Combined Adaptive Algorithms for States and Parameters Estimation in Control Systems

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Key Words: *Adaptive control; recursive estimation; adaptive Kalman filter.*

Abstract. *In this paper two adaptive algorithms for the recursive parameters and the state estimation in linear discrete-time systems are proposed. These algorithms combine the standard least square method with the conventional stochastic state observer (Kalman filter). The Kalman filter improves estimates quality, obtained with the least square method, in the sense of their consistency. These algorithms can be part of a stochastic control system with indirect adaptation software. The comparison with another often used in practice estimation algorithms as well as algorithms for adaptive control (with indirect adaptation) based on conventional algorithms are presented.*

I. Introduction

The parameters and thestate estimation is a central problem in the control theory. The parameters estimation is an identification problem. Independently from the control problem, solutions for a wide class dynamic plants exist [1,2,3]. It is well known that by taking into account, the information available a priori it is possible to obtain an optimal parameters' estimation method in sense of optimal model structure, an optimal criteria for estimates' quality and an optimal estimation algorithm [4]. This „optimal" estimation algorithm can be used for comparison with a wide variety of the known particular methods. Unfortunately, the a priori information for plant or surrounding environment, in most of the practical cases is incomplete, uncertain or difficult for use. Due to its simple algorithmic structure and the fact that some of the a priori information is not directly incorporated, the conventional recursive least square method (RLSQ) is still one of the most often used method in practice. In fact the RLSQ method is quite sensitive to the disturbance deviation from the Gaussian distribution, which cause decrease of its performance and eventually can cause its unworkability. With the exception of some particular cases, without any practical significance, the RLSQ method can not ensure consistent parameters' estimate. In case of the closed-loop estimation, all unfavorable effects are intensified. Most of the existing RLSQ method modifications do not provide complete problem solution. Moreover, some of the complicated algorithmic realization of RLSQ modifications leads to some numerical problems which harm their real-time usage.

Instead of traditional approaches which ensure whitening of residual error (Extended least square method), the approach proposed in this paper combines conventional RLSQ method with optimal filtration of the measurement noise. This algorithm has another advantage namely that beside of plant's parameters estimation it also provides state estimates. Later on, those estimates can be directly used for the state regulator design.

II. Combined Adaptive Algorithm

The block diagram of proposed algorithm is presented in *figure 1*[9].

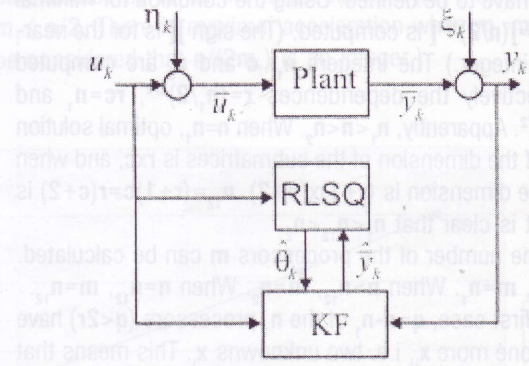

Figure 1. Structure scheme of combined adaptive algorithm

The plant is described with the equation

$$(1)\quad \bar{y}_k = \bar{\varphi}_k^T \theta,$$

where

$$(2)\quad \bar{\varphi}^T = [-\bar{y}_{k-1}\ \ -\bar{y}_{k-2}\ \ \dots\ \ -\bar{y}_{k-n}\ \ \bar{u}_{k-1}\ \ \bar{u}_{k-2}\ \ \dots\ \ \bar{u}_{k-n}];$$

$$\theta^T = [a_1\ \ a_2\ \ \dots\ \ a_n\ \ b_1\ \ b_2\ \ \dots\ \ b_n].$$

$\bar{\varphi}^T$ is a $2n$ dimensional vector composed from observations obtained at time k, $\bar{y}_{k-1}, \bar{y}_{k-2}, \dots \bar{y}_{k-n}$ and $\bar{u}_{k-1}, \bar{u}_{k-2}, \dots, \bar{u}_{k-n}$ are the actual data of the output and input signals respectively, θ is a $2n$ dimensional vector with parameter estimate.

