Time-dependent Solidification in a Square Cavity with a Temperature-modulated Liquid Layer Cooled from Above

S. Kimura, M. Ueda, K. Kanev

Key Words: *Phase change; dynamic solidification; perturbation solution; convection heat transfer; transient phase change; e-learning.*

Abstract. *In this work, we investigate the dynamic response of a solid phase formed during unidirectional solidification below a cooling top wall of a square cavity filled with distilled water and subjected to time-varying heating temperatures at its bottom wall. Assuming a quasi-steady state condition, we have formulated a one-dimensional model that predicts the average thickness of the forming solid phase. While non-dimensionalizing the model equations, three important non-dimensional parameters are identified, namely the Biot number based on the solid phase thickness at steady state, the Stefan number based on the temperature difference between the cooling upper wall and the liquid temperatures, and the Stefan number based on the heating bottom wall and the liquid temperatures. A perturbation solution of the quasi-steady state formulation has been developed for small amplitude temperature variations on the heating bottom wall. The perturbation solution has been extensively tested against a full two-dimensional numerical solution that uses boundary-tracking techniques for tracing the solid-liquid interface, and good general agreements have been confirmed. The solid phase thickness variation with the time and its phase delays have been expressed as a function of the non-dimensional angular frequency of the heating bottom temperature and the above-mentioned three non-dimensional parameters. The implications of this study and its potential for employment in education and practical engineering have been also addressed.*

Introduction

Solidification is widely seen both in our daily life and in large scale processes pertinent to geophysics and planetary physics. The formation of earth crust at mid-ocean ridges and crystallization of magma in magma chambers are great concerns in order to understand the structures of earth interior and plate kinematics [1,2]. The ice formation and melting in the earth polar regions is important in order to analyze the global climate changes, due to its great thermal impact on the earth's energy balance [3]. The amount of snow fall and its melting in the high mountain regions gives a significant influence over the hydrological circulation in the limnology, of which our environmental, agricultural and industrial water resources are greatly dependent [4]. In the field of industrial applications, the solidification also plays an important role in such areas as metal castings and production of single crystal silicon. Various metal alloys are formed as a result of solidification of multi-component molten phase [5].

As mentioned above, solidification phenomena are observed in a wide range of spectrum. However, most of the solidification researches have been conducted for the fixed thermal boundary conditions, i.e. the cooling and the liquid phase temperatures are assumed constant throughout the solidification processes. In reality, either of them is seldom satisfied in a strict sense. Periodic temperature change, for example, is one of the typical cases of temperature variations. Still solidification and melting subject to periodic temperature modulations has not been studied in full detail, and we can only mention a few relevant works. Most of them are concerned with thermal storage performance utilizing phase-change material (PCM). Bransier [6] analyzed conduction-dominated thermal behaviour of a PCM in contact with a fluid undergoing sinusoidal temperature variation. Kalhori and Ramadhyani [7] investigated the heat transfer in a vertical annular storage unit subject to a periodic steady state operation. Jariwana et al. [8] looked at a vertically-positioned cylindrical latent-heat storage container due to a similar periodic operation. More recently, Ho and Chu [9] investigated periodic melting in a square box numerically, when the temperature at the hot vertical wall is modulated in time. They reported the correlations of mean heat transfer rate as well as the oscillating heat transfer amplitude. In the similar scope of interest on the energy storage system, Viswanath and Jaluria [10,11] numerically investigated the transient nature of solidification and melting.

On the other hand, in our earlier papers [12,13] we are primarily concerned about the effect of cooling temperature variations on the dynamic response of solid layer thickness. Kimura and Vynnycky [12] also presented a one-dimensional model for downward solidification from the top boundary in a water layer with vigorous convection. It turns out that the model predicts the experimental results rather poorly. Therefore, the same problem has been revisited recently by Kimura and Kanev [13] in order to improve the one-dimensional solidification model. The newly proposed model has demonstrated a striking improvement, when the theoretical prediction is compared with the experiment. It is found that the key issue in the degenerated model is to evaluate the correct convective heat flux on the solid-liquid boundary. However, the difficulty to estimate the heat flux is particularly pronounced when water is used for working fluid, despite that many heat transfer correlations are compiled in heat transfer handbooks, e.g. Bejan and Kraus [14]. This is due to the fact that convecting flows adjacent to the ice front become extremely complex by the presence of density maximum near $4\,^{\circ}\mathrm{C}$. Convection heat transfer near $4\,^{\circ}\mathrm{C}$ in a horizontal water layer heated from below was studied numerically by Blake et al. [15]. They encountered non-uniqueness in the heat transfer rates dependent on the convecting patterns realized in the horizontal layer. Freezing of conduits is a classical problem, often discussed in the context of forced convection heat transfer and cooling temperature influence, that may cause detrimental impact on water supply systems in cold region. The complex ice formation processes depending on the cooling temperature of pipe wall have been reviewed in a book by Lock [16].

