Design and Analysis of Fractional $\mathcal{ML}$- DTC Control Systems

E. Nikolov, N. Nikolova, V. Trashlieva

Key Words: *Fractional Repetitive and Dead-Time Compensation Control – Configuration; Design; Analysis and Applications; Robust Stability and Performance; Robust Margins.*

Abstract. *One essentially new class of fractional $\mathcal{ML}$-DTC control systems is proposed in the work. It is configured through combinations of strategy repetitive control, dead-time compensation control and fractional control. Repetitive control is an effective strategy for periodic disturbances suppression by filtering their influence into the control system, assuming that the period of disturbances is known. The use of fractional dead-time compensators in the systems provides advantages in quality control of industrial plants with a variable delay. The control with fractional operators of integration and differentiation joins the control systems in the class of robust control systems. In the work are given methods, criteria and synthesis algorithms for fractional $\mathcal{ML}$-DTC control systems. Their application and the analysis of quality are examined.*

1. Introduction

The following control systems are known: fractional control systems, fractional dead-time compensation control systems (***DTC – Dead-Time Compensation***) [2-11] and repetitive control systems with $\mathcal{ML}$-memory [12-24]. The present work proposes new class ***$\mathcal{ML}$-DTC fractional control systems.***

The method and the algorithm for their synthesis consist of:

- effective control strategy combination with suppression of:
  - internal re-parameterized/restructured perturbations in the control plant;
  - external (a priory known) disturbances in the control system;
- using a fractional control algorithm, in order to set these systems in the class of robust control systems.

Problems in achieving this purpose are: a structure configuration, methods systematization, design criteria and algorithms for ***$\mathcal{ML}$-DTC fractional control systems***. They are applied to a specific numerical example, in order to analyze and estimate the effectiveness of the new class systems that are being proposed.

Concerning the systems in this work structural configuration, methods, criteria and algorithms for analytical synthesis are presented, also are shown results from the application, analysis, robust analysis of ***$\mathcal{ML}$-DTC*** fractional control systems and conclusions and literature in use.

2. Structural Configuration

For the considered control system structure shown on *figure 1* with control plant G integral I^{α} and the differential D^{β} operators of fractional order [1] in the algorithm of controller R_{NE} (1) are used. R_{NE} provides the system with a feature of invariance of the stability margins GM (gain margin) and PM(phase margin) to the plant model G re-parameterized/ re-structured in an a priori uncertainty condition i.e. the system has a robust properties. The presence of delay τ in the plant poses conventional control difficulties. Some of the „hardest“ restructuring plant disturbances are those associated with the variations of the delay value τ. An effective and applicable strategy for solving these problems is the one based on the delay ***DTC***-compensation (*figure 2*) using a robust filter F_{NE} [2-11]. The F_{NE} design is related with the size of the delay τ^* in the nominal plant G^*. The presence in series of F_{NE} in the control algorithm structure (*figure 3*) transforms the regulator R_{NE} into a fractional regulator with a pre-filter, which is determined by the relation between F_{NE} and R_{NE}. The repetitive control is an effective strategy in the presence of periodic external signal disturbances in industrial environment. The repetitive control systems (*figure 5*) can be distinguished from the traditional feedback systems (*figure 4*), due to the fact that they contain $\mathcal{ML}$-filter with a memory F_{ML} (***Memory Loop***) [12-24]. It is assumed that the reference y^o and/or any other signal disturbances of the system (ν, f) demonstrate periodic features with a known a priori (during the design) constant period value T_p. The repetitive system structure (*figure 5*) contains a basic regulator (in this case fractional $F_{NE} R_{NE}$), an $\mathcal{ML}$-filter F_{ML} and a control plant G. The re-parameterized/ restructured perturbations in G are denoted by ξ. The $\mathcal{ML}$-filter with memory F_{ML} is a cut-off filter in the system for frequency $\omega_p = 2\pi / T_p$ of harmonic signals with a period T_p coinciding with those of ν, f or y^o .

After equivalent transformations of the structure (fig.5), the proposed and analyzed further in this paper ***$\mathcal{ML}$-DTC fractional control systems*** is shown at *figure 6*.

This structural configured system (*figure 6*) combines effectively the control strategies implemented to act against: internal re-parameterized/ restructured perturbations in the control plant; external (known in advance) disturbances towards the control system. A fractional control algorithm is used in order to set repetitive control systems in the class of the robust control systems [1]. The structure of regulator with input ε and output u is shown in *figure 7*. The algorithm comprises the three components in series.

$$R_{NE}^{ML\circ DTC} = F_{ML} F_{NE} R_{NE}$$

3. Methods, Criteria and Algorithms for $\mathcal{ML}$-DTC Fractional Control Systems Analytical Synthesis

For each component in the $R_{NE}^{ML\circ DTC}$ algorithm (*figure 7*) of the ***$\mathcal{ML}$-DTC fractional systems*** (*figure 6*) the following features are specified: main descriptions; dynamical control parameters; methods, criteria and analytical synthesis requirements.

3.1. ***The design*** (1a) ***of ID – component*** R_{NE} (1b) is a function of the nominal plant G^* and is distinguished [1] by the following:

- ***dynamical parameters for adjustment of*** R_{NE}**:**
  - α, β – fractional order of integral I^{α} and the differential D^{β} operators used;
  - $\omega_{b,I}, \omega_{h,I}, \omega_{b,D}, \omega_{h,D}$ – cut-off frequencies of horizontal profile in the integral I^{α}_{app} and differential D^{β}_{app} fractional order (α, β) operator approximations;
- **a synthesis method – °polynomial recursive approximation°**;
- ***criteria – °vertical profile with predefined stability margins°***;
- ***analytical requirements*** of R_{NE} synthesis (2 ÷11)**.**

(1a) $R_{NE} \equiv \left(I^{\alpha} D^{\beta}\right)_{app} \underset{\{\sigma = const\}}{\Leftrightarrow} G^*, \left(G^* = \hat{G}^* e^{-\tau^* p}\right)$

(1b)
$$R_{NE} = \left(I^{\alpha} D^{\beta}\right)_{app} = \left(\frac{1+p\left(\omega_{bI}\right)^{-1}}{1+p\left(\omega_{hI}\right)^{-1}}\right)^{\alpha} \prod_{i=1}^{N} \left(\frac{1+p\left(\omega_{Ii}\right)^{-1}}{1+p\left(\omega'_{Ii}\right)^{-1}}\right) + \left(\frac{1+p\left(\omega_{bD}\right)^{-1}}{1+p\left(\omega_{hD}\right)^{-1}}\right)^{\beta} \prod_{j=1}^{M} \left(\frac{1+p\left(\omega'_{Dj}\right)^{-1}}{1+p\left(\omega_{Dj}\right)^{-1}}\right),$$
$$\forall \omega\left(\bar{\ell}_a, \bar{\ell}_m\right) \in \left[\omega_{IA}, \omega_{DB}\right], \{0 < \alpha < 1\}; \{0 < \beta < 1\};$$

(2) $N \geq 5;\ n' = 2\left(1 - (\pi)^{-1} PM_m^{nom}\right)$

(3) $\omega_u > 250\ \omega_c;\ \alpha = n - n' = n - 2(\pi)^{-1} \arcsin\left(GM_m^{nom}\right)^{-1}$

(4) $\omega_{IA} = 0.1\ \omega_u;\ \omega_{DA} = 1.1\ \omega_u;\ \lambda = \left(\omega_h \omega_b^{-1}\right)^{(0.1+\alpha)}$

(5) $\omega_{IB} = 0.9\ \omega_u;\ \omega_{DB} = 10\ \omega_u;\ \eta = \left(\left(\omega_h \omega_b^{-1}\right)^{N^{-1}}\right)^{(0.9-\alpha)}$

(6) $\omega_b = 0.2\ \omega_A;\ \ \omega_o = 0.85\ \omega_{Ib};\ \omega'_{i+1} = (\lambda\eta)^{i}.\eta^{0,5}\ \omega_b$