In respect to measurable quantities of the output $y_k = \bar{y}_k + \xi_k$ and the input signal $u_k = \bar{u}_k - \eta_k$, the equation (1) takes form:

$$(3)\quad y_k = \varphi_k^T \theta + \varepsilon_k;$$

$$\varphi^T = [-y_{k-1}\ \ -y_{k-2}\ \ \dots\ \ -y_{k-n}\ \ u_{k-1}\ \ u_{k-2}\ \ \dots\ \ u_{k-n}];$$

$$\varepsilon_k = \xi_k + a_1\xi_{k-1} + a_2\xi_{k-2} + \dots + a_n\xi_{k-n} + b_1\eta_{k-1} + \dots + b_n\eta_{k-n},$$

where ε_k is the residual error, ξ_k is the measurement noise and η_{k-1} is the input noise.

The parameter's vector θ is estimated recursively by the conventional least square method

(4) $\hat{\theta}_{k+1} = \hat{\theta}_k + L_{k+1}(y_{k+1} - \varphi^T_{k+1}\hat{\theta}_k)$;

where L_{k+1} is the RLSQ gain matrix.

In the purposed estimation algorithm, noised outputs y_{k-i}, $i = 1,2,\ldots,n$ are replaced with their estimates $\hat{\bar{y}}_{k-i}$, obtained by the Kalman filter (KF)

(5) $$\hat{\bar{y}}_k = C\hat{x}_k;$$
$$\hat{x}_{k+1} = F\hat{x}_k + Gu_k + K_{k+1}V_{k+1};$$
$$V_{k+1} = y_{k+1} - CGu_k - CF\hat{x}_k$$

where F, G and C are matrixes from appropriate state-space description of equation (3). K_{k+1} is the Kalman filter matrix gain. V_{k+1} is the Kalman's filter innovation process. The matrices F, G and C are replaced by their estimates $\hat{F}, \hat{G}$ and $\hat{C}$. They are obtained by the least square algorithm (4) in which the vector φ_k^T is replaced by

(6) $$\hat{\varphi}_k^T = \left[-\hat{\bar{y}}_{k-1} \;\; -\hat{\bar{y}}_{k-2} \ldots -\hat{\bar{y}}_{k-n} \;\; u_{k-1} \;\; u_{k-2} \ldots u_{k-n}\right]$$

The combined algorithm consists of the following steps:

Step 1. The algorithm is started with an arbitrary initial value of parameters $\hat{\theta}_0$, (such as $\hat{\theta}_0 = 0$).

Step 2. The matrices $\hat{F}_0$, $\hat{G}_0$, $\hat{C}_0$ and $\hat{K}_0$ are formed from the initial value $\hat{\theta}_0$ and the estimate $\hat{y}_k$ is computed according to algorithm (5) (for $k = 0$).

Step 3. The new estimate $\hat{\theta}_1$ is obtained according to equation (4) (for $k = 0$).

Step 4. The algorithm is restarted from step 2, with the estimate $\hat{\theta}_1$ instead of the estimate $\hat{\theta}_0$.

Step 5. Described above steps are repeated until the estimates are stabilized or a priori maximum number of iterations is exceeded.

III. Combined Multiple Model Algorithm

The block diagram of the proposed algorithm is presented on *figure 2*. The multiple model Kalman filter (MMFK) is presented on *figure 3* [5,6].

q particular state estimates $x_k^{(i)}, i = 1,2,\ldots,q$ are computed from q working in parallel Kalman filters. They are designed for q different plant's models, which are obtained for different parameters θ from the parameter's uncertainty domain. The weighted estimate is formed from

(7) $\hat{\bar{y}} = C\hat{x}_k^*$, where the combined plant's state estimate $\hat{x}_k^*$ is obtained from

$$\hat{x}_k^* = \sum_{i=1}^{q} c_i x_k^{(i)},$$

$$\sum_{i=1}^{q} c_i = 1.$$

The weight coefficients c_i are obtained recursively from the minimal variance condition of the combined estimate's innovation process V_k^*. The recursive procedure is similar to the algorithm presented in section II, but the used estimate is computed from equation (7).

