Nonlinear Path Following Control for a Bi-steerable Vehicle

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Abstract. *This paper proposes a nonlinear feedback path control law for a bi-steerable vehicle (a four-wheel-steering vehicle designed to steer the rear wheels always in opposite direction to the front ones* in function of the front steering angle*). First, a kinematic model of the vehicle in error coordinates expressed in a moving reference frame, which is partially linked to the vehicle is developed. The control law is constructed using a backstepping recursive design technique yielding exponential stability of the closed-loop system in error coordinates and invariant properties with respect to the vehicle speed. Simulation results illustrate the effectiveness of the proposed controller.*

1. Introduction

In recent years, there has been considerable effort in the development of tracking controllers for automated vehicles with several conventional steering wheels. As the control of nonholonomic wheeled mobile robots (WMRs), in most cases, the control problem is stated for WMRs with two *independently* steerable wheels. In this case, the motivation of this problem is reflected in the fact that the WMR in the plane possesses three degrees of freedom of motion and should be able to have any desired orientation angle along the path (the WMRs can be controlled to follow a path with independent orientation). In [1], two trajectory tracking control methods for a wheeled mobile robot with two steering wheels based on a linearizing feedback approach and Lyapunov oriented control design were proposed. Transformation of the kinematic model into two-chain single generator chained form was also given. The admissible wheels configurations were investigated to prevent the pure rolling and non-slipping conditions in the case of more than two steering wheels. In [3], feedback control of a mobile robot vehicle with two independently steering wheels has been studied. It has been shown that the input-output linearization cannot be achieved by any static feedback. Dynamic feedback has been used for trajectory tracking in order to independently control the position and orientation of the robot vehicle.

Regarding passenger vehicles, four-wheel-steering (4WS) systems which are able to control way rotation and lateral motion independently by controlling the steering angle of both front and rear wheels have been designed for many years [4,5]. In [6], control algorithms for parallel steering maneuver (crabbing) and following a path with desired orientation for a four-wheel-steering vehicle with independent steerable wheels were presented.

Recently, there has been increasing interest in designing a new type automatic car for public individual transport, the so called bi-steerable vehicle [7,8]. The particularity of the steering system of this vehicle is that the front and rear wheels are steered always in opposite direction in order to offer better vehicle maneuverability. The rear wheels are *not independently* steered from the front ones and as a consequence, the vehicle has only two degrees of freedom in the plane as a conventional front-wheel steering car (FWS). However, comparing the kinematic models of these vehicles, different complexity arises from the bi-steerable car, in particular, when trying to determine a flat (linearizing) output and convert the kinematic model into chained form. Although the kinematic model of the vehicle is flat [9], and can be converted into chained form, to apply feedback control based on the chained form representation of the system, we must overcome the problem of finding functions that generate a chained set of coordinates for this kind of vehicle. In this case, to solve the control problem for a bi-steerable vehicle, an alternative which becomes attractive, is a control scheme based on a reduced vehicle model.

In this paper, we present a nonlinear path following controller for a bi-steerable vehicle. The proposed control law is constructed using a backstepping design technique [10] and is based on the reduced-order model of the system. Exponential stability of the closed-loop subsystem for the vehicle lateral and orientation errors is achieved and, at the same time, the system is invariant with respect to the vehicle velocity. We prove that the internal dynamics, associated with a part of the system which has not be taken into account in the feedback control design, is locally exponentially stable.

The paper is organized as follows: In Section 2, the kinematic model of the vehicle is presented. In Section 3, we state the path following problem using error coordinates expressed in a moving reference frame partially linked to the vehicle. In Section 4, the design of the proposed controller and stability analysis are given. Simulation results are presented in Section 5. Section 6 contains some conclusions.

2. Vehicle Model

A plan view of the vehicle considered in this paper, is shown in *figure 1*.