(7) $\omega_h = 1.2\ \omega_B;\ \ \omega_{i+1} = (\lambda\eta)^{i}.\lambda.\eta^{0,5}\ \omega_b$

(8) $\left(\omega'_{Ii}\right)^{-1} > \left(\omega_{Ii}\right)^{-1} > \left(\omega_0\right)^{-1};\ \left(\omega'_{Di}\right)^{-1} > \left(\omega_{Di}\right)^{-1}$

(9) $\left(\omega'_{Di}\right)^{-1} > \left(\omega_{Di}\right)^{-1} > \left(\omega'_{Ii}\right)^{-1};\ \left(\omega'_{Ii}\right)^{-1} > \left(\omega_{Ii}\right)^{-1} > \left(\omega_0\right)^{-1}$

(10) $\omega_u > \omega_c;\ \omega_u \geq 250\ \omega_c;\ \omega_A > \omega_c;\ \omega_B >> \omega_c;$
$0.5\left(\omega_{IA} - \omega_{DB}\right) \leq \left(\omega_u - \omega_c\right)$

(11) $\omega_b > \omega_c;\ \omega_b < \omega_A;\ \omega_h > \omega_B;\ (\lambda\eta)_{opt} = 3.98;$
$(\omega_h/\omega_b)_{opt} = 250 \div 600$

where:

I^{α}, D^{β} – fractional operators (original functions, irrational functions);

$I^{\alpha}_{app}, D^{\beta}_{app}$ – approximating operators (original function approximations, rational functions);

i, j – component counter of the approximating polynomial (entire number);

M, N – the number of the participating forced units in the approximating polynomial (entire number);

GM_m^{nom}, PM_m^{nom} – desired gain and phase stability margins values of the designed nominal system;

$(\omega_i)^{-1}, (\omega'_i)^{-1}$ – time constants in the forced units in the approximating polynomial (real, positive numbers);

$\omega_u, (\omega_u)^{-1}$ – the unit frequency and basic fractional regulator time constant;

n' – the plant model order;

$\hat{G}^*, e^{-\tau^* p}$ – rational and irrational components in the nominal model G^* of the plant G;

ω_b, ω_h – lower and upper frequencies of the approximation;

ω_A, ω_B – lower and upper frequencies of the range of approximation;

λ, η – recursive factors (recursion indexes).

3.2. ***The design*** (12a) ***of DTC – component*** F_{NE}(12.b) is a function of delay τ^* in the nominal plant model G^* and has the following characteristics [6]**:**

- ***a dynamical parameter for*** F_{NE} ***adjustment***:
  - τ_F – pre-filter time constant, plant delay nominal value;
  - ζ – fractional differentiation D^{ζ}_{app} operator order;
  - $\omega_{b,\zeta}$, $\omega_{h,\zeta}$ – cut-off frequencies of the horizontal profile in the filter F_{NE} module;
- ***a synthesis method – °polynomial recursive approximation of fractional frequency compensation°***;
- ***criteria – °frequency characteristics adequacy of the rational approximating system and frequency characteristics adequacy of the irrational component in the nominal plant model in predefined frequency range°*** ;
- ***analytical dependences***, determining the F_{NE} synthesis (13÷14), where Φ^{DTC}_{NE} (13a) is the transfer function of the closed-loop system with fractional delay compensation (*figure 3*), which is distinguished from the control system transfer function Φ_{PID} (13b) described by same plant and *PID*-regulator, due to the fact that it is not a function of the plant delay $e^{-p\tau^*}$.

3.3. ***The design*** (15a) ***of ML – component*** F_{ML} (15b) is a function of the external harmonic disturbances frequency ω_ρ in the system and is determined [19,20] by the following**:**

- ***dynamic parameters for*** F_{ML} ***adjustment***:
  - $\omega_{b,i}$, $\omega_{h,i}$ – cut-off frequencies of the horizontal profile in the F_{ML} – filter module. These frequencies define the effectiveness of the filter frequency bandwidth in case of value fluctuations of $\omega_\rho = 2\pi / T_\rho$ – cut-off frequency, determined by the frequency of harmonic disturbance toward the system, known in advance;
  - $l_i(\omega_\rho, \Omega_i)$ – groups of n elements with a deay (elements with memory) connected in series and in parallel;
- ***a synthesis method – °band- pass filter equation°***;
- ***criteria – °cut-off module°***;
- ***analytical requirements*** for the F_{ML} filter synthesis with memory (16÷17) as a dependence between dynamical adjustment parameters (17)**.**

Using these dynamical parameters F_{ML} can be synthesized analytically with respect to the criteria realization defined by the module (18) requirements where:

– T_ρ – periodical disturbances time constant, F_{ML} time constant;

– $\Omega_i = (\omega_\rho/\omega_{b,i}) = (\omega_{h,i}/\omega_\rho)$ – F_{ML} own relative frequency;

– $\omega_{b,i}$, $\omega_{h,i}$ – lower and upper frequencies of the ***band-pass*** filter approximation.

(12a) $$F_{NE} \underset{\{e^{-p\tau^*}F_{NE}=1\}}{\Leftrightarrow} \tau^*, \left(F_{NE} = F_\tau D^\zeta \,\hat{=}\, F_\tau D^\zeta_{app}\right)$$

(12b) $$F_{NE} \,\hat{=}\, F_\tau D^\zeta_{app} \equiv \frac{(1+p)}{(1+\tau_F p)} \left(\frac{\omega_u}{\omega_{h,\xi}}\right)^\zeta \prod_{i=1}^{N} \frac{\left(1+p(\omega'_i)^{-1}\right)}{\left(1+p(\omega_i)^{-1}\right)}$$

(13a) $$\Phi^{DTC}_{NE}(p) = \frac{R_{NE}(p)\hat{G}^*(p)e^{-p\tau^*}F_{NE}(p)}{1+R_{NE}(p)\hat{G}^*(p)e^{-p\tau^*}F_{NE}(p)} = R_{NE}\hat{G}^*\left(1+R_{NE}\hat{G}^*\right)^{-1}, \ \left(e^{-p\tau^*}F_{NE}(p)=1\right)$$

(13b) $$\Phi_{PID}\left(p, e^{-p\tau^*}\right) = R_{PID}\,\hat{G}^* e^{-p\tau^*}\left(1+R_{PID}\,\hat{G}^* e^{-p\tau^*}\right)^{-1}$$

(14) $$e^{-p\tau^*}F_{NE}(p)=1 \leftrightarrow \begin{cases} |\exp(-j\omega\tau^*)|\,|F_{NE}(j\omega)| \,\hat{=}\, 1, \ \forall\omega\in[\omega_{b,\xi}, \omega_{h,\xi}] \\ arg(\exp(-j\omega\tau^*)) + arg(F_{NE}(j\omega)) \,\hat{=}\, 0, \ \forall\omega\in[\omega_{b,\xi}, \omega_{h,\xi}] \end{cases}$$

(15a) $$F_{ML} \underset{\{y(\omega)\neq\varsigma(y^0(\omega_P), v(\omega_P), f(\omega_P))\}}{\Leftrightarrow} \omega_p, \left(\omega_p = 2\pi / T_p\right)$$

(15b) $$F_{ML} = \left(2-\sum_{k=1}^{l} W_k(j\omega)e^{-j\omega k T_p}\right)^{-1} = \left(2-\sum_{k=1}^{l} \kappa_k (j\omega T_k+1)^{-1} e^{-j\omega k T_p}\right)^{-1},$$
$$\left(\sum_{k=1}^{l} |W_k(j\omega)| \equiv 1, \ \left(W_k(j\omega) = \kappa_k (j\omega T_k+1)^{-1}\right), (2 \le l \le 20)\right)$$

(16) $$y_{(j\omega)} \neq \varsigma\left(y^0_{(j\omega_P)}, v_{(j\omega_P)}, f_{(j\omega_P)}\right)$$