In the present work, it is the heating temperature that is varied periodically. The oscillating heating boundary condition is employed in an attempt to simulate thermal and momentum surges occurring in the liquid layer. Such situations arise in many engineering and environmental problems. For example, high temperature molten materials are often needed to stay in the vessels for some time and to flow out through the channels and piping systems in various process engineering. In these cases, it is often desirable to have the vessel and pipe walls coated with the solidified thin layers, in order to prevent direct contact between the walls and highly reactive and high temperature molten materials. There is a great interest to determine a parametric range where the solidified layer on the vessel wall can survive safely under a certain level of thermal and momentum surges in the liquid region. In the environmental issue, development of thermal structure and mixing process in the lake water also act as the thermal and momentum surges during the ice formation and melting time. Such environmental issues have been addressed in a book by Kantha and Clayson [17].

We continue to seek for a possible development of one-dimensional simple model for predicting the response of ice-layer front, when the heating temperature oscillates at a steady period and amplitude. A horizontal water layer subject to downward solidification from the top is assumed as before, but this time it is the heat flux from the liquid region that oscillates in time. First quasi-steady one-dimensional model, describing the energy conservation on the solid-liquid boundary, is presented, and a perturbation solution follows. We also provide a limited number of two-dimensional numerical results in order to test the validity of the proposed degenerated model. In our view, developing a simple one-dimensional model plays an important role, in order to identify nondimensional parameters that characterize the behaviour of the system, and their relative importance. It should be also mentioned that the degenerated model can be computed very easily, and a rough picture of parametric dependency will be obtained quickly.

Mathematical Formulation

Problem Description. Referring to *figure1*, a square cavity of two vertical insulating walls and constant temperature top and the bottom walls, is filled with water. The temperature of the water is initially set to a higher than the solidification point value, then the temperature of the top wall is quickly lowered to an arbitrary subzero value, while the bottom is kept at the initial temperature, so that one could expect vigorous convection in the liquid region during the development of ice layer from the top wall. In order to avoid an unnecessary complexity due to the density extremum at $4\,^\circ\mathrm{C}$, we assume that the initial temperature is much higher than $4\,^\circ\mathrm{C}$, typically 10 to $15\,^\circ\mathrm{C}$. After a steady state has been reached, the bottom temperature is modulated with a specific temperature amplitude and oscillation frequency. We investigate a dynamic solid layer thickness response against the bottom temperature modulation, which simulates a temperature and momentum surge arising in the liquid.

Quasi-Steady State One-Dimensional Model. Assumption of quasi-steady state of temperature in the solid layer greatly simplifies the mathematical formulation, since we can get rid of second-order partial differential equation for heat diffusion. This assumption is valid as long as the solid-liquid boundary moves more slowly than the heat diffusion time within the solid layer. The condition can be expressed by

$$\frac{l^2}{\alpha_S} \ll \omega^{-1} \tag{1}$$

in a strict sense, where α_s, l, ω are the thermal diffusivity of solid, the average solid layer thickness at steady state and oscillation angular frequency of the bottom heating wall respectively. One-dimensional model can be a good simplification as long as we are only concerned about the average solid layer thickness, as already demonstrated by Kimura and Kanev [13].

The bulk of liquid below the solid layer is assumed to be dominated with vigorous convection due to a large enough temperature difference between the solidification front and the bulk of liquid or the bottom wall temperature. Then the energy balance on the solid-liquid boundary can be written as follows:

$$\rho_S L \frac{dz^*}{dt^*} + h(T_H - T_0) = k_S \frac{T_0 - T_C}{z^*} \tag{2}$$

where * denotes dimensional quantities, and ρ_s, L, z, t, h, k_s, T_C, T_0, T_H are the solid-layer density, latent heat, vertically downward coordinate, time, convective heat transfer coefficient, thermal conductivity of solid layer, the cooling wall temperature, the liquidus temperature, and the liquid body temperature respectively. It should be noted that a linear temperature profile is assumed in the solid. Equation (2) yields the steady state solid-layer thickness, which forms the fundamental length scale of the present problem. Based on this length scale, the dimensionless version of equation (2) becomes

