain, K_u is the ultimate gain, T_u is the ultimate period.

Other authors suggest to use different control structures and different methods. Liu proposed two control structures for cascade control systems (Liu, 2005). Lestage used serial-cascade, parallel-cascade and pseudo-cascade structures (Lestage, 1999). Kaya proposed usage of different control structure in inner and outer loop, (Kaya, 2001).

In this paper a method for online adaptive tuning of PI controllers in cascade structure is presented. All necessary information for controller parameters tuning is received from the pulse response of the plant.

2. Cascade Systems

Cascade control can be used when there are several measurement signals and one control variable. It is particularly useful when there are significant dynamics, e.g., long dead time or large time constants, between the control variable and the process variable. More tight control can be achieved by using an intermediate measured signal that responds faster to the control signal. Cascade control is built up by nesting the control loops, as shown in *figure 2.1*.

The system in this figure has two loops. The inner loop is called *the secondary loop*; the outer loop is called *the primary loop*. The reason for this terminology is that the outer loop deals with the primary measured signal. It is also possible to have a cascade control with more nested loops. The performance of a system can be improved with a number of measured signals, up to a certain limit. If all state variables are measured, it is often not worthwhile to introduce other measured ones. It such a case the cascade control is the same as state feedback.

It is important to be able to judge whether cascade control can give improvement and to have a methodology for choosing the secondary measured variable. This is easy to do, because the key idea of cascade control is to arrange a tight feedback loop around a disturbance. In the ideal case the secondary loop can be so tight so that the secondary loop is a perfect servo wherein the secondary measured variable responds very quickly to the control signal.

3. The Adaptive Tuning Method

By combining methods for determination of process dynamics with method for computing the controller parameters, methods for adaptive tuning of controllers can be obtained. A method for adaptive tuning (or automatic tuning) means a method where the controller is tuned automatically on demand from a user. An adaptive tuning procedure consists of the following

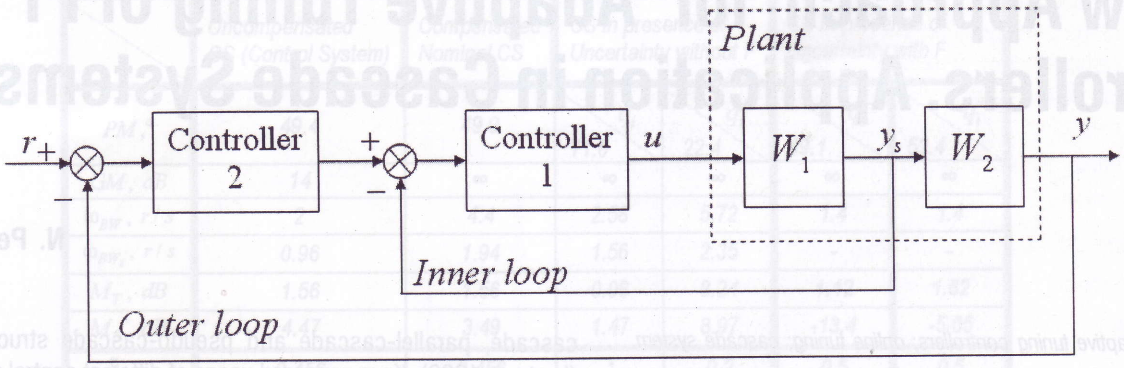

Figure 2.1 Block diagram of a cascade control system

r is the reference signal, u is the control signal, y_s is the secondary output signal, y is the primary output signal

steps:

- Generation of a process disturbance.
- Evaluation of the disturbance response.
- Calculation of controller parameters.

This is the same procedure that an experienced operator uses when tuning a controller manually.

Many stable processes can be represented by a second order system plus dead time (SOPDT) with sufficient accuracy

(2) $$W_P(s) = \frac{1}{a_2 s^2 + a_1 s + a_0} e^{-\tau_P . s};$$

where the parameters a_2, a_1 and a_0 are calculated directly from plant pulse response in the open loop system (Petkov 1972, Krug and Minina 1962).