(17) $$l_i(\omega_p, \Omega_i) = \frac{\log_{10}(\omega_p - \omega_{b,i})}{\log_{10}(\omega_{h,i} - \omega_p)} = \frac{\log_{10}(\omega_p - \omega_p\Omega_i^{-1})}{\log_{10}(\omega_p\Omega_i - \omega_p)},$$
$$\left(\begin{matrix} \omega_{b,i} < \omega_p < \omega_{h,i} \ ; \ \omega_{h,i} - \omega_{b,i} = \Delta\omega_i > 0 \ ; \ 2 \le l_i \le 20 \ ; \\ \Omega_i = (\omega_p/\omega_{b,i}) = (\omega_{h,i}/\omega_p) \ ; \ 1{,}5 \le \Omega \le 3{,}0 \end{matrix}\right)$$

(18) $$|F_{ML}(j\omega)| \equiv \begin{cases} 0, & \forall\omega\in[\omega_{b,i}, \omega_{h,i}], (\omega_{b,i} < \omega_p < \omega_{h,i}) \\ 1, & \forall\omega\in[0, \omega_{b,i}], \forall\omega\in[\omega_{h,i}, \infty) \end{cases}$$

3.4. *The analytical design of* $R^{ML\circ DTC}_{NE}$ (19) *in* ***ML-DTC fractional control system*** (*figure 6*, *figure 7*) using ***ID***- fractional ***ML***-repetitive regulator with ***DTC*** – delay compensation that uses:

• ***method*** – ***°polynomial recursive approximation of fractional frequency compensation and band-pass filter equation°,***

• ***criteria*** (20) – ***°a vertical profile with given stability margins°, °frequency characteristics adequateness of the rational approximating system and of the irrational component in the nominal plant model in an a priori given frequency range° and °cut-off module°.***

The analytical synthesis of $R^{ML\circ DTC}_{NE}$ (19) in ***ML-DTC fractional system*** (*figure 6*, *figure 7*) follows the algorithm (21÷34), where:

– Π – a functional set of variations describing an a priori uncertainty in the control plant;

– $G^{\blacksquare}$ – „perturbed on upper limit“ control plant model;

– ℓ_m, ℓ_a – multiplicative and additive internal perturbations in the control plant.

(19) $$R^{ML\circ DTC}_{NE} = F_{ML}F_{NE}R_{NE} \equiv \left(2-\sum_{k=1}^{l}\kappa_k(1+T_k p)^{-1}e^{-pkT_p}\right)^{-1}\left(F_\tau D^\xi_{app}\right)\left(I^\alpha D^\beta\right)$$

(20)
$$a)\text{−>>} \quad \Pi : \begin{Bmatrix} \ell_a(j\omega) = G^{\blacksquare}(j\omega) - G^*(j\omega) \ ; \ |\ell_a(j\omega)| \le \bar{\ell}_a(j\omega) \\ \ell_m(j\omega) = \ell_a(j\omega)(G^*(j\omega))^{-1} \ ; \ |\ell_m(j\omega)| \le \bar{\ell}_m(\omega) \end{Bmatrix}$$
$$b)\text{−>>} \quad R_{NE} : \begin{cases} GM \equiv 20\log_{10}\left|W^{\substack{gabarit\\vertical}}_{NE}(\xi, j\omega_\pi)\right| \equiv const, [dB] \\ \omega^{nom}_\pi : arg\, W^{nom}_{NE}(j\omega^{nom}_\pi) = -\pi \ \Leftrightarrow \ GM^{nom}_m(\omega^{nom}_\pi) \\ PM \equiv -\left(arg\left(W^{\substack{gabarit\\vertical}}_{NE}(\xi, j\omega_u)\right) + 180°\right) \equiv const, [deg] \\ \omega^{nom}_u : \left|W^{nom}_{NE}(j\omega^{nom}_u)\right| = 1 \ \Leftrightarrow \ PM^{nom}_m(\omega^{nom}_u) \end{cases}$$
$$c)\text{−>>} \quad F_{NE} : \begin{cases} |\exp(-j\omega\tau^*)|\,|F_{NE}(j\omega)| \equiv 1, \ \forall\omega\in[\omega_{b,\xi}, \omega_{h,\xi}] \\ arg(\exp(-j\omega\tau^*)) + arg(F_{NE}(j\omega)) \equiv 0, \ \forall\omega\in[\omega_{b,\xi}, \omega_{h,\xi}] \end{cases}$$
$$d)\text{−>>} \quad F_{ML} : \left\{ |F_{ML}(j\omega)| \equiv \begin{cases} 0, & \forall\omega\in[\omega_{b,i}, \omega_{h,i}], (\omega_{b,i} < \omega_p < \omega_{h,i}) \\ 1, & \forall\omega\in[0, \omega_{b,i}], \forall\omega\in[\omega_{h,i}, \infty) \end{cases} \right.$$

• *the* synthesis of ***ID***-fractional regulator $\boldsymbol{R_{NE}}$ (21÷26)

(21) $$n' = 2\left(1-(\pi)^{-1}PM^{nom}_m\right)$$

(22) $$\alpha = n' - n = n' - 2\left(1-\left(PM^{nom}(j\omega^{nom}_u)\right)/\pi\right); \ \alpha = \log\lambda\,(\log(\lambda\eta))^{-1}$$

(23) $$N \ge 5 \ ; \omega_u > 250\ \omega_c \ ; \ \omega_{IA} = 0.1\,\omega_u, \ \omega_{DA} = 1.1\,\omega_u, \ \omega_{IB} = 0.9\,\omega_u, \ \omega_{DB} = 10\,\omega_u$$

(24) $$\omega_b = 0.2\,\omega_A, \omega_0 = 0.85\,\omega_{Ib}; \omega_h = 1{,}2\,\omega_B; \lambda = \left(\omega_h\,\omega_b^{-1}\right)^{(0.1+\alpha)}; \eta = \left(\left(\omega_h\,\omega_b^{-1}\right)^{N^{-1}}\right)^{(0.9-\alpha)}$$

(25) $$\omega'_{i+1} = (\lambda\eta)^i \,.\, \eta^{0,5}\,\omega_b \ ; \ \omega_{i+1} = (\lambda\eta)^i \,.\, \lambda \,.\, \eta^{0,5}\,\omega_b$$

(26) $$R_{NE} = \left(I^\alpha D^\beta\right)_{app} = \left(\frac{1+p(\omega_{bI})^{-1}}{1+p(\omega_{hI})^{-1}}\right)^\alpha \prod_{i=1}^{N}\left(\frac{1+p(\omega_{Ii})^{-1}}{1+p(\omega'_{Ii})^{-1}}\right) +$$
$$+\left(\frac{1+p(\omega_{bD})^{-1}}{1+p(\omega_{hD})^{-1}}\right)^\beta \prod_{j=1}^{M}\left(\frac{1+p(\omega'_{Dj})^{-1}}{1+p(\omega_{Dj})^{-1}}\right),$$
$$\forall\omega\left(\bar{\ell}_a, \bar{\ell}_m\right) \in [\omega_{IA}, \omega_{DB}], \{0<\alpha<1\}; \{0<\beta<1\};$$
$$(\omega'_{Ii})^{-1} > (\omega_{Ii})^{-1} > (\omega_0)^{-1}; (\omega'_{Di})^{-1} > (\omega_{Di})^{-1};$$
$$(\omega'_{Di})^{-1} > (\omega_{Di})^{-1} > (\omega'_{Ii})^{-1} > (\omega_{Ii})^{-1} > (\omega_0)^{-1}$$

• the synthesis of fractional ***DTC***-delay compensator F_{NE} (27÷30):

(27) $$\zeta \equiv \left(-arg\left(G^*(j\omega^*_C)\right)/(\pi/2)\right); \ \tau_F \equiv \tau^*; \zeta = \log\lambda\,(\log(\lambda\eta))^{-1}; \lambda\eta = 3{,}58$$