$$\frac{dz}{dt} + Ste \cdot Bi = \frac{S}{z}\,, \tag{3}$$

where the following non-dimensionalization has been carried out

$$l = \frac{k_S(T_0 - T_C)}{h(T_H - T_0)}\,,\ t = \frac{t^*}{l^2/\alpha_S}\,,\ z = \frac{z^*}{l}\,, \tag{4}$$

and each dimensionless number is defined by

$$Bi = \frac{hl}{k_S}\,,\ S = \frac{c_S(T_0 - T_C)}{L}\,,\ Ste = \frac{c_S(T_H - T_0)}{L}\,. \tag{5}$$

The equations (3) and (5) explicitly indicate that the important non-dimensional numbers of the present problem are the Biot number based on convective heat flux on the solid-liquid boundary, the Stefan number based on the super-cooling of the cold wall, and the Stefan number based on the super-heating of the liquid region.

A Perturbation Analysis. Equation (3) looks very simple, but unfortunately no analytical solution is possible. Therefore we seek for a perturbation solution based on the smallness of the liquid body temperature modulation. If the above assumption is made, the associated heat flux modulation can be represented with either temperature or heat transfer modulation. Here we employ the latter case, and the associated heat flux modulation can be expressed as

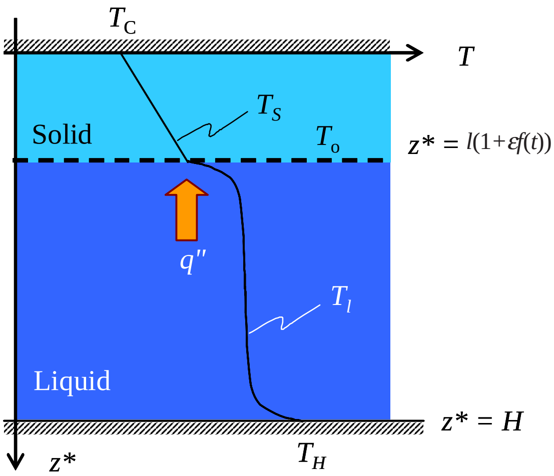


Figure 1. Schematic diagram of one‐dimensional model

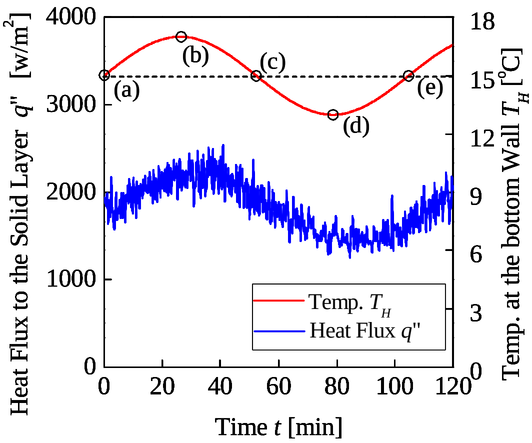
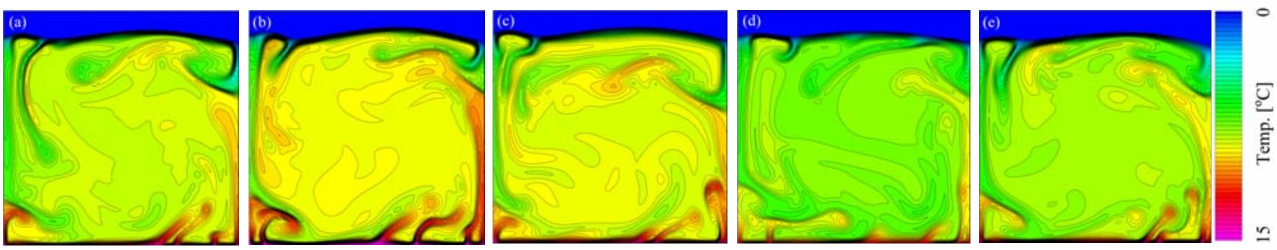


Figure 2. Bottom wall temperature and the heat flux modulation in a single period of time(S=0.049)



(a) (b) (c) (d) (e)

Figure 3. Oscillating temperature fields generated by two‐dimensional simulations. (a)‐(e) are corresponding to those in figure 2