The bi-steerable vehicle has four steering and driving wheels. The wheels are assumed to roll without lateral sliding. The four wheel steering system has the ability to steer the rear wheels always in opposite direction to the front ones with a rear-to-front steering angle ratio n, ($0<n\leq 1$). To simplify the derivation of the vehicle kinematic model, we consider the so called „two-wheel vehicle model“ composed of two virtual wheels placed

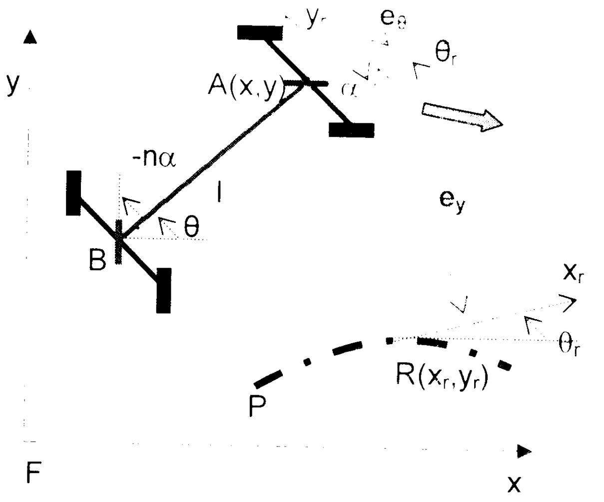

Figure 1. A plan view of a bi-steerable vehicle

at the mid-points of the front and rear wheel axles (points A and B, respectively), and oriented in the direction to the wheels. The coordinates of a reference point A placed at the center of the front vehicle axle, with respect to an inertial frame Fxy, are denoted by (x,y). The angle θ is the orientation angle of the vehicle with respect to the frame Fxy. The angle α is the steering angle of the front virtual wheel measured with respect to the vehicle body. The length of the vehicle is denoted by l. Using the coordinates of the reference point A, the configuration of the vehicle is described by four generalized coordinates, $q=[x, y, \theta, \alpha]^T$. The nonholonomic constraints can be written in the form

$$A(q)\dot{q} = 0 \tag{1}$$

where $A(q)$ is a $2x4$ full rank matrix of the form

$$A(q) = \begin{bmatrix} -\sin(\theta+\alpha) & \cos(\theta+\alpha) & 0 & 0 \\ -\sin(\theta-n\alpha) & \cos(\theta-n\alpha) & l\cos(n\alpha) & 0 \end{bmatrix} \tag{2}$$

and $\dot{q}$ is the vector of generalized velocities.

The constraint equation (1) can be converted into an affine driftless control system

$$\dot{q} = B(q)\eta \tag{3}$$

where the columns of the 4x2 matrix $B(q)$

$$B(q) = \begin{bmatrix} \cos(\theta+\alpha) & 0 \\ \sin(\theta+\alpha) & 0 \\ \dfrac{\sin[(n-1)\alpha]}{\cos(n\alpha)} & 0 \\ 0 & 1 \end{bmatrix} \tag{4}$$

form a basis of the null space of $A(q)$. The control input $\eta=[v_A, \omega_\alpha]^T$ is a $2x1$ vector of independent quisi-velocities which parameterizes the degree of freedom of the system, where v_A is the velocity of point A (the mid-point of the front virtual wheel) and ω_α is the steering angular velocity of the front virtual wheel.

3. Problem Formulation

The path following geometry used in this paper is represented in *figure 1*. Consider a bi-steerable vehicle moving on a flat surface. We assume that the path P is a smooth planar curve. A reference coordinate frame Rxy, $(R(x_r, y_r))$, is defined such that the Rx axis is tangent to the path and oriented in the direction of motion to follow, and the Ry axis passes through the reference point A of the vehicle (a reference frame partially linked to the vehicle). We suppose that the distance between the points A and R is smaller than the reference curvature radius ρ_r in point R and, in that way, ensuring that the reference frame Rxy is uniquely defined (see [11]).

We introduce a new variable $\theta_\alpha \overset{\text{def}}{=} \theta + \alpha$. Using the reference frame Rxy, the error coordinates of the vehicle $e = [e_x, e_y, e_\theta]^T$, i.e., the position and orientation of the front vehicle wheel with respect to the moving reference frame Rxy are given by (see also [12]).

$$\begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r & 0 \\ -\sin\theta_r & \cos\theta_r & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - x_r \\ y - y_r \\ \theta_\alpha - \theta_r \end{bmatrix} \tag{5}$$

where $e_x(t) \equiv 0$ (in this paper, we are interested in designing a path tracking (lateral) controller for the vehicle).