(28) $$1000\,\omega^*_C \le \omega_u \le 2000\,\omega^*_C$$

(29) $$\omega_A = 0{,}10\,\omega_u \ ; \ \omega_B = 10{,}00\,\omega_u \ ; \ \omega_{b,\zeta} = 0{,}2\,\omega_A = 0{,}02\,\omega_u \ ; \ \omega_{h,\xi} = 1{,}2\,\omega_B = 12{,}00\,\omega_u$$
$$\omega'_1 = \lambda^{-0,5}\,\omega_u \ ; \ \omega_1 = \lambda^{+0,5}\,\omega_u \ ; \ \omega_N = \eta^{-0,5}\,\omega_{h,\xi} = \eta^{-0,5}\,12{,}00\,\omega_u$$

(30) $$F_{NE}(j\omega) \,\hat{=}\, F_\tau D^\zeta_{app} \equiv \frac{(1+j\omega)}{(1+j\omega\tau_F)}\left(\frac{\omega_u}{\omega_{h,\xi}}\right)^\zeta \prod_{i=1}^{N}\left(1+j\frac{\omega}{\omega'_i}\right)\left(1+j\frac{\omega}{\omega_i}\right)^{-1}$$

• the synthesis of repetitive ***ML***-filter with memory F_{ML} (31÷34):

(31) $$\omega_{b,i} < \omega_p < \omega_{h,i} \ ; \ \omega_{h,i} - \omega_{b,i} = \Delta\omega_i > 0$$

(32) $$\Omega_i = (\omega_p/\omega_{b,i}) = (\omega_{h,i}/\omega_p); \ 1{,}5 \le \Omega \le 3{,}0$$

(33) $$l_i(\omega_p, \Omega_i) = \frac{\log_{10}(\omega_p - \omega_{b,i})}{\log_{10}(\omega_{h,i} - \omega_p)} = \frac{\log_{10}(\omega_p - \omega_p\Omega_i^{-1})}{\log_{10}(\omega_p\Omega_i - \omega_p)}; 2 \le l_i \le 20$$

(34) $$F_{ML}(j\omega) = \left(2-\sum_{k=1}^{l} W_k(j\omega)e^{-j\omega kT_p}\right)^{-1} = \left(2-\sum_{k=1}^{l}\kappa_k(j\omega T_k+1)^{-1}e^{-j\omega kT_p}\right)^{-1},$$
$$\sum_{k=1}^{l}|W_k(j\omega)| \equiv 1, \left(W_k(j\omega) = \kappa_k(j\omega T_k+1)^{-1}\right), (2 \le l \le 20)$$

The algorithm described for $R^{ML\circ DTC}_{NE}$ (19) design in ***ML-DTC fractional system*** (21÷34) consists of three basic stages.

The solutions obtained for each of the algorithms are not a function of the solution of the other stages. Each one of the

