PID Control Systems with Maximum Gain Margin and Increased Speed Performance

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Abstract. *Novel sub-optimal easy to apply tuning rules for maximum gain margin control are derived for general-form two-degree of freedom (2DOF) PID controllers. As an additional step thus estimated controller parameters are detuned by weighting coefficients for the reference signal in the P and D controller parts according to an extra performance criteria – maximum controlled signal speed with no serious changing of the achieved system gain and phase margins. The approximated solutions obtained after optimization and simplification by Symbolic Toolbox of MATLAB are compared with well-known design methods*

1. Introduction

Minimum-time control is often critical demand for the PID controller in practice [1,2]. There is fundamental tradeoff between the requirements for control system maximum speed performance and its robustness due to the non-linearity and/or time-varying dynamics, disturbances and other factors [3]. It is well-known that PID controllers are very popular [4,5] because of its relatively simple structure which allows intuitive understanding of its behavior but also on its high working effectiveness for handling step responses and load disturbances, giving relatively good robustness of the control system.

Gain and phase margins are basic performance indices for system quality and they are used in PID control design for years. Some of them assure desired margins by polynomial analysis calculating stabilizing regions of the PID parameters [6,7]. Others apply inverse function mapping through fuzzy neural network [8]. According to the authors' experience [10] the approach based on Arctan-approximations can be used and the derived tuning formulae are powerful enough to be competitive to the most popular tuning rules.

Our experience shows that the gain margin is a function both of the plant model and PID controller parameters and to find its maximum is not a simple task. In the paper we will avoid one of the biggest problems in PID controllers – the over-tuning of the D-part [9] taking into account the two degrees of freedom (2DOF) PID as a more complex and generalized form of these controllers with good potential to give control systems robustness. In order to realize new control properties a two-step procedure is proposed and demonstrated: in the first step the basic PID parameters are tuned according to the maximum gain margin criteria, and in the second step additional weighting coefficients for the reference signal in the PID controller P and D parts are tuned according to maximum speed performance criteria. The authors also show how the increasing of the D-part in the ref-

erence-control channel can give increasing speed performance with no influence on the system stability and robustness. The approximated solutions are obtained after optimization and simplification by Symbolic Toolbox of MATLAB.

This paper is structured in the following way. The theoretical foundations of the proposed novel two-steps procedure are given in section 2 and 3, respectively. A test example shows the effectiveness of the proposed approach in section 4. There is a comparative analysis of this method with the well-known „tau-method" for PID tuning [1]. Future development of the problems in the research area is discussed.

2. A Novel Method for Standard PID Controller Tuning

2.1. Preliminary Theoretical Notes

The proposed method for PID tuning is applied mainly for plants with self-stabilization behavior. Their dynamics can be presented by the following three-parametric model

$$(1) \qquad W_m(s) = \frac{K_o}{(Ts+1)} e^{-Ls} \ ,$$

where K_o, T and L are model gain, time-constant and time delay, respectively.

The first step of the tuning procedure is addressed to the standard PID controller with transfer function:

$$(2) \qquad W_{PID}(s) = \frac{K_p \left(T_d T_i s^2 + T_i s + 1\right)}{T_i s}$$

with well-known parameters to be tuned: proportional gain K_p, integral time-constant T_i and derivative time-constant T_d .

In order to simplify the next mathematical tasks and also to limit the undesirable control signal behavior due to the high values of K_p and the influence of the derivative ingredient of the control, the novel method puts two constraints on the controller dynamics:

Constraint 1. The PID controller has only real zeros just the same as the well-known serial PID.