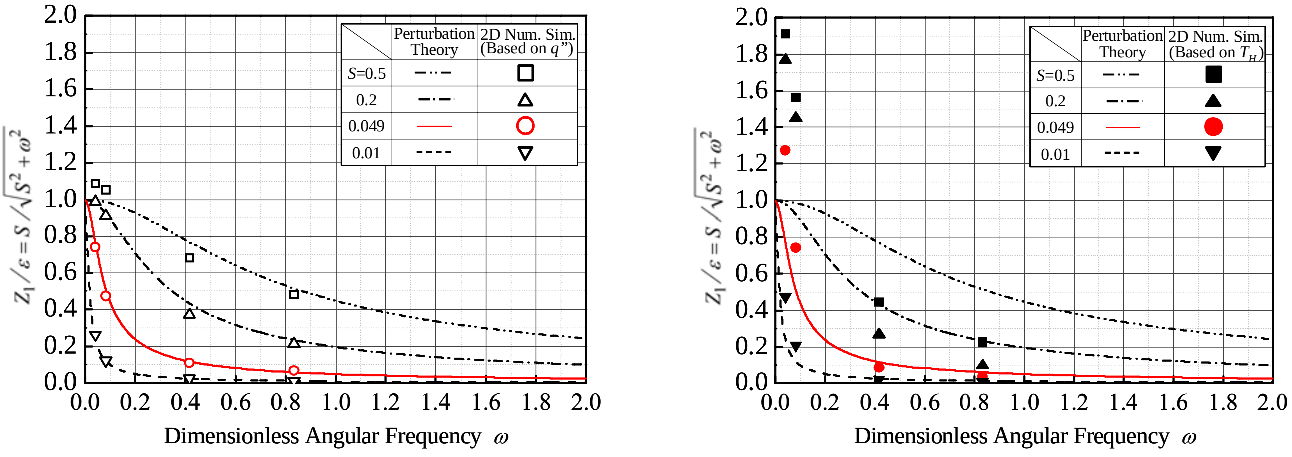


Figure 4. Oscillation amplitude of Ice-layer thickness as a function of nondimensional angular frequency for different values of *S*. The perturbation results are compared with the two-dimensional results shown with symbols for *S*=0.049. (a) Numerical results are correlated with the heat flux modulation at the solid-liquid boundary, (b) Numerical results are correlated with the bottom temperature modulations

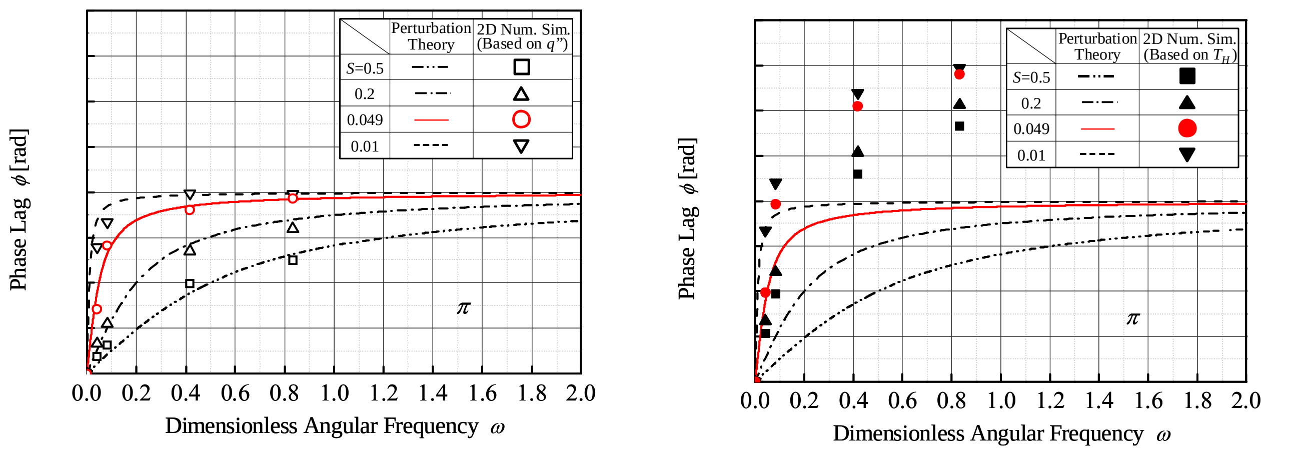


Figure 5. Phase delay of ice layer oscillation relative to that of heat flux on the ice-water interface predicted by perturbation analysis for different values of *S*. Two-dimensional numerical results are shown with symbols for *S*=0.049 for comparison, (a) Numerical results are correlated with the heat flux modulation at the solid-liquid boundary, (b) Numerical results are correlated with the bottom temperature modulations