Differentiating the equality (5) and using (3), in conformity with the nonholonomic constraints (1), after simple calculations, we obtain

$$\begin{aligned} 0 &= -v_r + v_A \cos e_\theta + v_r c_r e_y \\ \dot{e}_y &= v_A \sin e_\theta \\ \dot{e}_\theta &= v_A \frac{\sin[n(\alpha+1)]}{l\cos(n\alpha)} - v_A \frac{c_r \cos e_\theta}{1 - c_r e_y} + \omega_\alpha \end{aligned} \tag{6}$$

where $c_r = 1/\rho_r$ is the curvature of the reference path P at the point R, and v_r is the velocity of point R. We assume that the error coordinates (e_x, e_y) as well as the front-wheel steering angle α are measured.

The curvilinear coordinate s_r along the reference path can be determined from the first equation of (6) in the form

$$\dot{s}_r = v_r = v_A(\cos e_\theta)/(1 - c_r e_y). \tag{7}$$

Combining eq. (7), the last two equations of (6) and the last equation of (3), we obtain a kinematic model of the vehicle in error coordinates, (e_y, e_θ), in the form

$$\begin{aligned}
\dot{s}_r &= v_r \\
\dot{e}_y &= v_r(1 - c_r e_y)\tan e_\theta \\
\dot{e}_\theta &= v_r\left[\frac{1 - c_r e_y}{\cos e_\theta}\frac{\sin[(n+1)\alpha]}{l\cos(n\alpha)} - c_r\right] + \omega_\alpha \\
\dot{\alpha} &= \omega_\alpha
\end{aligned} \tag{8}$$

We assume that the velocity $v_A(t)$ of the vehicle is strictly positive, bounded, continuous, that does not converge to zero, and the orientation error $|e_\theta| \in [0, \pi/2)$. Also, we will assume that the reference path is a circle ($c_r = cte$) or a straight line ($c_r = 0$). In this case, using the parameterization (e_y, e_θ) and given a path P, the path following problem consists of finding a feedback control law for the subsystem composed of the last three equations of (8) with control input ω_α, such that the state vector $[e_y, e_\theta, \alpha]^T$ tends to $[0, 0, \alpha_r]^T$, as $t \to \infty$ where $\alpha_r = \alpha(c_r) = cte$.

4. The Controller

4.1. Feedback Control Design

In this section, we present a nonlinear path following controller for a bi-steerable vehicle described by the last three equations of (8). The reference velocity $v_r(t)$ could be considered as a function of time and, from (7), it follows that it is strictly positive for $|e_\theta| \in [0, \pi/2)$ and $|e_y| \leq |\rho_r|$. To obtain a time-invariant system, the differentiation with respect to time is replaced by differentiation with respect to s_r, ($ds_r = v_r dt$), where s_r is the reference path length drawn by point R of the reference coordinate frame Rxy (*figure 1*). In that way, we express the vehicle equations of motion in terms of s_r and we denote the derivation with respect to s_r by " ′ ". Using s_r as an independent variable instead of the time-index t, the last three equations of (8) can be written in the form

$$\begin{bmatrix} e'_y \\ e'_\theta \\ \alpha' \end{bmatrix} = \begin{bmatrix} (1 - c_r e_y)\tan e_\theta \\ \dfrac{1 - c_r e_y}{\cos e_\theta}\dfrac{\sin[(n+1)\alpha]}{l\cos(n\alpha)} - c_r \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u \tag{9}$$

where $u = \omega_\alpha / v_r$

Remark 1. The path following problem stated in Section 3, can be formulated in terms of s_r, namely, the problem consists of finding feedback control law for the subsystem composed of the last three equations of (8) with control input ω_α, such that

$$\lim_{s_r \to \infty} e_y(s_r) = 0; \ \lim_{s_r \to \infty} e_\theta(s_r) = 0; \ \lim_{s_r \to \infty} \alpha(s_r) = \alpha_r$$

The design strategy used in this paper consists in constructing a stabilizing control u for the subsystem composed of the first and second equations of (9). Assuming that α is bounded (exponential stability of α to the equilibrium state α_r will be established) and measured, we introduce α into the stabilizing control u. Applying the designed control law to the system (9), we prove that the dynamics of α, (the third equation of (9)), which has not been taken into account in the feedback control design, converges exponentially to its equilibrium state α_r. The design procedure is based on a backstepping recursive design methodology, which yields exponential convergence of e_y and e_θ to zero.

First, we present the design of the stabilizing control u for the subsystem composed of the first and second equations of (9). In our case, the beckstepping procedure is completed at the second step by finding a control law which makes the derivative of the constructed Lyapunov function negative definite.