IV. Simulation Results

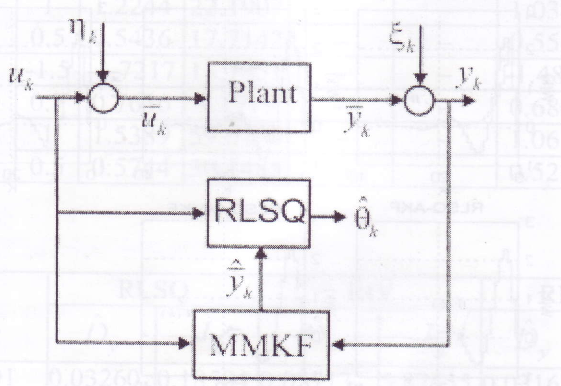

Figure 2. Structure scheme of combine multiple algorithm

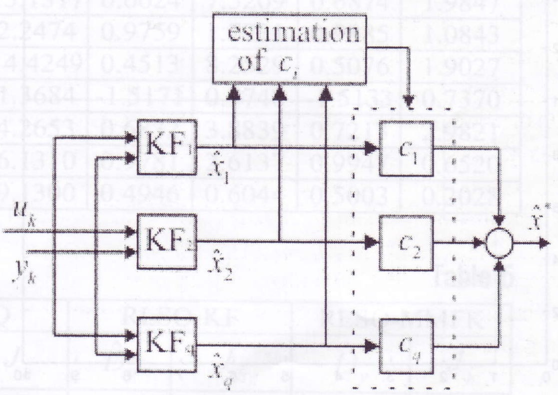

Figure 3. Structure scheme of multiple model KF

The simulation experiments with proposed algorithms for combined parameters and the state estimation are carried out. For the evaluation purpose, a comparison with other most often used in practice methods, such as the conventional recursive least square method (RLSQ), the recursive least square method with an instrumental value (RIV) (instrumental variables are chosen as past values of input signal [8]) and the recursive extended least square method (RELSQ) (with an auto-regressive whitening filter of second order [7]), is performed. For the simulation purpose a second order continuous-time plant is used. It has a time-constant $T = 4s$ and a damping gain $\xi = 0.36$.

The discrete-time model with sample time $T_0 = 2s$ is obtained. The received model has parameters

$$\theta^T = [-1.5 \quad 0.7 \quad 1 \quad 0.5].$$

The input disturbance η_k is a white-gaussian noise with variance $D_\eta = 0.02$. The output is measured in presence of the white-gaussian noise ξ_n with a variance D_ξ. In different experiments the signal to noise ratio varies from 0.02 up to 0.25 (the variance D_ξ varies from 0.0225 up to 3.5625). The simulation experiments are performed for two cases:

a) The parameter estimation in a open-loop system (Fig.1-2) with the measured white-gaussian noise with a variance

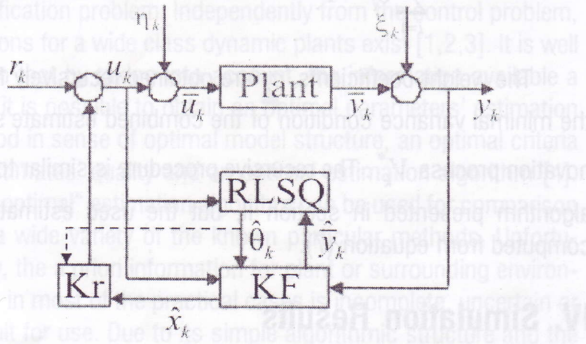

Figure 4. Structure scheme of adaptive control system

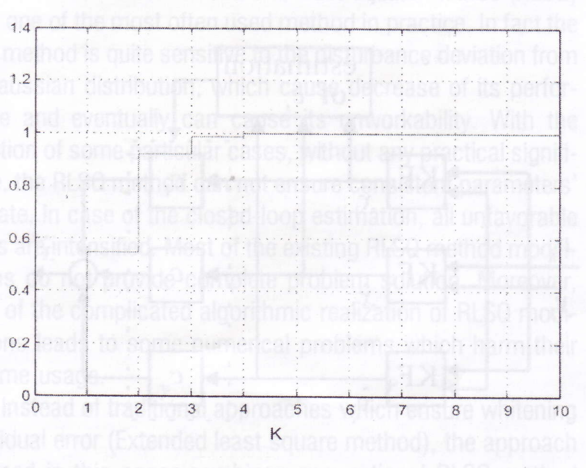

Figure 5. Step-response of system with actual regulator

$D_u = 1$ as an input signal.