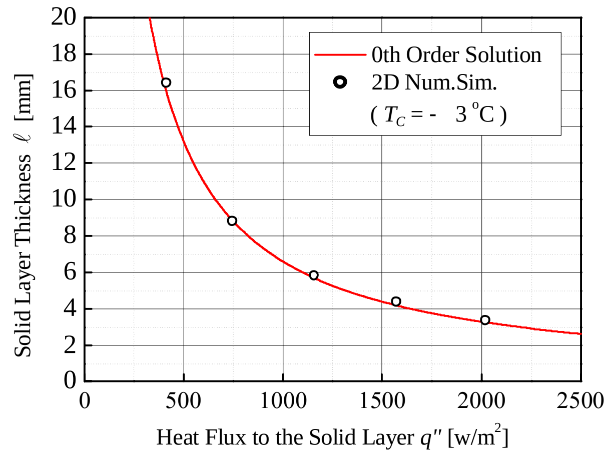


Figure 6. Ice-layer thicknesses at steady state for different values of heat fluxes on the ice-water interface and a fixed cooling temperature predicted by perturbation analysis. The two-dimensional numerical results are shown with symbols for comparison

(6) $h(t) = h_0(1 - \varepsilon f(t))$,

where h_0, ε, $f(t)$ are the convective heat transfer coefficient at steady state, a small dimensionless amplitude, and a function of order $O(1)$ expressing the modulation. Assuming that the solid-layer thickness can be also expanded in terms of a power series of ε,

(7) $z = z_0 + \varepsilon z_1 + O(\varepsilon^2)$

Then, equation (3) becomes

(8) $$\frac{dz_0}{dt} + \varepsilon \frac{dz_1}{dt} + \cdots - S\left(\frac{1}{z_0} - \varepsilon \frac{z_0}{z_1^2} + O(\varepsilon^2)\right) = -C_0(1 - \varepsilon f(t)),$$

where

(9) $$C_0 = \frac{c_S(T_H - T_0)}{L} \cdot \frac{h_0 l}{k_S} = Ste \cdot Bi$$

is the dimensionless number defined at a steady state.

Collecting terms of 0th order of ε, and by setting the time derivative to zero a steady state solution can be obtained as

(10) $$z_0 = \frac{S}{C_0} = \frac{S}{Ste \cdot Bi} = 1.$$

The first order correction $z_1(t)$ can be obtained by collecting terms of 1st power of ε

(11) $$\frac{dz_1}{dt} + S\frac{z_1}{z_0^2} = C_0 f(t).$$

Here, in order to demonstrate the dynamic behaviour of the solid layer, we introduce a simplest modulation, namely one expressible in terms of trigonometric function, $f(t) = \sin \omega t$. The solution of the above correction equation can be readily obtained as a solution of non-homogeneous linear equation. The solution for $z(t)$up to the first order correction term is, therefore, obtained as

(12) $$z(t) = 1 + \varepsilon\left\{\frac{S\omega}{S^2 + \omega^2}e^{-St} + \frac{S}{S^2 + \omega^2}(S \sin \omega t - \omega \cos \omega t)\right\}$$

The first term in the bracket will decay exponentially with time, and it is the second term that will eventually dominate the modulation of solid thickness. It should be noted that the modulation of solid layer thickness is independent of the Biot number and the super-heating Stefan number, but a function of the super-cooling Stefan number and the angular frequency. Immediate obtainability of such important results by inspection is a good example to demonstrate the power of analytical solution. This solution will be compared with the full two-dimensional numerical results.

Convection Heat Flux. At this stage it is helpful to review the convective heat transfer coefficient on the solid-liquid boundary. We assume that the convective heat transfer from the liquid region is due to buoyancy-driven flows caused by the temperature differences between the top and the bottom, such as Rayleigh-Benard convection between two parallel plates. The Rayleigh number in the liquid region Ra and the horizontally-averaged Nusselt number Nu at the cooling side are defined as follows in the present case, e.g.Turner [18]:

(13) $$Ra = \frac{g\beta\Delta T(H - l)^3}{\alpha_l \nu_l}, \quad Nu = \frac{q''(H - l)}{k_l}$$

where, g is the gravitational acceleration, β is the fluid thermal expansion coefficient, H is the height of the square cavity, $\Delta T = T_H - T_0$ is the temperature difference between the liquidus and the liquid region, q'' is the convective heat flux on the solid-liquid boundary, is the kinematic viscosity, and the subscript l denotes properties of liquid.