(3)
$$a_0 = \frac{A_p . T_p}{A_1};$$
$$a_1 = \frac{1}{A_1}\left(a_0 . A_2 - A_p \frac{T_p^2}{2} \right);$$
$$a_2 = \frac{1}{A_1}\left(a_1 . A_2 - a_0 A_3 + A_p \frac{T_p^3}{6} \right);$$

where: A_p — the pulse amplitude; T_p — the pulse width; A_1, A_2, A_3 — the areas shown in *figure 3.1* and Eq. (4).

For plants with a small time delay, the parameter τ_P is not explicitly determined during the identification step. The influence on the system behaviour is taken into account by appropriate values of a_0, a_1 and a_2.

The specification of the pulse amplitude and the pulse width depends on plant dynamics. It is equal to the amplitude of the maximum permissible input signal. The width T_p is included in the adaptive tuning algorithm. The end of the pulse is determined when the process output signal reaches a certain predetermined deviation from the steady-state value.

For calculation of A_1, A_2 and A_3 the following equations (Petkov, 1972, Krug and Minina, 1962) are used:

(4)
$$y_1(t) = \int_0^t [y(\tau) - y(\infty)]\, d\tau \qquad A_1 = y_1(\infty);$$
$$y_2(t) = \int_0^t [A_1 - y_1(\tau)]\, d\tau \qquad A_2 = y_2(\infty);$$
$$y_3(t) = \int_0^t [A_2 - y_2(\tau)]\, d\tau \qquad A_3 = y_3(\infty);$$

•Controller

The controller specified as PI has the following form:

(5) $$G_{PI}(s) = K_C \left(1 + \frac{1}{T_i s} \right) = K_C \frac{T_i s + 1}{T_i s},$$

where: K_c is the controller gain; T_i is the integral time constant.

The criterion for tuning PI controller parameters is determined by the desired damping ratio (ξ = 0.707) of the closed loop system (Garnov, Rabinovich and Vishnevetzkiy, 1971).

When plant damping ratio is no smaller than one ($\xi \geq 1$ ($a_1 \geq 2\sqrt{a_0 a_2}$)) the tuning results for tuning of controller parameters are:

(6)
$$K_C = \frac{2.a_0^2.a_2}{\left(a_1 - \sqrt{a_1^2 - 4.a_0.a_2} \right)^2};$$
$$T_i = \frac{2.a_2}{a_1 - \sqrt{a_1^2 - 4.a_0.a_2}}.$$

If the plant damping ratio is between zero and one ($0 < \xi < 1$ ($a_1 < 2\sqrt{a_0 a_2}$)), the PI controller parameters

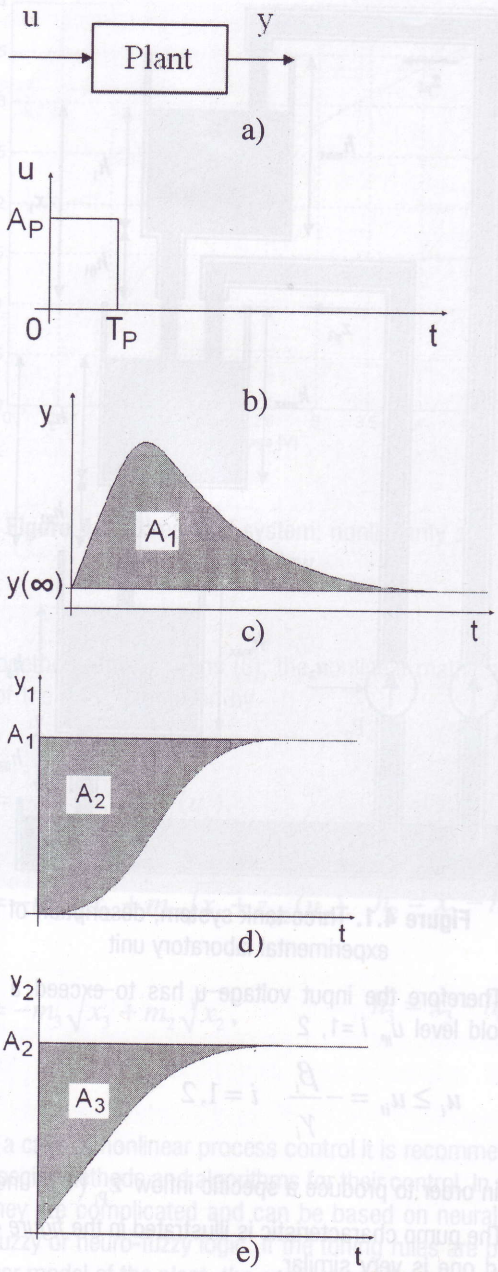

Figure 3.1 Illustration of input signal b) and areas: c) – A_1, d) – A_2 and e) – A_3

has been received by (7).