6) The parameter and state estimation in a closed-loop control system (*figure 4*). The system with an actual LQG regulator has a step-response shown on *figure 5* .

The multiple model Kalman filter is designed for worse case scenario. It consists of two models $(q = 2)$ which are considerable different from the actual plant model (the first Kalman filter is design for $\xi = 0.25$ and the second is designed for $\xi = 0.55$).

1. Parameters estimation in an open-loop system

The comparative analysis is based on the following indicators:

- The absolute value of the estimation bias $\Delta\theta = M\left\{\hat{\theta}\right\} - \theta$ is given in *table 1* as a percentage of the actual value $\theta^T = [-1.5 \quad 0.7 \quad 1 \quad 0.5]$.
- The estimate of the plant's gain $\hat{k}_p = \dfrac{\hat{\theta}_3 + \hat{\theta}_4}{1 + \hat{\theta}_1 + \hat{\theta}_2}$ is given in *table 2* as a percentage of the actual value $k_p = 7.5$.
- The impulse-response function of the estimated model $\hat{W}(k)$ is presented on *figure 6* and the general estimate error $J_{\hat{w}} = \dfrac{1}{N}\sum_{k=1}^{N}(w(k) - \hat{w}(k))^2$ is given in *table 2*. Where $w(k)$ is the discrete-time values of $W(k)$,for $k \geq 36$, $W(k) \approx 0$.
- The floating point operations count (flops), needed for one iteration, is given in *table 3*.

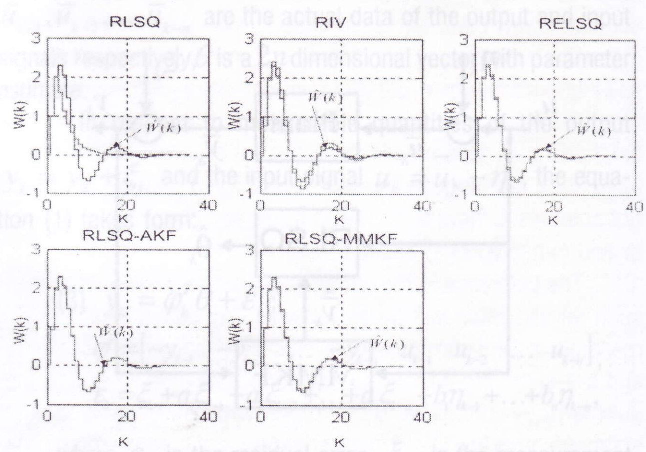

Figure 6. Impulse-response function of the estimated model

Table 1

D_ξ	RLSQ $\hat{\theta}$	RLSQ $\Delta\theta$	RIV $\hat{\theta}$	RIV $\Delta\theta$	RELSQ $\hat{\theta}$	RELSQ $\Delta\theta$	RLSQ-KF $\hat{\theta}$	RLSQ-KF $\Delta\theta$	RLSQ-MMFK $\hat{\theta}$	RLSQ-MMFK $\Delta\theta$
0.0225	-1.4873	0.7670	-1.4897	0.1851	-1.4995	0.0086	-1.4995	0.0191	-1.5024	0.2403
	0.6865	1.8268	0.6931	0.1209	0.6985	0.2267	0.6981	0.2082	0.7006	0.2151
	0.9899	0.4898	0.9719	1.4757	0.9986	0.5317	0.9997	0.5043	0.9972	0.1219
	0.5170	2.0890	0.5386	2.4659	0.4982	0.8702	0.4976	1.0761	0.4964	1.0896
1.2656	-0.9908	34.6527	-1.5376	1.9929	-1.4852	2.0561	-1.4926	1.9225	-1.4923	0.6220
	0.2293	68.8358	0.7435	3.6302	0.6856	3.6901	0.6952	4.1229	0.6926	1.3377
	0.9784	1.2843	0.8077	5.9168	0.9772	1.0641	0.9384	0.8628	0.9848	1.1856
	1.0446	102.5759	0.6844	14.4228	0.5719	18.0697	0.5598	8.4477	0.4956	1.1814
2.2500	-0.8270	45.7123	-1.5326	3.7567	-1.4789	2.7106	-1.5003	1.9643	-1.4931	0.6414
	0.0890	89.1272	0.7405	8.8804	0.6781	4.7727	0.7027	4.2520	0.6935	1.3608
	0.9813	0.6232	0.7567	5.2125	0.9693	1.6181	0.9045	2.5282	0.9824	1.5534
	1.2104	134.5035	0.7708	3.2573	0.5962	23.3327	0.5782	10.4677	0.4906	1.9324