$$(3) \qquad W_{PID}(s) = \frac{K_p (T_1 s + 1)(T_2 s + 1)}{(T_1 + T_2)s} .$$

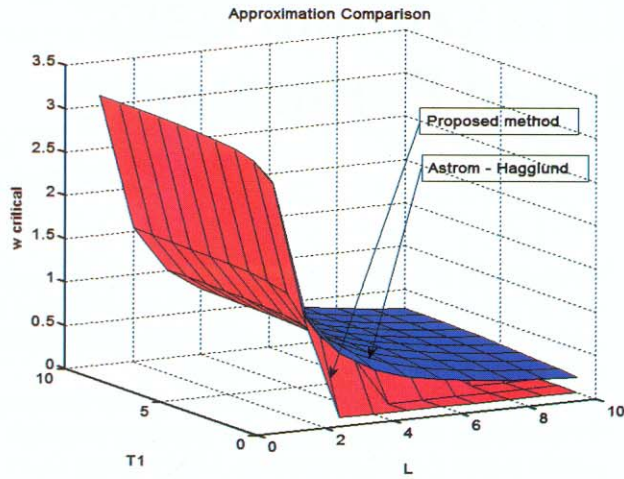Taking into account (2) and (3) the new parameters are

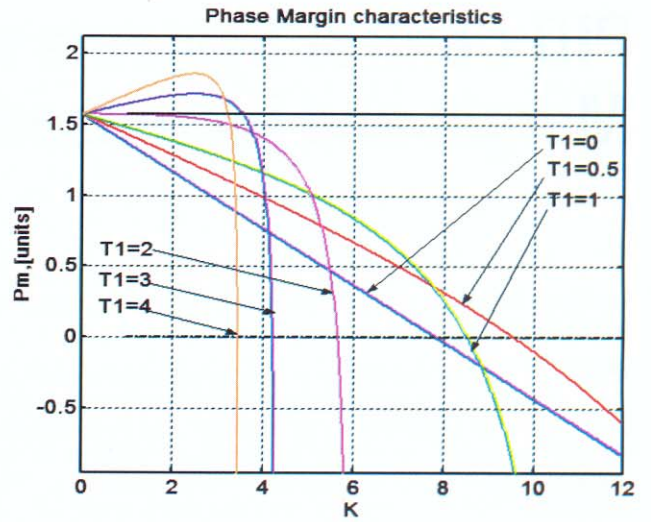**Figure 1.** Comparison between (8) and (9) for $\varphi(\omega) = -\pi$

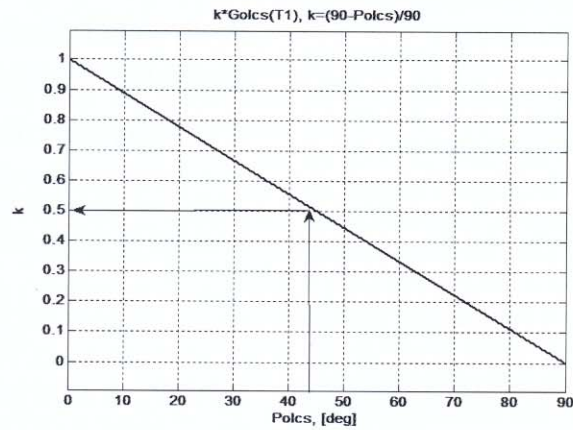**Figure 2.** Phase margin for different T_1

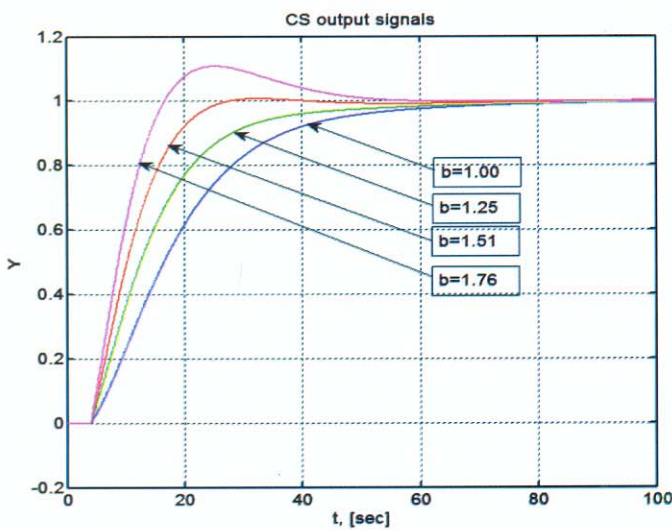**Figure 3.** Tuning rule for k (y axis) based on P_{olcs}^{des} (x axis)