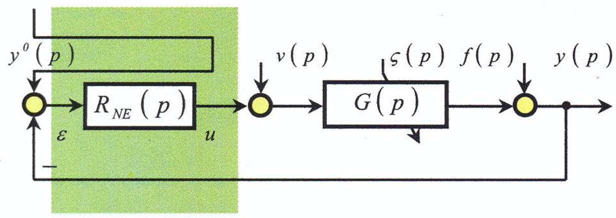


Figure 1

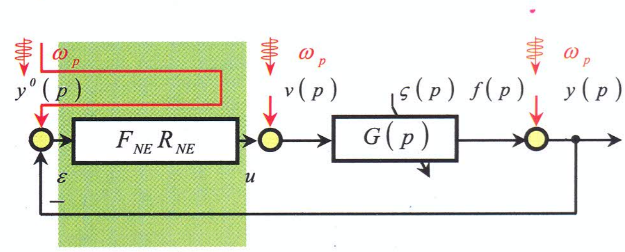


Figure 4

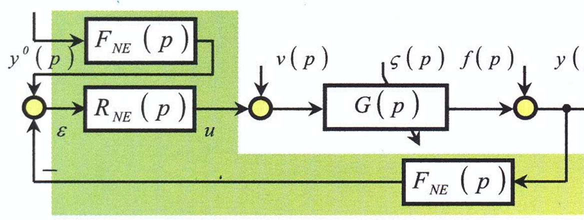


Figure 2

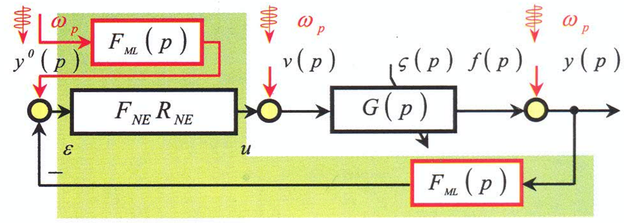


Figure 5

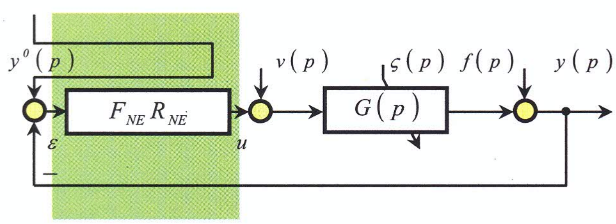


Figure 3

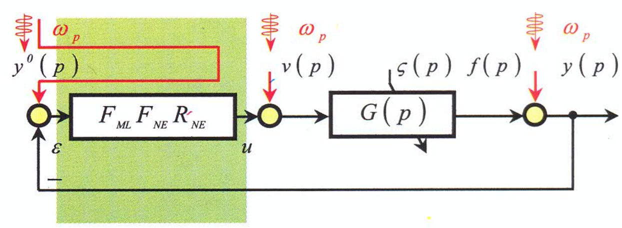


Figure 6

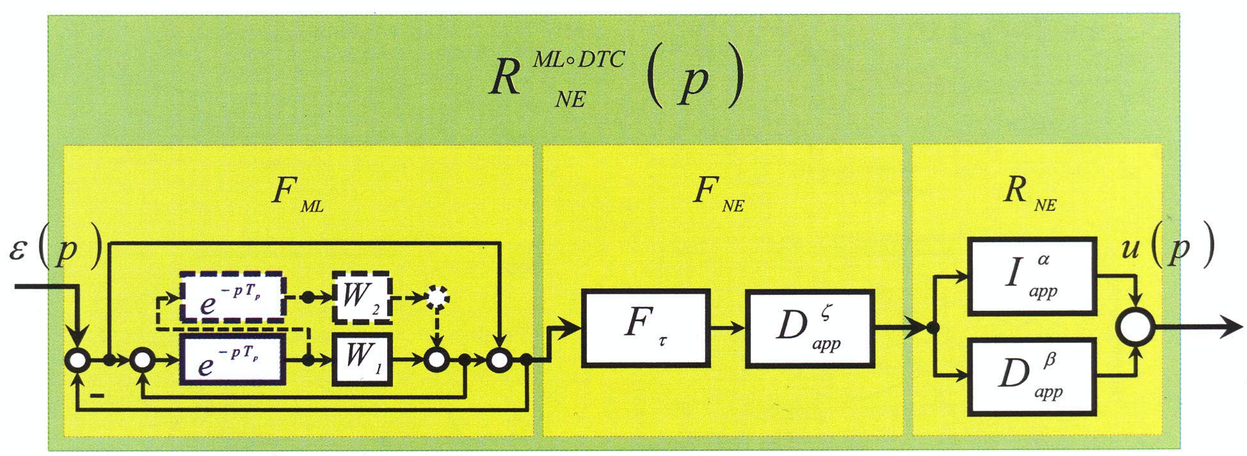


Figure 7

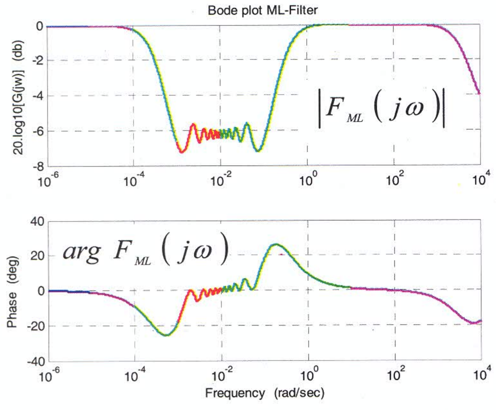


Figure 8a

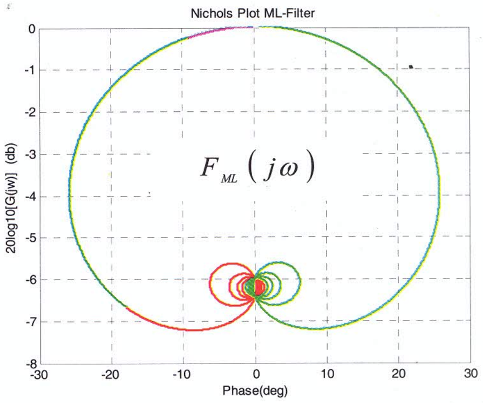


Figure 8b

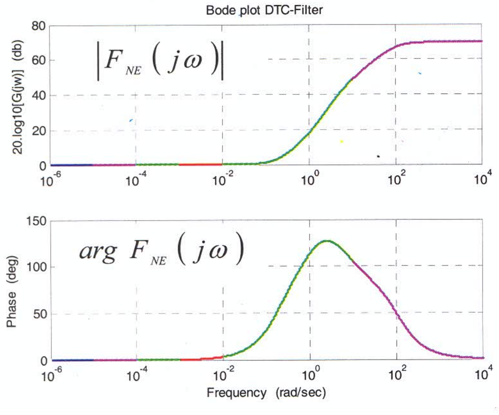


Figure 9a

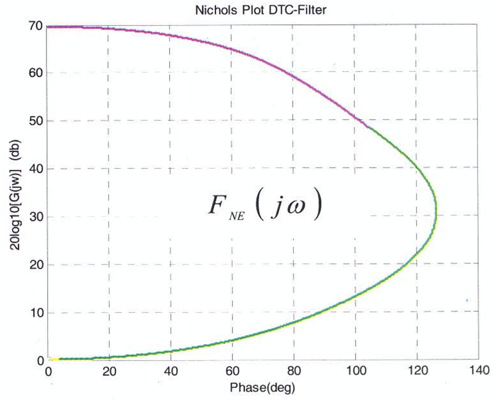
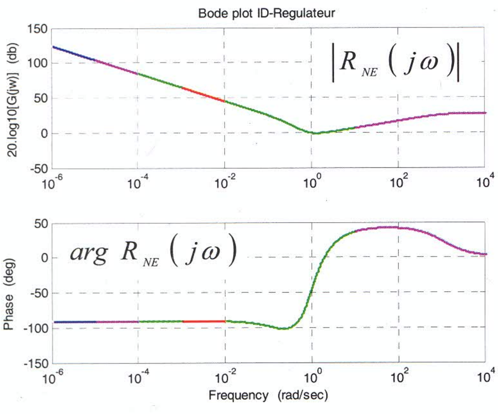


Figure 9b



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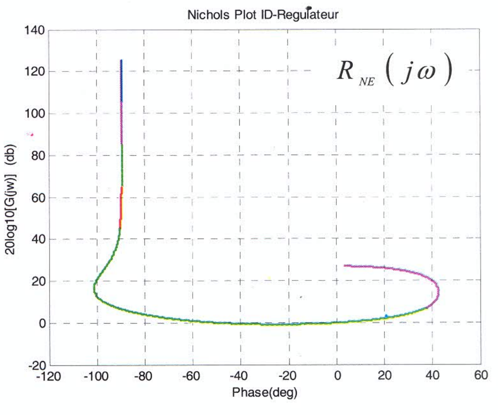


Figure 10b

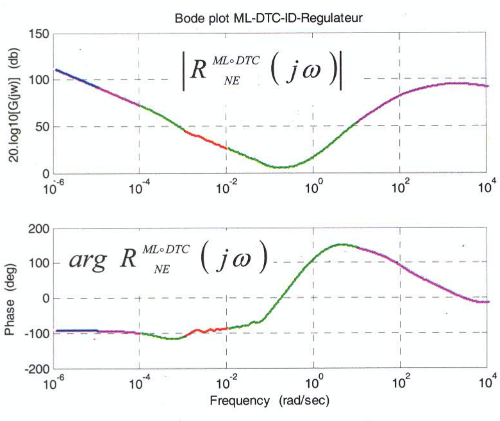
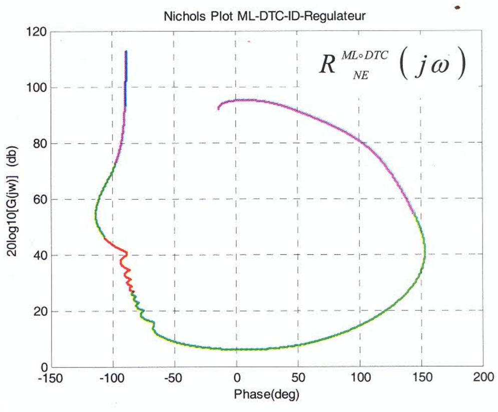


Figure 11a



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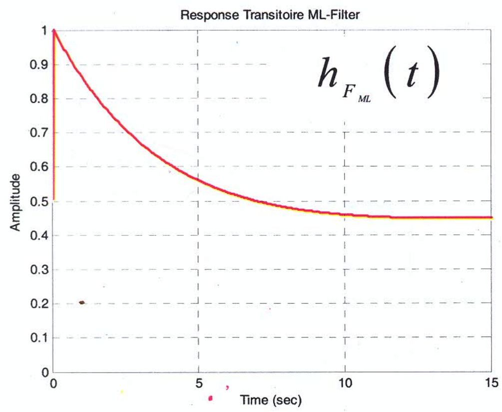

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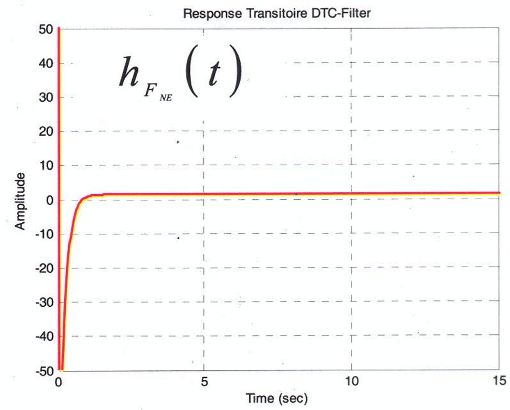

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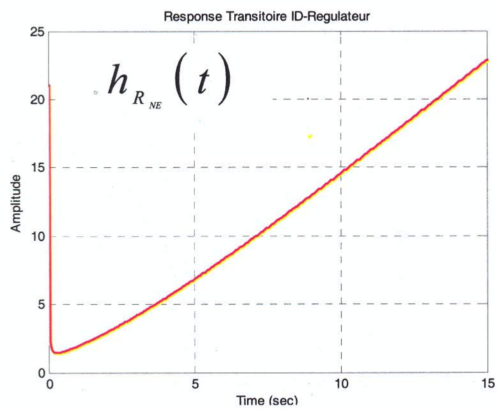

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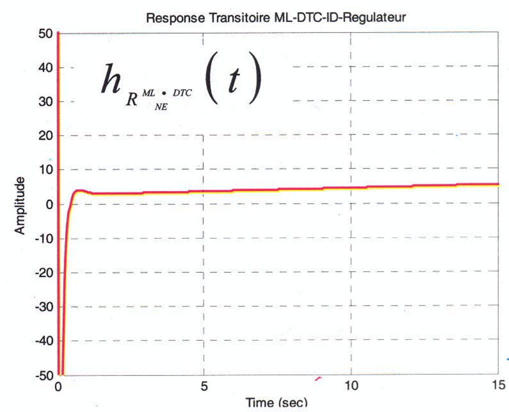