Table 2

D_ξ	RLSQ $J_{\hat{w}}$	RLSQ $k_{об}$	RIV $J_{\hat{w}}$	RIV $k_{об}$	RELSQ $J_{\hat{w}}$	RELSQ $k_{об}$	RLSQ-KF $J_{\hat{w}}$	RLSQ-KF $k_{об}$	RLSQ-MMFK $J_{\hat{w}}$	RLSQ-MMFK $k_{об}$
0.0225	0.0443	0.8680	0.0209	0.9933	0.0026	0.3266	0.0039	0.5140	0.0051	0.4898
1.2656	9.9849	13.1170	0.8404	3.3642	0.0363	3.0421	0.0244	1.4186	0.0424	1.4689
2.2500	12.4595	11.5313	1.0401	4.0054	0.0933	4.7682	0.0414	2.3255	0.0509	1.9826

Table 3

	RLSQ	RIV	RELSQ	RLSQ-KF	RLSQ-MMFK
flops	315	365	812	494	575

Table 4

D_ξ	θ	RLSQ $\hat{\theta}$	RLSQ $\Delta\theta$	RIV $\hat{\theta}$	RIV $\Delta\theta$	RELSQ $\hat{\theta}$	RELSQ $\Delta\theta$	RLSQ-KF $\hat{\theta}$	RLSQ-KF $\Delta\theta$	RLSQ-MMFK $\hat{\theta}$	RLSQ-MMFK $\Delta\theta$
0.001	-1.5	-1.6767	11.1138	-1.0427	29.4040	-1.5604	4.0810	-1.5198	1.1860	-1.4980	0.1775
	0.7	0.7060	0.2610	0.8284	17.3413	0.6883	1.8893	0.7220	2.9745	0.6981	0.3562
	1	1.0509	4.7898	0.7813	21.3270	1.0107	1.0968	0.9974	0.2100	1.0116	1.2628
	0.5	0.5162	3.1852	0.3390	31.3433	0.5002	0.1746	0.5028	0.7608	0.5047	1.1468
0.006	-1.5	-1.6481	11.2364	–	–	-1.4845	2.8822	-1.5214	1.1888	-1.4853	1.06800
	0.7	0.6933	4.0021	–	–	0.6472	13.1317	0.6624	7.5209	0.6874	1.9847
	1	1.2244	22.1907	–	–	1.0380	2.2474	0.9759	1.7731	0.9885	1.0843
	0.5	0.5436	17.7142	–	–	0.5533	14.4249	0.4513	8.2529	0.5076	1.9027
0.01	-1.5	-1.7217	18.7851	–	–	-1.4838	1.3684	-1.5171	0.9748	-1.5133	0.7370
	0.7	0.7636	9.1719	–	–	0.6868	4.2653	0.6832	3.8839	0.7213	2.9821
	1	1.5389	59.0194	–	–	1.0610	6.1310	0.9781	2.6137	0.9947	0.6520
	0.5	0.5744	30.1485	–	–	0.5287	9.1300	0.4946	0.6044	0.5003	0.2028

Table 5

D_ξ	RLSQ $\hat{D}_y$	RLSQ J_h	RIV $\hat{D}_y$	RIV J_h	RELSQ $\hat{D}_y$	RELSQ J_h	RLSQ-KF $\hat{D}_y$	RLSQ-KF J_h	RLSQ-MMFK $\hat{D}_y$	RLSQ-MMFK J_h
0.001	0.03260	0.18504	0.02713	5.87555	0.03165	0.02743	0.03198	0.00067	0.03229	0.00071
0.006	0.03849	0.47892	–	–	0.03340	0.02486	0.02973	0.01305	0.03160	0.00121
0.01	0.04987	1.57776	–	–	0.03417	0.01789	0.03080	0.00292	0.03189	0.00088