Figure 4a. CS output signals for various b and c
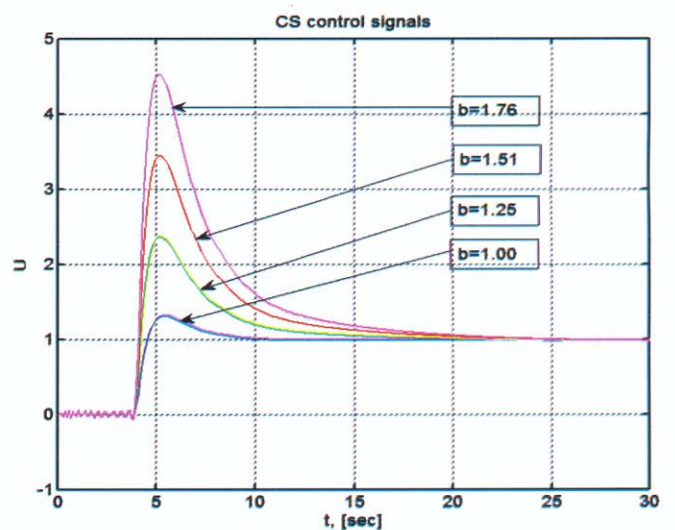

Figure 4b. Control signals for various b and c

Figure 5a. CS output signals for proposed and AH PID tuning

Figure 5b. Control signals for proposed and AH PID tuning

Figure 6. Sensitivity functions