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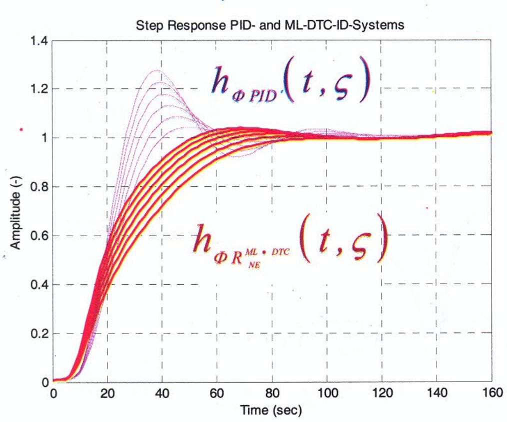

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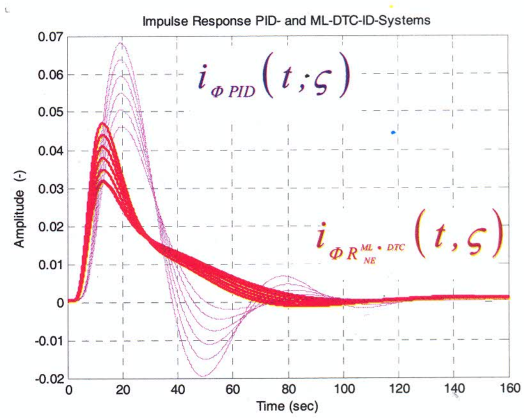

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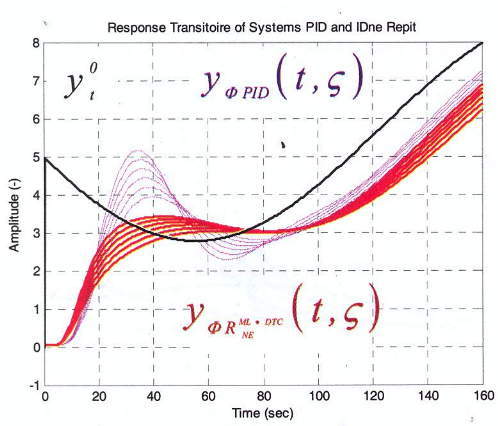

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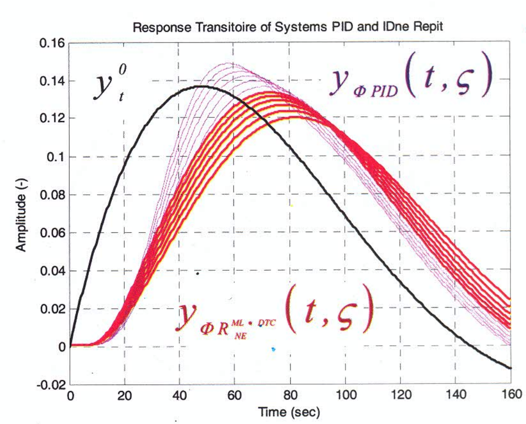

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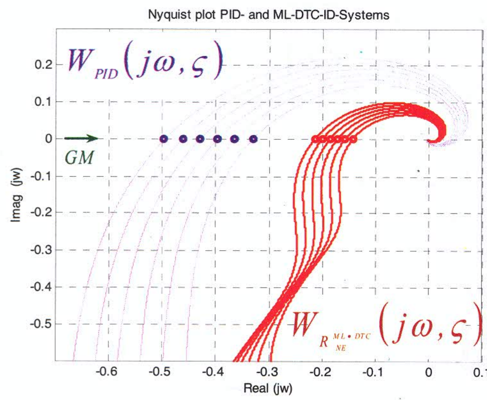

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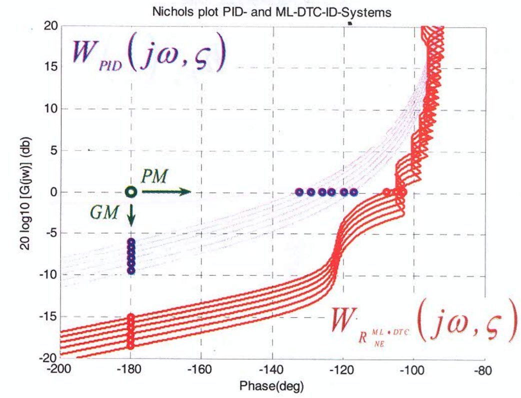
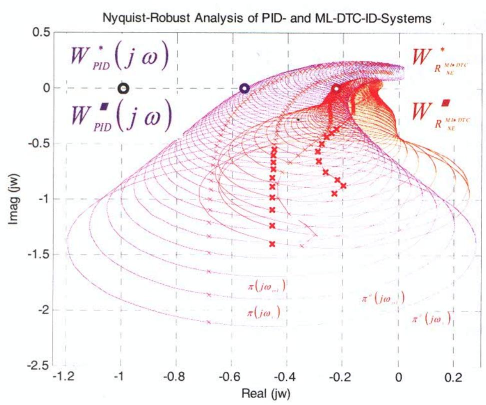


Figure 21



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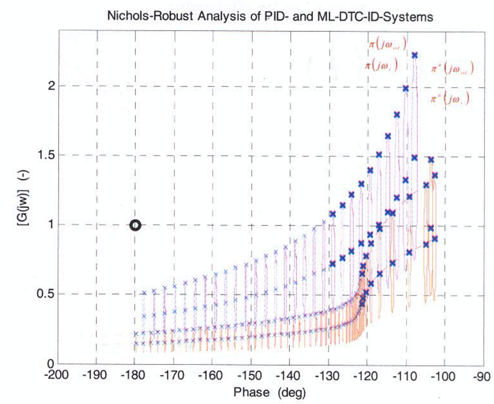

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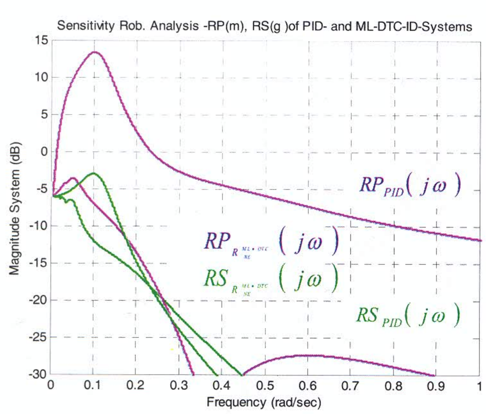

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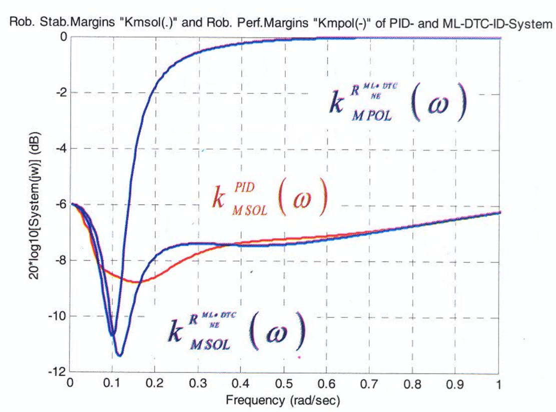

Figure 25

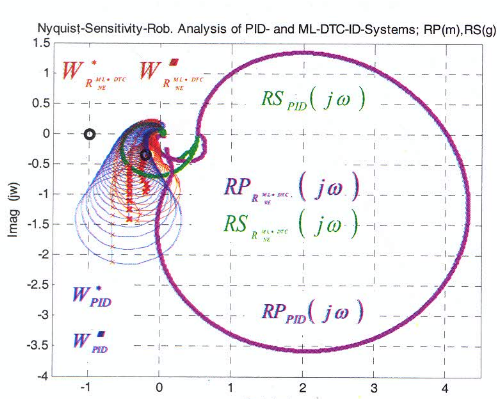

Figure 26

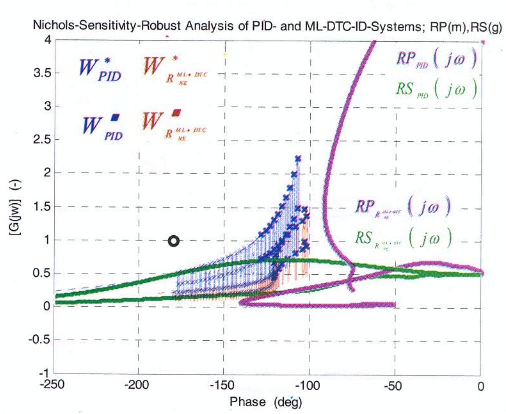

Figure 27

stages as well as their consecutiveness are independent in the system design process.