Step 1. We form the function

$$V_1 = \frac{1}{2}e_y^2. \tag{10}$$

Using an intermediate virtual control η_1

$$\eta_1 := \tan e_\theta = -\frac{k_1 e_y}{1 - c_r e_y}, \quad k_1 > 0 \tag{11}$$

we obtain for the derivative of (10)

$$V_1' = -k_1 e_y^2 < 0. \tag{12}$$

Step 2. Consider the augmented function

$$V_2 = V_1 + \frac{1}{2}(\tan e_\theta - \eta_1)^2. \tag{13}$$

Choosing the control u in the form, ($k_2 > 0$)

$$\begin{aligned}
u = c_r &- \frac{1 - c_r e_y}{\cos e_\theta}\frac{\sin[(n+1)\alpha]}{l\cos(n\alpha)} - \frac{k_1 \tan e_\theta}{1 - c_r e_y}\cos^2 e_\theta \\
&- e_y(1 - c_r e_y)\cos^2 e_\theta - k_2(\tan e_\theta - \eta_1)\cos^2 e_\theta
\end{aligned} \tag{14}$$

the derivative of (13) results in

$$V_2' = -k_1 e_y^2 - k_2\left(\tan e_\theta + \frac{k_1 e_y}{1 - c_r e_y}\right)^2 < 0. \tag{15}$$

The control law (15) yields the following closed-loop system for the subsystem composed of the first and second equations of (9)

$$\begin{aligned}
e'_y &= (1 - c_r e_y)\tan e_\theta \\
e'_\theta &= -\frac{k_1 \tan e_\theta}{1 - c_r e_y}\cos^2 e_\theta - e_y(1 - c_r e_y)\cos^2 e_\theta \\
&\quad - k_2\left(\tan e_\theta + \frac{k_1 e_y}{1 - c_r e_y}\right)\cos^2 e_\theta.
\end{aligned} \tag{16}$$

4.2. Stability Analysis

From (13) and (15), using the Lyapunov stability theory [13, Theorem 3.1, p. 101], it follows that the origin (0,0) is an asymptotically stable equilibrium point for the closed-loop system (16) obtained by applying the control law (14) to the subsystem composed from the first and second equations of (9). Furthermore, exponential stability is also achieved. Indeed, using (10), (13) and (15), and choosing $m = 2k_2$ and $k_1 \geq k_2$, the following inequality holds

$$V_2' + mV_2 \leq 0. \tag{17}$$

Application of Convergence Lemma [14, p. 91] indicates the exponential convergence for V_2 to zero, i.e.,

$$V_2(s_r) \leq V_2(0)e^{-ms_r}, \tag{18}$$

and this in turn implies that e_y and e_θ converge to zero exponentially.

Since the dynamics of α has not been taken into account in the feedback control design, the effectiveness of the proposed controller based on the reduced-order model depends on the dynamics of α. Our next step in the stability analysis is to establish exponential convergence of α to the equilibrium state α_r, (α_r is obtained from the third equation of (9) and equation (14), setting $\alpha' = 0$ and solving the corresponding trigonometric equation with respect to $\alpha = \alpha_r$ for $e_y = e_\theta \equiv 0$). To study the stability of the internal dynamics of the closed-loop system (9)-(14), we analyze the zero dynamics of α. Assuming that $e_y(s_r) = e_\theta(s_r) \equiv 0$, $\alpha(0) \neq 0$, and substituting u from (14) in the third equation of (9), we obtain the following equation for the zero dynamics of the system

$$\begin{aligned}\alpha' &= -\frac{\sin[(n+1)\alpha]}{l\cos(n\alpha)} + c_r \\ &:= f(\alpha).\end{aligned} \tag{19}$$

From the geometrical argument, one can show that

$$c_r = \frac{\sin[(n+1)\alpha_r]}{l\cos(n\alpha_r)}. \tag{20}$$

To establish exponential stability of (19) to α_r, we use the Lyapunov linearization method, ([13], Theorem 4.4., p.179). Let α_r be an equilibrium point for equation (19). Expanding the right site of (19) into a Taylor series about α_r and using (20), we obtain