$$K_C = \frac{2.a_0^2.a_2}{\left(a_1 + \sqrt{\left|a_1^2 - 4.a_0.a_2\right|}\right)^2};$$

$$T_i = \frac{2.a_2}{a_1 + \sqrt{\left|a_1^2 - 4.a_0.a_2\right|}} \qquad (7)$$

This adaptive tuning algorithm calculates the parameters of inner controller first and after that the parameters of the outer controller. The adaptive tuning procedure for every one controller includes the same steps as the procedure for first controller that is realized as follows, *figure 3.2*:

1. The pulse signal with amplitude A_P of the plant input is applied.
2. The end of the pulse is determined when the process output signal reaches a certain predetermined deviation from the steady-state value, if T_P has not been set in step 1.
3. The parameters A_1, A_2 and A_3 are calculated by expressions (4) from the pulse transfer function.
4. The parameters a_2, a_1 and a_0 are calculated by expressions (3).
5. $a_1 \geq 2\sqrt{a_0 a_2}$.
6. If a step 5 is true, than the controller parameters K_C and T_i are calculated by expressions (6).
7. If step 5 is false, than the controller parameters K_C and T_i are calculated by expressions (7).

For calculation of the outer controller parameters, a tuned inner controller is use. In this case the input signal will be the reference signal for inner loop. The proposed algorithm is applicable for self-control plants with time delay, which could be

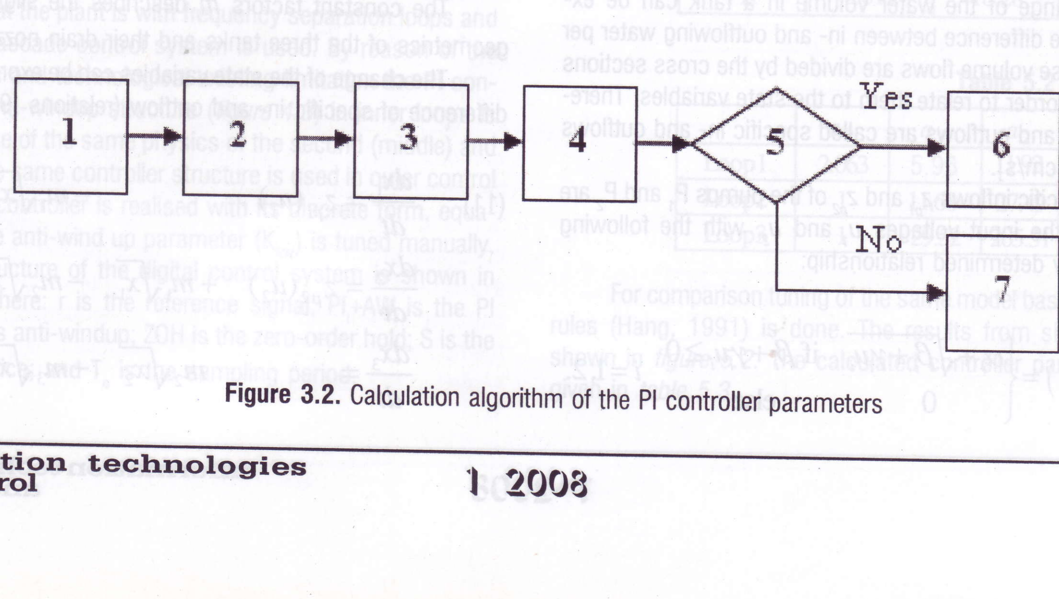

Figure 3.2. Calculation algorithm of the PI controller parameters

approximated by transfer function of a second order system plus dead time.