The initial conditions for analytic design of the regulator $R_{NE}^{ML\circ DTC}$ in the system are known or predefined

$$G^*, G^{\blacksquare}, GM_m^{nom}, PM_m^{nom}, \tau^*, \omega_p.$$

The algorithm (21÷34) is used to evaluate $R_{NE}^{ML\circ DTC}$ ***dynamical adjustment parameters*** in the ***ML-DTC fractional system:***

- α, β – the fractional order of the integration I^{α} and differentiation D^{β} operators used in R_{NE};
- ω_{bI}, ω_{hI}, ω_{bD}, ω_{hD} – cut-off frequencies of the horizontal profile in the approximations of the integral I_{app}^{α} and differential D_{app}^{β} fractional order α, β operators in R_{NE};
- τ_F – the time constant of F_{NE};
- ζ – the order of the fractional differentiation D_{app}^{ζ} operator in F_{NE};
- $\omega_{b\zeta}$, $\omega_{h\zeta}$ – cut-off frequencies of the horizontal profile in F_{NE} – filter module;
- ω_{bi}, ω_{hi} – cut-off frequencies of the horizontal profile in F_{ML} – filter module;
- $l_i(\omega_p, \Omega_i)$ – groups of n elements with a delay (elements with memory) in F_{ML} connected in series and in parallel.

4. Applications, Analysis and Robust Analysis of *ML-DTC* Fractional Control Systems

For a specific example of control plant G described with a nominal G^* (35) and perturbed on upper limit $G^{\blacksquare}$ (36) models, are synthesized the following:

- ***ML-DTC fractional control system*** (*figure 6*) containing $R_{NE}^{ML\circ DTC}$ (37÷40), according to the algorithm (21÷34), the values of the dynamical adjustment parameters are also indicated;
- the classic ***PID -system*** (*figure 1*) containing R_{PID} (41) regulator, where the values of the dynamical adjustment parameters are also indicated .

The two systems using control algorithms $R_{NE}^{ML\circ DTC}$ (40) and R_{PID} (41) are modeled. Results obtained from their parallel simulation are shown as follows ***for***:

- ***the control algorithm***
  - •• frequency characteristics and step response of: F_{ML} (39) – component (*figure 8*, *figure 12*), F_{NE} (38) – component (*figure 9*, *figure 13*), R_{NE} (37) – component (*figure 10*, *figure 14*) and ***ML-DTC regulator*** $R_{NE}^{ML\circ DTC}$ (*figure 11*, *figure 15*);
- ***characteristics*** of the open-loop and the closed-loop ***ML-DTC fractional system*** and ***PID-system*** as function of the in–dicated (20a) range of parameter variations ξ in the plant G
  - •• step responses $h_{\Phi R_{NE}^{ML\circ DTC}}(t,\xi)$ and $h_{\Phi R_{PID}}(t,\xi)$ (*figure 16*);
  - •• impulse responses $i_{\Phi R_{NE}^{ML\circ DTC}}(t,\xi)$ and $i_{\Phi R_{PID}}(t,\xi)$ (*figure 17*);
  - •• responses $y_{\Phi R_{NE}^{ML\circ DTC}}(t,\xi)$ and $y_{\Phi R_{PID}}(t,\xi)$ of an arbitrary input signal y_t^t (*figure 18*, *figure 19*);
  - •• frequency responses $W_{R_{NE}^{ML\circ DTC}}(j\omega,\xi)$ and $W_{R_{PID}}(j\omega,\xi)$ (*figure 20*, *figure 21*);
- ***frequency Nyquist*** – and ***Black-Nichols***-robust analysis by the characteristics of the nominal W^* and the perturbed on upper limit $W^{\blacksquare}$ (42) open-loop systems (*figure 22*, *figure 23*) fulfilling the requirements for robust stability (43) and robust performance (44) in conditions of an a priori uncertainty, which is described by the set of $\pi(j\omega)$ (45) circles on $\pi^{o}(j\omega_i)$ (46) circumferences with radiuses $r^{o}(\omega_i)$ (47) and central points ω_i from the hodograph W^*;
- ***robust analysis*** determined by the closed-loop system sensitivity characteristics satisfying the requirements for achieving robust stability RS (48) and robust performance RP (49) (*figure 24*);
- ***general*** frequency robust analysis of the open-loop and closed-loop systems (*figure 26*, *figure 27*);
- robust stability ***margin*** $k_{M\,SOL}$ (50) and robust performance ***margin*** $k_{M\,POL}$ (51) shown on *figure 25*.

(35) $$G^* = \hat{G}^* e^{-\tau^* p} = 0{,}150\,(4p+1)^{-1}\,e^{-5p} = 0{,}150\,(4p+1)^{-1}(1{,}66p+1)^{-3}$$

(36) $$G^{\blacksquare} = \hat{G}^{\blacksquare} e^{-\tau^{\blacksquare} p} = 0{,}225\,(4p+1)^{-1}\,e^{-10p} = 0{,}225\,(4p+1)^{-1}(3{,}33p+1)^{-3}$$

(37) $$R_{NE} \hat{=} \left(I^{1,2}D^{0,5}\right)_{app} = \frac{(2p+1)}{2p}\frac{(2{,}18p+1)(0{,}61p+1)}{(4{,}13p+1)(1{,}15p+1)} + \frac{(0{,}41p+1)(0{,}11p+1)}{(0{,}21p+1)(0{,}06p+1)},\ (\alpha=1{,}2;\beta=0{,}5)$$

(38) $$F_{NE} \hat{\equiv} F_{\tau} D_{app}^{1,5} = \frac{(p+1)}{(5p+1)}\left(\frac{(0{,}41p+1)(0{,}11p+1)}{(0{,}21p+1)(0{,}06p+1)}\right)^3,\ (\tau_F = 5,s\ ;\varsigma = 1{,}5)$$

(39) $$F_{ML} = \left(2 - \sum_{k=1}^{20} \frac{0{,}1\,(0{,}0001p+1)_k^{-1}}{(133{,}33p+1)_k^3}\right)^{-1},\ (\omega_p = 10^{-2}, rad/s\ ;\ T_p = 400, s)$$

(40) $$R_{NE}^{ML\circ DTC} = \left(2 - \sum_{k=1}^{20} \frac{0{,}1\,(0{,}0001p+1)_k^{-1}}{(133{,}33p+1)_k^3}\right)^{-1} \frac{(p+1)}{(5p+1)}\left(\frac{(0{,}41p+1)(0{,}11p+1)}{(0{,}21p+1)(0{,}06p+1)}\right)^3 \times$$
$$\times\left(\frac{(2p+1)}{2p}\frac{(2{,}18p+1)(0{,}61p+1)}{(4{,}13p+1)(1{,}15p+1)} + \frac{(0{,}41p+1)(0{,}11p+1)}{(0{,}21p+1)(0{,}06p+1)}\right)$$

(41) $$R_{PID} = 2{,}35\,(8p+1)(8p)^{-1}(2p+1)(0{,}4p+1)^{-1},\ (k_p = 2{,}35;\ T_i = 8,s;\ T_d = 2,s)$$