Then, if Ra is large enough, namely, $Ra > 5 \times 10^5$, the Rayligh number and the Nusselt number satisfy the following relationship,

(14) $Nu \propto Ra^{1/3}$

Therefore, in connection with q'', we obtain the following equation (15) by combining equations (13) and (14).

(15) $$q'' = \frac{k_l \Delta T}{(H - l)} Nu \propto k_l \Delta T \left\{\frac{g\beta\Delta T}{\alpha_l \nu_l}\right\}.$$

Equation (15) implies that heat flux to the cooling boundary from the liquid region is constant, and independent of the distance between the two plates. Consequently, in the one-dimensional model, during unidirectional solidification, the convective heat transfer to the solid-liquid interface from liquid region can be taken as a constant value, independent of the solid layer thickness, as long as the Rayleigh number remains large enough to satisfy the condition $Ra > 5 \times 10^5$. By a careful two-dimensional numerical simulation, the horizontally-averaged heat flux on the solid-liquid boundary in the 10cm square cavity filled with distilled water heated by $T_H = 15°C$ is determined as $q'' = 1813 W/m^2$.

Two-dimensional Numerical Simulation

Description of Two-Dimensional Numerical Code. Two-dimensional full numerical simulations have been performed extensively to test the above-developed analytical solution. The numerical code has been developed based on finite difference method. Water was assumed to be incompressible and viscous, and a laminar and unsteady model was employed. In order to deal with the moving boundary problem, the boundary tracking method which performs coordinate transformation in the vertical direction was adopted. The governing equations were the equation of continuity, the Navier-Stokes equation, the energy equations in the solid and liquid regions, the equation for energy conservation on the solid-liquid interface. The Boussinesq approximation was also adopted in the Navier-Stokes equation. The 3rd-order upwind scheme and the 4th-order central difference scheme were respectively employed for the convection and diffusion terms. The 2nd-order Crank-Nicolson method was applied to time-integration.

The computational domain was divided into solid and liquid regions, and the governing equations were discretized on non-uniform grid networks with 51×201 and 201×201 (height by width) grid points respectively. In the horizontal direction, the grid space was symmetric with respect to the centerline: the grid points were dense near the walls and the grid becomes coarser as departing from the side walls, in the both solid and liquid regions. In the vertical direction, the grid points were dense near the boundaries in the liquid. On the other hand, the vertical grid points were spaced evenly in the solid.

Verification of the Numerical Code. The present numerical code has been tested extensively against the experimental results. The experiment has been performed using a 10 cm cubic container filled with distilled water. The bottom wall temperature was fixed to $T_H = 15°C$, and the cooling top wall was varied from -3 °C to -7 °C, in order to produce several different ice-layer thicknesses at steady state. We compared the horizontally averaged heat flux obtainable by measuring the average ice-layer thickness at steady state and the corresponding numerical results. It was found that both agree well within 1%. Thus the code validation has been completed.

Representative Numerical Results. We show one representative series of two-dimensional temperature field variations during one period of sinusoidal heating temperature oscillation in *figure 2* and *figure 3*. *Figure 2* indicates at what time each picture of *figure 3* was generated during the entire period of temperature oscillation. Small scale temperature structures and the deformed ice-water boundary are clearly seen, which cannot be captured by the degenerated one-dimensional model.

Comparison between Perturbation Solution and Numerical Results

Oscillation Amplitudes of Ice-Layer Thickness. Oscillation amplitudes of Ice-layer thickness are shown as a function of nondimensional angular frequency for different values of S in *figure 4*. Perturbation predictions are indicated with lines for several values of S. The perturbation results are compared with the two-dimensional results shown with symbols for S=0.049, corresponding to the solidification of distilled water. A good agreement between the analytical and numerical results can be seen in the *figure 4(a)*. The numerical results based on the heating temperature modulation - *figure 4(b)* show some discrepancy with the perturbation analysis. This is due to the fact that the temperature variation does not linearly influence the heat flux on the ice-water interface.