4. Example

The system under consideration consists of three cuboidal water tanks that are arranged one atop the other, where the most upper one can be filled with water through pump P_1 and the middle one with water from pump P_2. Both pumps use the water from the reservoir that is located under the third tank. Each water tank has got drain nozzle that lets the water flow into the tank underneath or into the reservoir, see *figure 4.1.* The pumps are driven by input voltages u_1 and u_2. The voltage is limited from 0 V to 5 V. The water level (h_i) of each tank is limited to 32 cm.

•Modelling Issues

The pumps P_1 and P_2 are controlled by the input voltages u_1 and u_2 [V]. The resulting inflows of the tanks (divided by the cross section of the tanks) are labelled with z_{p1} and z_{p2} in *figure 4.1.* The output variables of the system are the three fill levels h_1, h_2 and h_3 [cm].

$$h^T = [h_1 \quad h_2 \quad h_3].$$

They can be measured by pressure sensors on the bottom of each tank. The state variables x_1, x_2 and x_3 are defined as water heights measured from the edges of the drain nozzles.

$$x^T = [x_1 \quad x_2 \quad x_3].$$

The heights of the drain nozzles are defined as h_{01}, h_{02} and h_{03} [cm], so there exist the following relations between state variables and output variables:

$$(8) \quad \begin{aligned} x_1 &= h_{01} + h_1; \\ x_2 &= h_{02} + h_2; \\ x_3 &= h_{03} + h_3. \end{aligned}$$

For all further experiments, the first pump will be used to control the system while the second pump is used to apply disturbances to the system.

•Nonlinear Model

The change of the water volume in a tank can be expressed as the difference between in- and outflowing water per time unit. These volume flows are divided by the cross sections of the tank in order to relate them to the state variables. Therefore these in- and outflows are called specific in- and outflows with the unit [cm/s].

The specific inflows z_{p1} and z_{p2} of the pumps P_1 and P_2 are functions of the input voltages u_1 and u_2, with the following experimentally determined relationship:

$$(9) \quad z_{p_i}(u_i) = \begin{cases} \alpha_i + \sqrt{\beta_i + \gamma_i u_i} & \text{if } \beta_i + \gamma_i u_i \geq 0 \\ 0 & \text{else} \end{cases} \qquad i = 1,2.$$

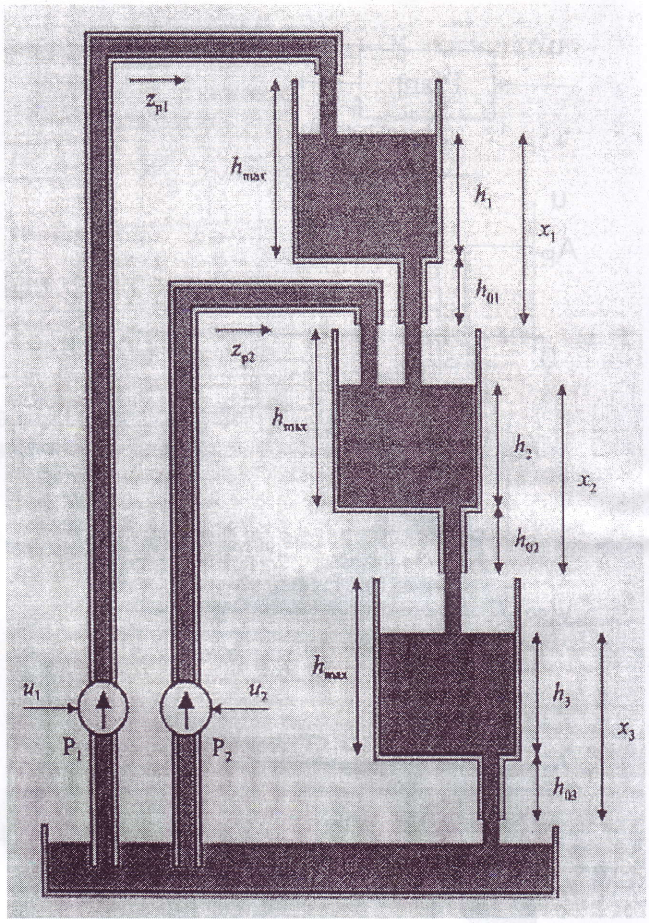

Figure 4.1. Three tank system, description of the experimental laboratory unit

Therefore the input voltage u_i has to exceed a certain threshold level u_{it}, $i = 1, 2$