$$\beta' = -f(\alpha_r)\beta \tag{21}$$

where $\beta \overset{\text{def}}{=} \alpha - \alpha_r$,

and

$$\begin{aligned}f(\alpha_r) &= \left.\frac{\partial f}{\partial \alpha}\right|_{\alpha=\alpha_r} \\ &= -\frac{n\cos\alpha_r + \cos[(n+1)\alpha_r]\cos(n\alpha_r)}{l\cos^2(n\alpha_r)}\end{aligned} \tag{22}$$

is the derivative of $f(\alpha)$ with respect to α for $\alpha = \alpha_r$. For simplicity, we assume that n is a constant ($0 \leq n < 1$). Choosing $n = 0.69$, (this value is adopted from the rear-to-front angle ratio of the CYCAB vehicle[7]), it can be shown that $f(\alpha_r) < 0$ for $|\alpha_r| \leq 1.15rad$ and the exponential stability at the origin of the linear equation (21) follows readily. Since the linearized equation (21) is exponentially stable, applying the aforementioned theorem, we can conclude that the nonlinear equation (19) is also locally exponentially stable in the neighborhood of α_r.

Remark 2: It should be noted that the bound of $1.15rad$ for $|\alpha_r|$ (corresponding to the reference path with minimal curvature radius), is much larger than the maximal admissible value for the front steering angle of the CYCAB vehicle, which is $|\alpha_{max}| \leq 0.4rad$ [6]. From a practical point of view, $|\alpha_r|$ should be smaller than $0.4rad$ and inequality $f(\alpha_r) < 0$ is always satisfied for a bi-steerable vehicle with $n = 0.69$.

5. Simulation Results

Simulation results were performed to illustrate the effectiveness of the proposed controller. The algorithm developed in Section 4 was implemented in MATLAB. A circular reference path with radius $\rho_r = 5m$ was chosen for the simulations. The parameters were chosen to be: the base length of the vehicle $l = 2m$; $n = 0.69$; the control parameters $k_1 = 3$, $k_2 = 0.2$. Simulation results of the planar vehicle path in the x-y plane with initial conditions $e_y(0) = 1m$, $e_\theta(0) = 0rad$ and $\alpha(0) = 0$ are shown in figure 2. The evolution of the error coordinates e_y and e_θ with respect to the reference path length s_r is presented in figure 3. The evolution of the front and rear steering angles α and $-n\alpha$ with respect to the reference path length s_r is presented in figure 4.

When the vehicle is traveling along a circle of radius ρ_r (in this case, the instantaneous center of rotation of the vehicle body coincides with the center of the reference path), a circle of minimal radius ρ_{rmin}, ($\rho_{rmin} = \rho_r \cos\alpha_r$), is drawn by a point which is placed at the vehicle longitudinal axis between points A and B. This point can be seen as a point which cuts corner during the turning maneuver. For $\rho_r = 5m$ (the magnitude of the reference radius which was used for simulation), one obtains $\rho_{rmin} = 4.86m$ which is quite acceptable from a practical view point.

6. Conclusions and Future Work

In this paper, a nonlinear path controller for a bi-steerable vehicle has been presented. Exponential convergence to zero of vehicle lateral and orientation errors with respect to the reference path has been achieved. It has been shown that the internal dynamics, associated with a part of the system which has not

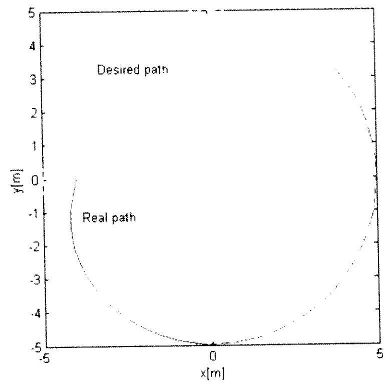

Figure 2. Following a circular path. The path drown by the vehicle guide point A in the x-y plane (solid red line), and the desired path (dashed green line). Initial conditions $e_y(0) = 1m$, $e_\theta(0) = 0\ rad$ and $\alpha(0) = 0\ rad$

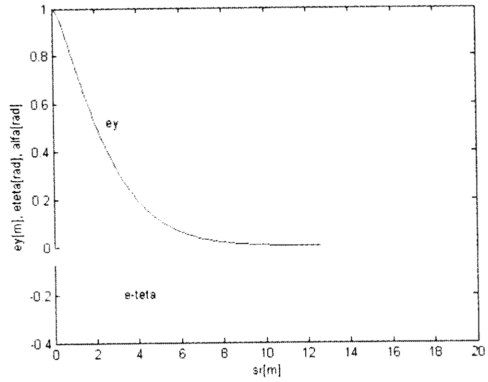

Figure 3. Following a circular path. Evolution of the error coordinates e_y and e_θ of system (9) with control law (14) with respect to the reference path length s_r. Initial conditions $e_y(0) =- 1m$, $e_\theta(0) = -\ 0.1rad$ and $\alpha(0) = 0$