Phase-Delay of Ice-Layer Oscillation. Another important observation is the phase-delay of the oscillating ice-layer thickness relative to the heat flux or the bottom wall temperature modulation. Again the phase-delays predicted by perturbation analysis are shown with lines, while the two-dimensional numerical results are displayed with symbols for S=0.049. The numerical results agree well with the perturbation results when they are correlated with the heat flux modulation on the ice-water boundary as seen in *figure 5(a)*. *Figure 5(b)*, on the other hand, shows the same results when they are correlated with the bottom temperature modulations. It is clear that the perturbation analysis fails to predict the phase delays. This indicates that there is already a time-lag between the heating bottom temperature and the resulting heat flux on the ice-water boundary.

Steady State Ice-Layer Thickness. The 0th order solution of perturbation analysis gives a steady state ice-layer thickness in nondimensional sense. The results can be translated to dimensional ones, in order to compare with the numerical simulations. The compared results are shown in *figure 6*, and, as it is seen from the figure, the two agree very well.

Heat Transfer e-learning Environment

Heat transfer is an important subject for many of the engineering disciplines. It is thus imperative for engineering students to understand heat flows and the associated processes, such as phase changes described in the present paper. However, because of the invisible nature of heat flows and temperature fields, it should be very helpful to provide engineering students with learning materials rich in visual content not limited to static pictures but also including motion and video. In particular, since phase change is a complex phenomenon involving both conduction and convection, it is hard to predict what happens when the boundary conditions, such as the cooling temperature, are altered over the time. This phenomenon has important applications in energy technology and material processing so both engineering students and practicing thermal design engineers can take advantage of such graphical interfaces. Through such interfaces and graphical environments understanding of the basics of phase change processes for a wide range of parameters, such as cooling temperatures, thermal properties, and strengths of convection heat flux could be greatly facilitated and enhanced. The graphical e-learning environment we have been developing and into which simulations discussed in the present paper are being integrated is applicable to many areas where solidification takes place not only as a learning but also as a design tool. It should be noted that a similar software to visualize discharged jets from tank orifice as an e-learning tool was reported by Kanev et al. [19] and is also considered for integration in the current e-learning environment.

Conclusion

A perturbation solution has been developed for one-dimensional model of solid-layer thickness oscillation due to the periodic temperature surges arising in the liquid region. The perturbation solution demonstrates that the relevant parameters are S (the Stefan number based on the super cooling of the cold wall temperature) and (the non-dimensional angular frequency based on the diffusion time in the solid layer.) A numerical code to compute dynamic solidification processes based on a two-dimensional model has been also developed. The good agreement between the two for a wide range of parameters proves that the present perturbation solution is capable of understanding the in-depth mechanism of dynamic solidifying process. It is proposed that the present perturbation analysis can be used in order to visualize the dynamic solidification processes in an e-learning „Heat Transfer“ class, providing a good sense of physics. A great potential of the degenerated simple model and full numerical simulator as a design tool in CAE (Computer Aided Engineering) is also suggested.

Acknowledgement. The authors gratefully acknowledge the help provided by Mr Tomohiro Kuratani, technician at School of Mechanical Engineering, Kanazawa University, in the course of preparing the present manuscript. This work was partially supported by Grant-in-Aid for Scientific Research (B) 20300269 and (C) 22560191.