$$u_i \geq u_{it} = -\frac{\beta_i}{\gamma_i} \qquad i = 1,2$$

in order to produce a specific inflow $z_{p_i}(u_i)$ unequal to zero. The pump characteristic is illustrated in the *figure 4.2*, the second one is very similar.

The outflows caused by the drain nozzles can be described approximately with the relations

$$(10) \quad m_i\sqrt{x_i} \qquad i = 1,2,3.$$

The constant factors m_i describes the slightly different geometrics of the three tanks and their drain nozzles.

The change of the state variables can be expressed as the difference of specific in- and outflow (relations (9) and (10)):

$$(11) \quad \begin{aligned} \frac{dx_1}{dt} &= z_{pi}(u_1) & & - m_1\sqrt{x_1}; \\ \frac{dx_2}{dt} &= z_{p2}(u_2) & + m_1\sqrt{x_1} & - m_2\sqrt{x_2}; \\ \frac{dx_3}{dt} &= & m_2\sqrt{x_2} & - m_3\sqrt{x_3}. \end{aligned}$$

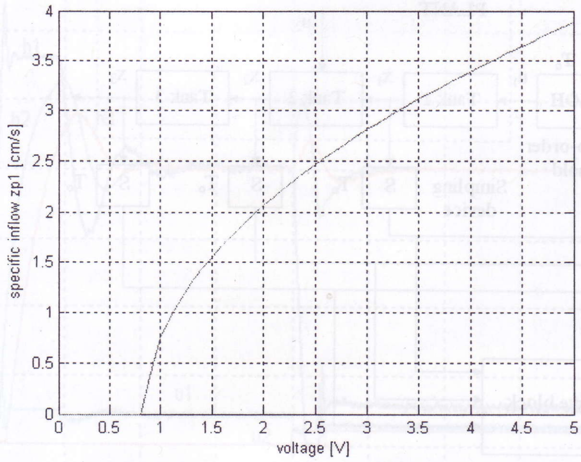

Figure 4.2. Three tank system, nonlinearity and the resulting inflow

Together with equations (8), the nonlinear mathematical model of the system is given by

$$\frac{dx_1}{dt} = -m_1\sqrt{x_1} + z_{pi}(u_1), \qquad h_1 = x_1 - h_{01}$$

$$\frac{dx_2}{dt} = -m_2\sqrt{x_2} + m_1\sqrt{x_1} + z_{p2}(u_2), \quad h_2 = x_2 - h_{02}$$

$$\frac{dx_3}{dt} = -m_3\sqrt{x_3} + m_2\sqrt{x_2}, \qquad h_3 = x_3 - h_{03}$$

In a case of nonlinear process control it is recommended to use special methods and algorithms for their control. In most cases they are complicated and can be based on neural networks, fuzzy or neuro-fuzzy logic. If the tuning rules are based on a linear model of the plant, the application of such methods to nonlinear plants may require retuning of the controller every time the set point of the control system is changed.

In this paper, the adaptive tuning method designed for linear systems is applied for control of a nonlinear plant. It is obviously, that the plant is with frequency separation loops and there for a cascade control system is used. By reason of the plant specific and technological existing limitation the PI controller plus anti-windup structure (*figure 4.3*) in inner loops is used. Because of the same physics of the second (middle) and third tank, the same controller structure is used in outer control loop. The PI controller is realised with its discrete form, equation (12). The anti-wind up parameter (K_{AW}) is tuned manually. The used structure of the digital control system is shown in *figure 4.4*, where: r is the reference signal; PI+AW is the PI controller plus anti-windup; ZOH is the zero-order hold; S is the sampling device; and T_0 is the sampling period.