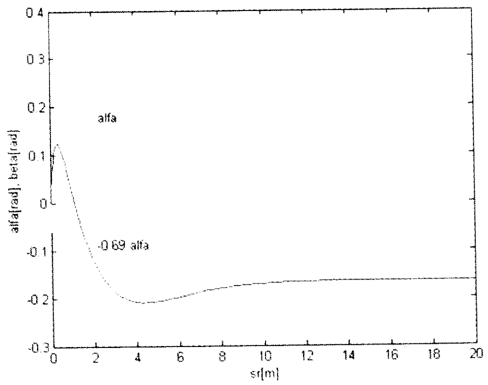

Figure 4. Following a circular path. Evolution of the front and rear steering angles α (green line) and $-n\alpha$ (red line, n = 0.69), respectively. Initial conditions $e_y(0) = -1m$, $e_\theta(0) = -\ 0.1\ rad$ and $\alpha(0) = 0$

be taken into account in the feedback control design, is locally exponentially stable. The results have provided an efficient and systematic approach to design a non-time based path controller for a bi-steerable vehicle yielding invariant properties with respect to the vehicle speed. Simulations confirmed the validity of the analysis and controller design.

Our future work will address the problems associated with the dynamical extension of the proposed controller in the presence of uncertainty in the dynamic model of the vehicle.

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References

1. Micaelli, A., C. Samson. Trajectory Tracking for Two-steering whees Mobile Robots. Proceedings of the 4th IFAC Symp. Robot Control (SYROCO), 1994, 249-256.
2. Thuilot, B., B. d'Andea-Novel, A. Micaelli. Modeling and Feedback Control of Mobile Robots Equipped with Several Steering Wheels.- IEEE *Transactions on Robotics and Automation*, No3, June 1996, 375-390
3. Yun, X., N. Sarkar. Dynamic Feedback Control of Vehicle with Two Steerable Wheels.- Proceedings of the IEEE Conference on Robotics and Automation, 1996, 3105-3110.
4. Furukawa, Y., N. Yuhara, S. Sano, H. Takeda, Y. Matsushita. A Review of Four-wheel Steering Studies from the Viewpoint of Vehicle Dynamics and Control.-*Vehicle System Dynamics*, 18, 1989, 151-186.
5. Whitehead, J. Four Wheel Steering: Maneuverability and High Speed Stabilization,. SAE 880642, 4.668-4.679.
6. Petrov, P., M. Parent. Nonlinear Path Control of a Four-Wheel-Steering Vehicle. Proceedings of the 11th International Conference on Advanced Robotics (ICAR'2003), Coimbra, Portugal, 2003, 1302-1307.
7. Baille, G., P. Garnier, H. Mathieu, R. Pissard-Gidolet. Le Sycab de l'INRIA Rhone-Alpes.-*Technical Report* No 0229, April 1999.
8. Parent, M., P. Daviet. Automated Urban Vehicles: towards a Dual Mode PRT (Personal Rapid Transit). Proceedings of the IEEE International Conference on Robotics and Automation, 1996, 3129-3134.
9. Sehavat, S., J. Hermosillo. The Cycab Robot: a Differentially flat System. Proceedings of the International Conference on Intelligent Robots and Systems, 2000, 312-317.
10. Krstic, M., I. Kanellakopoulos and P. Kokotovic. Nonlinear and Adaptive Control Design, John Wiley & Sons, New York, 1995.
11. Samson, C. Control of Chained Systems: Application to Path Following and Time-varying Point-stabilization of Mobile Robots.- *IEEE Transactions on Automatic Control*, 40, No.1, 1995, 64-77.
12. Petrov, P. and N. Shivarov. Path Control of an Autonomous Wheeled Mobile Robot,-*Problems of Engineering Cybernetics and Robotics*, 41, 1994,124-128.
13. Khalil, H. Nonlinear Systems, Macmillan, New York, 1992.
14. Slotine, J.-J., and W. Li. Applied Nonlinear - Control. Prentice Hall, 1991.

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