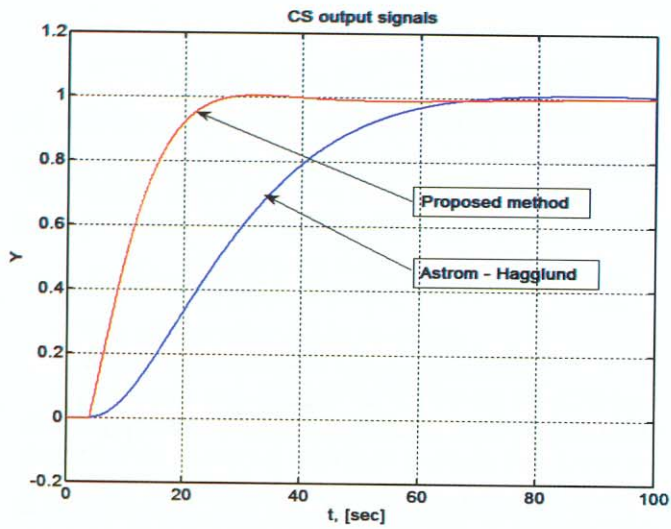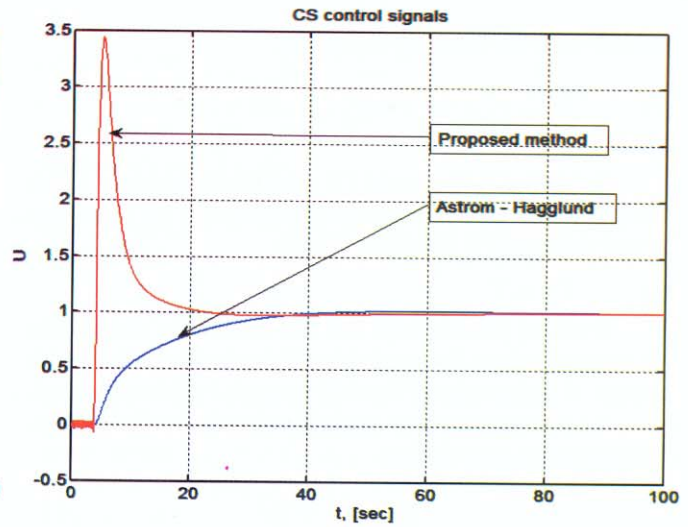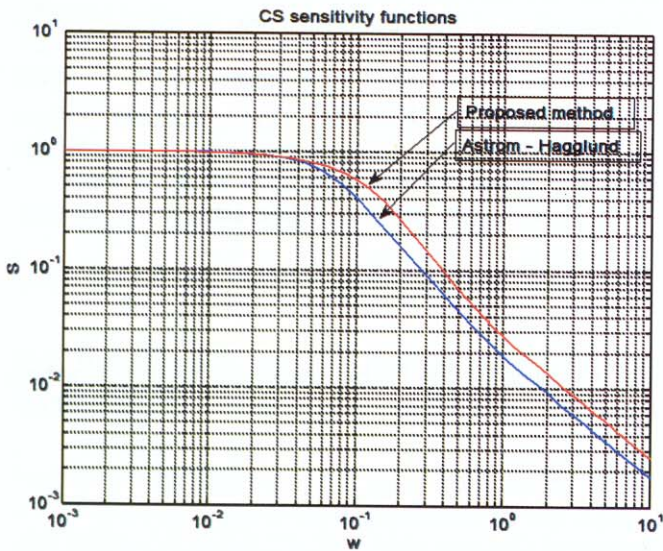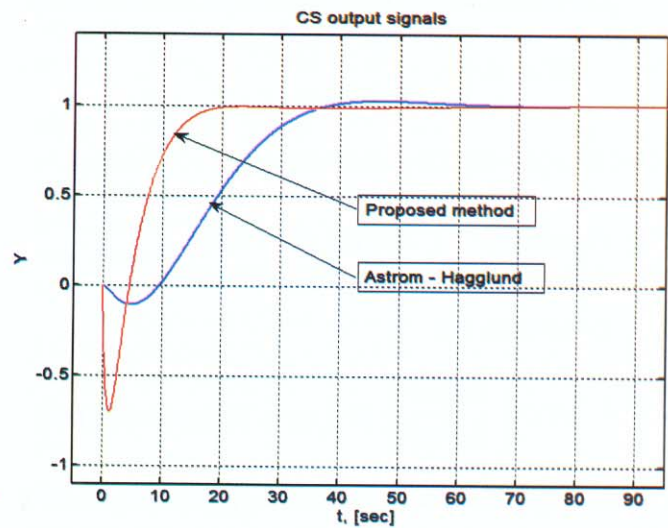**Figure 6a.** CS output signals for proposed and AH PID tuning
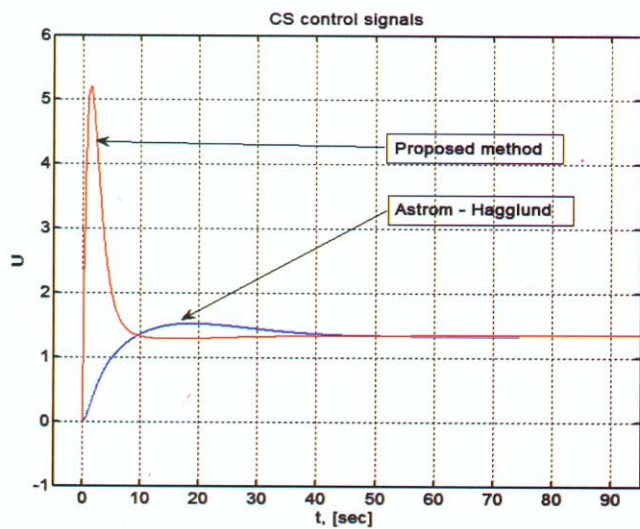

Figure 6b. Control signals for proposed and AH PID tuning
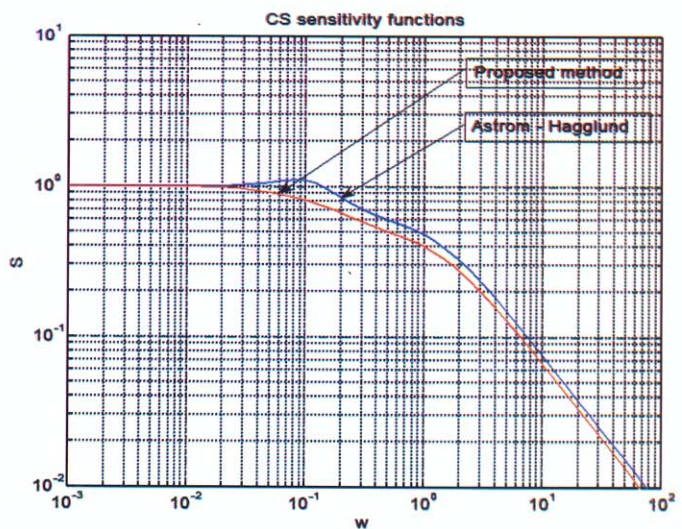