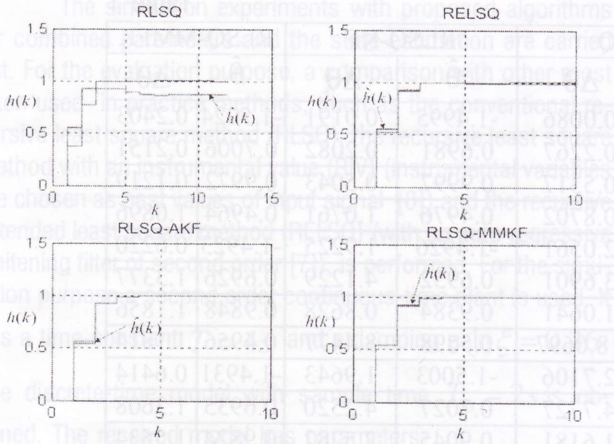

Figure 7. Step-response function of the control system

2. Parameters estimation in a closed-loop system

The comparative analysis is performed for the same indicators used in an open-loop (*table 4*). Only the indicator J_w is replaced by $J_{\hat{h}} = \frac{1}{N}\sum_{k=1}^{N}(h(k)-\hat{h}(k))^2$, where $h(k)$ is the step response function of the control system and $\hat{h}(k)$ is the step response function of the estimated control system. This indicator more clearly points out the influence of the plant's gain estimate $\hat{k}_p$ (*figure 7* and *table 5*). In this case one more indicator – the output signal $\bar{y}_k$ variance $\hat{D}_y$ is used. It characterizes the statistical accuracy of the control system (table 5).

The experimental results point out some advantages of the proposed algorithms in comparison with the traditional algorithms used. These advantages are more noticeable in case of a closed-loop estimation or/and in the presence of strong output noise. In a closed-loop regime some of the conventional estimation methods (as RIV) become unworkable in the presence of strong output noise.

V. Conclusion

Two algorithms for parameters and states estimation are proposed and studied in this paper. They are used for stochastic plant control in case of incomplete information. They combine the conventional least square method and the stochastic state observer (Kalman filter), which considerably improves the quality of the estimates. Simulation results of the proposed algorithms are presented. The comparative analysis with some of the most often used estimation methods are carried out and the obtained results are discussed. For better comparison different indicators are used. Each of them characterizes the plant's model and the control system accuracy.

References

1. Eykhoff, P. System Identification. Parameter and State Estimation. John Wiley and Sons Ltd, 1974.
2. Ljuing, L. System Identification. Theory for User. Prentice Hall, New Jersey, Englewood Chiffs, 1987.
3. Rastrigin, L. A., N. E. Madjarov, S. I. Markov. Parameters Estimation in Dynamics Objects. S.,Technika, 1978 (in Bulgarian).
4. Ya. Z. Tsypkin. Basis of Information Theory of the Identificatoion. M., Nauka, 1984 (in Russian).
5. Madjarov, N., L. Mihaylova. Adaptive Multiple Model Algorithms for Esimation and Control of Stochastic Systems.–*Automatics and Informatics*,1998, No. 2, 16-23 (in Bulgarian).
6. Slavov, T. Multiple Model Adaptive Linear Stochastic Control Systems Design. International Conference Automatics and Informatics' 04, Sofia, 2004.
7. Panuska, V. A Stochastic Approximation Method to Identification of Linear Systems Using Adaptive Filtering. Joint Automatic Control Conference. Ann Arbor, Mich., 1968,1014-1021.
8. Young, P. C., A. J. Jakeman. Refined Instrumental Variable Methods of Recursive Time Series Analysis. Part III: Extensions.–*Int J. Control*, 31,1981,741-764.
9. Madjarov, N., T. Slavov. Parallel Parameters and States Estimation in Linear Stochastic Systems.–*Information Technologies and Control*, 2003, №1, 43-47.

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