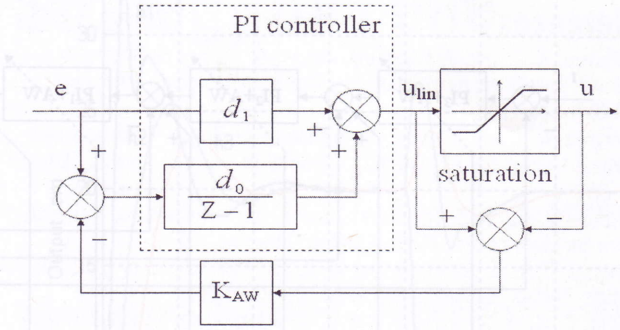

Figure 4.3. PI controller plus anti-windup structure.
Where: e is the controller input signal, u_{lin} is the linear output signal, u is the limited output signal, d_0 and d_1 is controller parameters

$$R(z) = \frac{d_1 z + d_0}{z + 1}; \qquad (12)$$

$$d_1 = K_C \ , \ d_0 = K_C\left(\frac{T_0}{T_i} - 1\right).$$

5. Results

- **Simulations**

For simulation the non-linear model of the three tank system is used. The control output is the level of the third water tank (h_3). The reference value is 20 cm. When the system is in steady state a disturbance is applied to the plant. The pump voltage (u_2) is changed from 0 V to 0.95 V. The simulation results from adaptive tuning with the method proposed in this paper, is shown in *figure 5.1*. The calculated controller parameters and calculated plant parameters for the controller tuning procedure are given in *table 5.1* and *table 5.2*, respectively.

Table 5.1

	Kc	Ti, s
PI1	0.5265	1.718
PI2	1.287	8.412
PI3	0.813	18.28

Table 5.2

	a_0	a_1	a_2
Loop1	2.063	5.96	11.92
Loop2	1	11.68	27.49
Loop3	1	29.52	205.51

For comparison tuning of the same model based on Hang's rules (Hang, 1991) is done. The results from simulation are shown in *figure 5.2*. The calculated controller parameters are given in *table 5.3*.

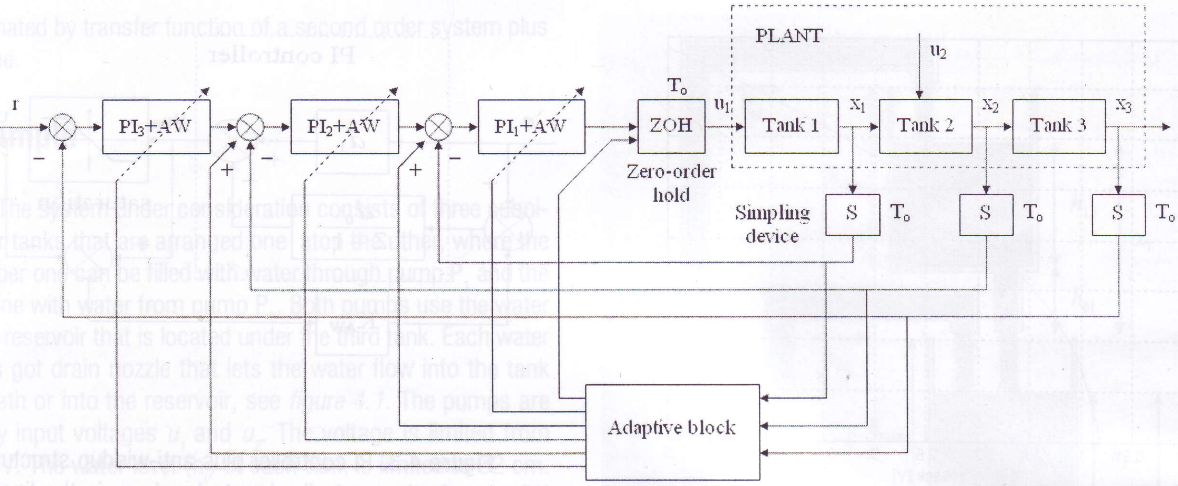

Figure 4.4. The control system structure

Table 5.3

	Kc	TI, s
PI1	0.17	3.32
PI2	0.849	16
PI3	0.71	27.79

In simulation as it can be seen from *figure 5.1* and *figure 5.2*, using the proposed method leads to smaller damping of the process (7%) and faster response of the controlled signal (settling time: $\chi_{st} = 160s$) than Hang's method (12% and $\chi_{st} = 250s$).