(42) $$W_i^* = R_i\,G^*\ ;\quad W_i^{\blacksquare} = R_i\,G^{\blacksquare}$$

(43) $$\left|1 + G^*(\omega)R_i(\omega)\right| > r^0(\omega),\ \forall\omega$$

(44) $$\left|1+G(\omega)R_i(\omega)\right| \geq \left|1+G^*(\omega)R_i(\omega)\right| - r^0(\omega),\ \forall G\in\Pi;\ \forall\omega$$

(45) $$\pi(j\omega) \in \mathcal{W}(j\omega),\ (\omega\in[0;\infty))$$

(46) $$\pi^0(j\omega_i) = \begin{cases} Re^0(\omega_i) = Re^*(\omega_i) + r(\omega_i)\cos\Omega,\ (\Omega\in[0,\infty)) \\ Im^0(\omega_i) = Im^*(\omega_i) + r(\omega_i)\sin\Omega,\ (\Omega\in[0,\infty)) \end{cases}$$

(47) $$r^0(\omega_i) = \left|l_a(\omega_i)R(\omega_i)\right| = \left|l_m(\omega_i)R(\omega_i)G^*(\omega_i)\right|$$

(48) $$RS_i \Rightarrow \left|\eta^*(\omega)\bar{\ell}_m(\omega)\right| < 1,\ (\forall\omega, \omega\in[0,\infty);\ \eta^* = RG^*(1+RG^*)^{-1})$$

(49a) $$RP_i \Rightarrow \left|\eta^*(\omega)\bar{\ell}_m(\omega)\right| + \left|e^*(\omega)\upsilon(\omega)\right| < 1,\ (\forall\omega, \omega\in[0,\infty);\ e^* = (1+RG^*)^{-1})$$

(49b) $$\upsilon(p) = \left[y^0(p)\quad v(p)\quad \varsigma(p)\right]^T = W_v(p)\,\upsilon'(p)$$

(50) $$k_{M\,SOL}(\omega) = r^0(j\omega)\left|1+R(j\omega)G^*(j\omega)\right|^{-1} \leq 1,\ (\forall\omega, \omega\in[0,\infty))$$

(51) $$k_{M\,POL}(\omega) = \left(\left|1+R(j\omega)G^*(j\omega)\right| - r^0(j\omega)\right)\left|1+R(j\omega)G^{\blacksquare}(j\omega)\right|^{-1} \leq 1,\ (\forall\omega, \omega\in[0,\infty))$$

5. Analysis and Conclusion

The obtained results in the present work prove the main conclusion which is that effective methods and structures exist for: fractional robust control of industrial plants in conditions of an a priori uncertainty, using a dead-time compensation and cut-off reaction against the external harmonic signal disturbances. They are structurally and parametrically controlled by the designer during the synthesis of the control systems with variable dead-time.

New and original issues results in the present paper are:

• The proposed robust structures of ***ID*** fractional regulators with ***DTC***- fractional dead-time compensators and ***ML***-cut-off band stop filters with memory and horizontal profile. Its application allows the effective control of objects with variable delay and harmonic noisy industrial environment in the class of robust systems.

• The proposed configuration solutions, method, criteria and algorithm for analytical synthesis of this new class robust fractional ***ML-DTC*** systems, which have an affirmative and proven working efficiency.

• The estimation, confirmation and demonstration of applicability of the proposed solutions and also working capacity of the methods. For this purpose, in the work is implemented general frequency robust ***Nyquist-*** and ***Black-Nichols*** analysis.

• The comparative analysis under same conditions showing the advantages for a specific numerical example of the proposed new class ***ML-DTC*** fractional control system in comparison with the classic system using a ***PID***-regulator. The results analysis shows:

•• one or more orders lower settling time (*figure 16 ÷ figure 19*);

•• considerable higher values of stability margins – gain margin ***GM*** and phase margin ***PM*** (*figure 20 ÷ figure 21*);

•• robust stability and robust performance (*figure 22 ÷ figure 27*) of ***ML-DTC fractional systems*** unlike the system using a ***PID***-regulator which has a robust stability but does not have a robust performance because

$$k_{M\,POL}^{PID}(\omega) > 1,\ (\forall\omega, \omega\in[0,\infty)).$$

They are also the essence of the results in the implementation of the works' purpose – to propose and analyze new class ***ML-DTC fractional control systems*** that give the possibility to reach effectiveness in the engineer practice in controlling plants with variable delay and harmonic industrial disturbances.

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Manuscript received on 11.11.2011

Prof. **Emil Nikolov**, D.Sc. Ph. D. (born 1950)
EDUCATION: M.Sc. (1973) Technical University of Sofia; SCIENTIFIC DEGREES: Ph.D. (1985) „Multivariable Control Systems-Analysis and Synthesis“; D.Sc. (1999) „Robust Control Systems“. EMPLOYMENT: Assistant Prof. (1974-1978); Senior Assistant Prof. (1979-1987); Associated Prof. (1988-1999); Full Professor (2000-Present); Professeur des Universités (invité) I-ere classe de l'Université d'Artois – Faculté de Sciences Appliquées, France (2001-Present); CAREER RELATED ACTIVITIES: Head of Department „Industrial Automation“ TUS-FA (1995-Present); Head of Chaire UNESCO TUS (2004-2008). UNIVERSITY TEACHING EXPERIENCE (TUS – Faculty of Automation and Filière Francophone de Génie Electrique; l'Université d'Artois – Faculté de Sciences Appliquées, France); Scientific director of laboratory „Control Instrumentation and industrial automation“. SCIENTIFIC INTERESTS: Fractional Control Algorithms and Systems; Robust- and Disturbances Absorbing Control and Industrial Applications; Control Instrumentation; Intelligent Control Instrumentation; Control Valves. MEMBER OF: UAI, IEEE, AFCEA, IFAC; Editorial Board of the International Journal of Automation and Control (IJAAC) Undersciences Publishers; PUBLICATIONS in international and Bulgarian journals – 400; BOOKS – 40.

Contacts:
Technical University of Sofia
Faculty of Automation
Department „Industrial Automation“
e-mail: nicoloff@tu-sofia.bg

Associated Professor **Nina Nikolova**, Ph. D. (born 1970)
EDUCATION: M.Sc. (1994) Technical University of Sofia – Faculty of Automation (TUS-FA); SCIENTIFIC DEGREES: Ph. D. (2004) „Research robust control systems“ TU-FA; EMPLOYMENT: Assistant Professor TUS-FA (2002-2003), Senior Assistant Professor TUS-FA (2003) Associated Professor TUS-FA (2007-Present). PUBLICATIONS IN INTERNATIONAL and BULGARIAN JOURNALS – 75; BOOKS – 2. UNIVERSITY TEACHING EXPERIENCE: Technical University of Sofia – Faculty of Automation and Filiure Francophone de Génie Electrique. Training Courses and consultaion – 10. DEFENDED STUDENTS: Scientific supervisor for the degree-projects of the 20 students. SCIENTIFIC INTERESTS: Robust Control and Industrial Applications; Fractional Control Algorithms and Systems; Disturbances Absorbing Control; Repetitive control Systems.

Contacts:
Technical University of Sofia; Faculty of Automation
Department „Industrial Automation“
e-mail: ninan@tu-sofia.bg

Vladimira Trashlieva, student (born 1988)
Education: finished in 2007 High school – 33 language school „Saint Sofia“ with French language; 2007 – Present – student at Technical University of Sofia, French Faculty (Faculté Francophone de Gŭnie Electrique) and University of Artois, Faculty of Applied Sciences, Laboratory of Computer Engineering and Automation (LGI2A-Laboratoire de Gŭnie Informatique et d'Automatique), Bethune (France) speciality Automation; SCIENTIFIC INTERESTS: Modeling and Control of Motorway Traffic Flow, Automatic Control and Industrial Applications.

Contacts:
Technical University of Sofia
French Faculty and Faculty of Automation
Department „Industrial Automation“;
e-mail: vladimira.trashlieva@gmail.com