References

1. Macdonald, K. C. Mid-Ocean Ridges: Fine Scale Tectonic, Volcanic and Hydrothermal Processes within the Plate Boundary Zone. - *Ann. Rev. Earth Planet. Sci.*, 10, 1982, 155-190.
2. Huppert, H. E., R. S. J. Sparks. Double-Diffusive Convection due to Crystallization in Magmas. - *Ann. Rev. Earth Planet. Sci.*, 12, 1984, 11-37.
3. Hibler, III D., J. E. Walsh. On Modelling Seasonal and Interannual Fluctuations of Arctic Sea Ice. - *J. Physical Oceanography*, 12, 1982, 1514-1523.
4. Freeze, R. A., J. A. Cherry. Groundwater. Prentice Hall, NJ, 1979.
5. Flemings, M. C., Solidification Processing. McGraw-Hill, New York, 1974.
6. Bransier, J. Stockage Periodique par Chaleur Latente Aspects Fondamentaux Lies a la Cinetique des Transferts. - *Int. J. Heat Mass Transfer*, 22, 1979, 875-883.
7. Kalhori, B., S. Ramadhyani. Studies on Heat Transfer from a Vertical Cylinder, with and without Fins, Embedded in a Solid Phase Change Medium. - *J. Heat Transfer*, 107, 1985, 44-51.
8. Jariwala, V. G., A. S. Mujumdar, M. E. Weber. The Periodic Steady State for Cyclic Energy Storage in Paraffin Wax. - *Can. J. Chem. Engng*, 65, 1987, 899-906.
9. Ho, C. J., C. H. Chu. Periodic Melting within a Square Enclosure with an Oscillatory Surface Temperature. - *Int. J. Heat Mass Transfer*, 36, 1993, 725-733.
10. Viswanath, R., Y. Jaluria. Comparison of Different Solution Methodologies for Melting and Solidification Problems in Enclosures, Numer. - *Heat Transfer, part A*, 24, 1993, 77-105.
11. Viswanath, R., Y. Jaluria, Numerical Study of Conjugate Transient Solidification in an Enclosed Region. - *Heat Transfer, part A*, 27, 1996, 77-105.
12. Kimura, S., M.Vynnycky. Time History of Ice-layer Formation at the Cooled Top Boundary and its Dynamic Response to the Time-varying Cooling Temperature, Moving Boundaries V. Edited by Sarler, B., C. A. Brebbia, H. Power, WIT press, 1999, 47-56.
13. Kimura, S., K. Kanev. e-Learning of Phase Change Processes under Vigorous Convection Heat Transfer. - I*nformation Technology and Control*, 8-3, 2010, 12-18.
14. Bejan, A., A.D .Kraus(editors). Heat Transfer Handbook. Wiley, 2003.
15. Blake, K. R., D. Poulikakos, A. Bejan. Natural Convection Near 4 in a Horizontal Water Layer Heated from Below. - *Phys. Fluids*, 27, 1984, 2608-2616.
16. Lock, G. S. H. The Growth and Decay of Ice, Cambridge University Press. Cambridge, 1990.
17. Kantha, L. H., C. A. Clayson. Small Scale Processes in Geophysical Fluid Flows. Academic Press, San Diego, 2000.
18. Turner, J. S. Buoyancy Effects in Fluids. Cambridge University Press, Cambridge, 1980.
19. Kanev, K., S. Kimura, M. Yoneya. e-learning Fluid Flows: Tank Discharge through an Orifice. Inter Academia 2008, 15-18 September 2008, Pecs-Hungary, 144-151.

Manuscript received on 20.12.2011

Shigeo Kimura *received the M.Sc. and the Ph.D. degrees in Mechanical Engineering, both from University of Colorado, Boulder, Colorado, USA in 1980 and 1983, respectively. He is a Professor with Institute of Nature and Environmental Technology, and the School of Mechanical Engineering, Kanazawa University, Japan, where he teaches fluid mechanics, thermodynamics and heat transfer. His main research interests are in transport processes due to environmental fluid motions. On this and related topics he has authored and coauthored more than 100 scientific journal and conference papers and patents. Dr. Kimura is a member of the Japan Society of Mechanical Engineers (JSME) and the American Society of Mechanical Engineers (ASME).*

Contacts:
e-mail: skimura@t.kanazawa-u.ac.jp

Masanori Ueda *received the B.Sc. degree in Mechanical Engineering from Kanazawa University, Japan. He is expecting to obtain his M.Sc. degree in Mechanical Engineering from the same university in March 2012 and to start working for the Komatsu, Ltd. manufacturer of construction and mining equipment in April 2012. In the last three years, he has been working on solidification problems numerically and experimentally. His study resulted in three presentations at domestic and two presentations at international conferences. He is a member of the Japan Society of Mechanical Engineers (JSME) and the Japan Society of Refrigerating and Air Conditioning Engineers (JSRAE).*

Contacts:
e-mail: ueda@ryuko.ms.t.kanazawa-u.ac.jp

Kamen Kanev *received the M.Sc. degree in mathematics and the Ph.D. degree in computer science in 1984 and 1989, respectively. He is a Professor with the Research Institute of Electronics, the Graduate School of Informatics, and the Graduate School of Science and Technology, Shizuoka University, Japan, where he teaches and supervises students majoring in computer and information science. His main research interests are in interactive computer graphics, vision information processing, and user interfaces and surface based interactions. On this and related topics he has authored and coauthored more than 100 scientific journal and conference papers and patents. Dr. Kanev is a member of the IEEE, the Association of Computing Machinery (ACM), and the Asia-Pacific Society for Computers in Education (APSCE).*

Contacts
e-mail: kanev@rie.shizuoka.ac.jp