•Laboratory Experiment in Real Time

For the real experiment WinCon Server is used, (WinCon 3.2). The control output is the level of the third water tank (h_3). A constant reference height of 20 cm is chosen. A disturbance is activated, when the steady state in the reference tracking is reached. The pump voltage (u_2) is changed from 0 V to 0.5 V. The results from adaptive tuning with the method proposed in this paper are shown in *figure 5.3*. The calculated controller parameters and calculated parameters from controller tuning procedure are given in *table 5.4* and *table 5.5*, respectively.

Table 5.4

	Kc	TI, s
PI1	0.3972	1.913
PI2	1.276	9.131
PI3	0.7711	22.11

Table 5.5

	a_0	a_1	a_2
Loop1	1.58	5.597	11.52
Loop2	1	12.71	32.67
Loop3	1	36.45	317

Results from tuning of the three tank system having used Hang's rules presented in (Hang, 1991), are shown in *figure 5.4*. The calculated controller parameters are given in *table 5.6*.

Table 5.6

	Kc	TI, s
PI1	0.17	3.32
PI2	0.849	16
PI3	0.71	27.79

Comparing *figure 5.3* and *figure 5.4* it becomes obvious, that the method of this paper gives better results than Hang's method. It leads to small damping of the process, 8%, and faster settling, $\chi_{st} = 200s$, to the desired reference values (Hang's method gives damping of the process 10%, and settling – $\chi_{st} = 300s$).

6. Conclusions

In this paper, a new method for online adaptive tuning of PI controllers is proposed. Additional plant information is not necessary for this method. All necessary information for calculation of the controller parameters is received directly through information derived from the pulse response of the plant. In order to demonstrate the performance of the adaptive tuning method that has been designed for linear systems an example with control of a three tank system (which has feebly nonlinear behaviour) is used. As it can be seen from the simulation (*figure 5.1*) and from the given real time experiment (*figure 5.3*), the controlled process is convergent. The simulation and laboratory experiment in real time confirm the application and working capacity of the proposed method. A comparison with Hang's method shows the benefits of the adaptive tuning method presented in this paper.

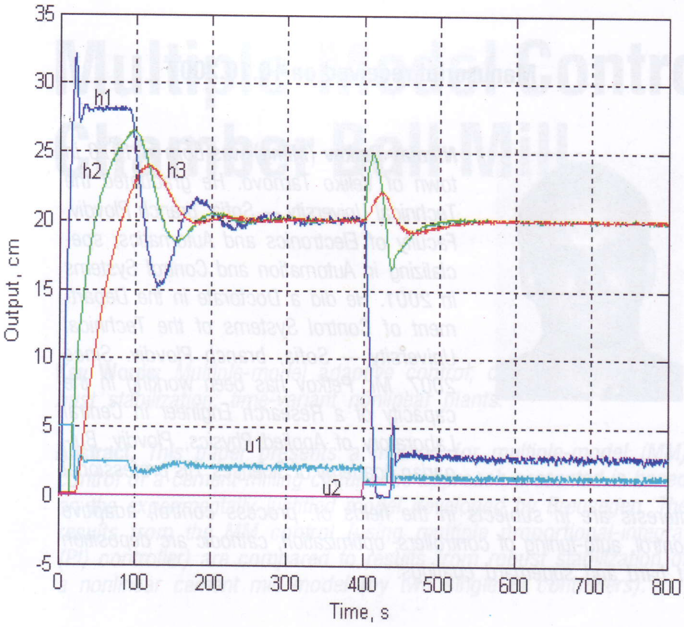

Figure 5.1. Simulation results from proposed adaptive tuning procedure in this paper