Figure 7. Sensitivity functions

defined under the following conditions:

$$(4) \qquad T_i = T_1 + T_2 , \qquad T_d = \frac{T_1 T_2}{T_1 + T_2} .$$

Constraint 2. The model (1) transfers its dynamics to the controller. Then zero-pole canceling happens in the open-loop control system (CS) in conformity with

$$(5) \qquad T_2 = T .$$

It will be obvious later that the constraints simplify the optimization criteria for maximizing the gain margin in CS so convenient tuning rules of the PID controller will be produced.

For that purpose PID controller description (3) is transformed according to (5) into

$$(6) \qquad W_{PID}(s) = \frac{K_p (T_1 s + 1)^2}{(T_1 + T)s} .$$

The transfer function of the open-loop CS takes the form

$$(7) \qquad W_{olcs}(s) = \frac{K}{(T_1 + T)} \frac{(T_1 s + 1)}{s} e^{-Ls} ,$$

where the open-loop CS gain is written as $K = K_o K_p$.

2.2. Parameter T_1 Tuning via Phase Characteristic Nonlinear Approximation

On condition that

$$\varphi(\omega) = \operatorname{arctg}\left(\frac{\operatorname{Im}(\omega)}{\operatorname{Re}(\omega)}\right), \ W(j\omega) = \operatorname{Re}(\omega) + j \operatorname{Im}(\omega),$$

the open-loop CS phase characteristic $\varphi_{olcs}(\omega)$ is the sum of the phase characteristics of its differential, integrate and delay parts.

$$\varphi_{diff}(\omega) = \frac{\pi}{2} - arctg\left(\frac{1}{T_1 \omega}\right), \quad \varphi_{int}(\omega) = -\frac{\pi}{2},$$

$$\varphi_{tau}(\omega) = -L\omega$$

Hence

$$(8) \qquad \varphi_{olcs}(\omega) = -arctg\left(\frac{1}{T_1 \omega}\right) - L\omega .$$

After nonlinear approximation using [10]

$$\frac{d}{dx} arctg\left(\frac{1}{x}\right) \approx \frac{d}{dx} \frac{1}{x}$$

expression (8) can be simplified, so

$$(9) \qquad \varphi_{olcs}^{nonlin}(\omega) \approx -\frac{1}{T_1 . \omega} - L\omega .$$

Statement 1. $\varphi_{olcs}^{nonlin}(\omega_{-\pi}) \approx \varphi_{olcs}(\omega_{-\pi})$

Graphical Demonstration. According to the definition, the gain margin G_{olcs} is reciprocal to the amplitude of W_{olcs} for the frequency $\omega_{-\pi}$. After the solution of the equation $\varphi_{olcs}^{nonlin}(\omega) = -\pi$, the value $\omega_{-\pi}$ is defined as

$$(10) \qquad \omega_{-\pi}^{nonlin} = \frac{\pi\left(1 + \sqrt{1 - \dfrac{4L}{\pi^2 T_1}}\right)}{2L} \quad \text{for} \quad T_1 \geq \frac{4L}{\pi^2} .$$

If the critical frequency $\omega_{-\pi} = f(T_1, L)$ is simulated as a function of, for example, $T_1 \in [1,10]$ and $L_1 \in [1,10]$ for integer values of these coefficients only, we find for $T_1 \geq \dfrac{4L}{\pi^2}$ that the calculated total integral square error for nonlinear approximation (9) is $J = 0.0231$. This is illustrated on figure 1 by complex values of $\omega_{-\pi}$ for the nonlinear approximation outside of region of validity replaced by zeros.