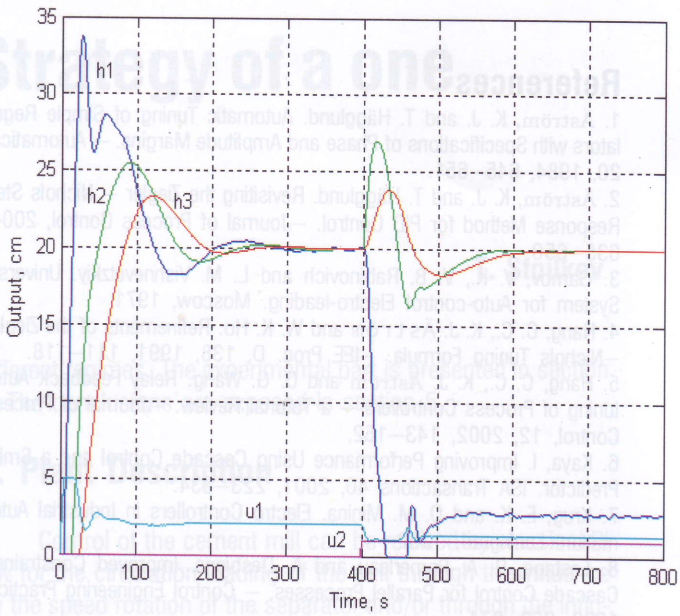

Figure 5.2. The simulation results obtained using (Hang, 1991) tuning method

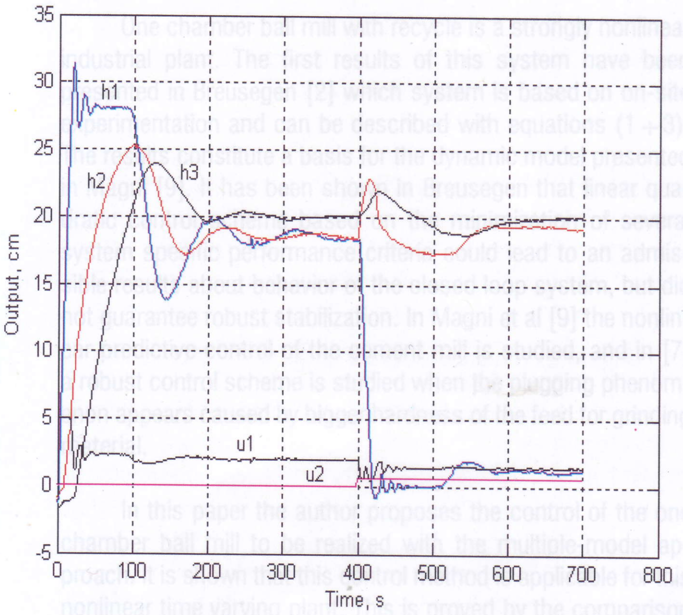

Figure 5.3. Real time experiment results obtained by proposed adaptive tuning procedure in this paper

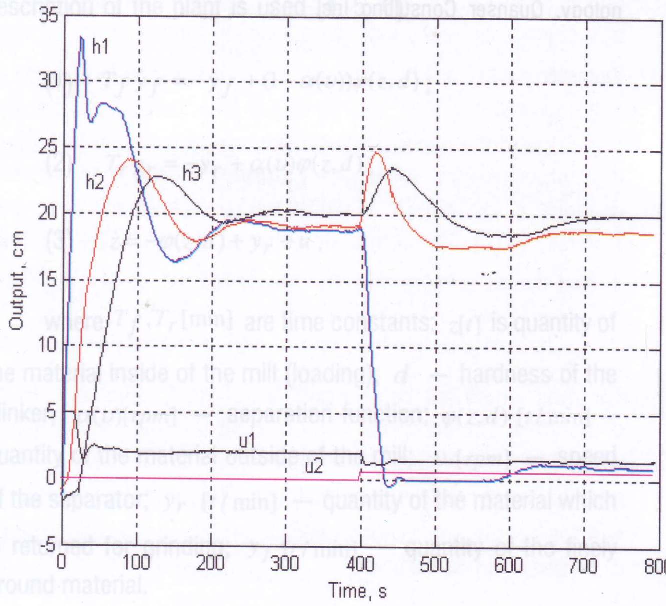

Figure 5.4. The real time experiment results obtained using (Hang, 1991) tuning method

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