Then the gain margin is worked out from

$$(11) \quad G_{olcs}^{nonlin}(\omega) = \frac{1}{|W_{olcs}(j\omega)|} = \frac{T + T_1}{K\sqrt{T_1^2 + \dfrac{1}{\omega^2}}} .$$

for the value (10) and the expression is a function of the tune parameter T_1

$$(12) \qquad G_{olcs}^{nonlin}(T_1) = \frac{T + T_1}{K\sqrt{T_1^2 + \dfrac{4L^2 T_1^2}{\left(\pi T_1 + \sqrt{\pi^2 T_1^2 - 4L T_1}\right)^2}}} .$$

The tuned parameter T_1 can give the maximum margin of (12) as a solution of the optimization task $T_1 = \arg\max G_{olcs}^{nonlin}$.

We have obtained the exact result of the optimization task, which is in the Appendix. It is extremely large and complex, which makes it hard for practical use (except in computer simulations). This is the reason why we have obtained an approximation. We do not claim that this is the best approximation, but it leads to good performance shown later by simulations in the work. We deduced this result in three heuristic steps:

Step 1. Solving a simpler problem

The main reason for the complexity of (A6) is that T_1 appears both in the numerator and the denominator of (12). If we remove it from the numerator, the solution would be simpler. There is a case – a PD controller and a plant as an integrator with time delay

$$W_{olcs}(s) = \frac{K(T_1 s + 1)}{s} e^{-Ls}$$

which gives the following approximation:

$$G_{olcs}^{nonlin}(T_1) = \cfrac{1}{K\sqrt{T_1^2 + \cfrac{4L^2T_1^2}{\left(\pi T_1 + \sqrt{\pi^2 T_1^2 - 4LT_1}\right)^2}}}$$

This result is almost the same as in (7).

The exact solution of this simpler task is

$$T_{1_{opt}}^{nonlin} = \frac{8291596482138231}{18014398509481984}L \approx 0.4603L$$

This approximation will be used next.

Step 2. Extension of the simpler task result

The simple solution does not depend on the plant time-constant T presented in (12). The heuristic view on the problem points that the solution of (12) must approach the simpler solution after increasing of T because the weight of T_i in the numerator of (12) will becomes smaller and the optimum value of T_i will be shifted on the real axis less from the value for the simpler solution. In this way the approximation has the next uncertain form:

$$T_{1_{opt}}^{nonlin} \approx 0.4603L\left(1 + \frac{?}{?T}\right)$$

Step 3. The final choice of expression by experiments

After series of simulations with different values of the parameters of (12) we obtained the following result

$$(13) \quad T_{1_{opt}}^{nonlin} \approx 0.4603L\left(1 + \frac{\pi - 1}{\pi T}\right).$$

2.3. Parameter T_1 Tuning via Phase Characteristic Linear Approximation

The nonlinear approximation of the phase characteristic leads to definitely restrictions for T_1 together with the need to use a second approximation of the very complex nonlinear expression before working out (13). This is the reason to try another approach to solve the problem via linear approximation of the differential part in phase characteristic $\varphi_{olcs}(\omega)$ in the form [10]

$$\varphi_{diff}(\omega) = arctg(T_1.\omega) \approx T_1.\omega,$$

so

$$(14) \quad \varphi_{olcs}^{lin}(\omega) \approx T_1.\omega - \frac{\pi}{2} - L.\omega.$$

Although this approach is not as accurate as the nonlinear one, it gives a simpler usable solution. Proceeding with the same manner as in section 2.2, for $\omega_{-\pi}^{lin} = \frac{\pi}{2(L - T_1)}$ we get the gain margin as function of T_1

$$(15) \quad G_{olcs}^{lin}(T_1) = \frac{T + T_1}{K\sqrt{T_1^2 + \frac{4(L - T_1)^2}{\pi^2}}}.$$

The gain margin can be maximized for the optimal parameter

$$(16) \quad T_{1_{opt}}^{lin} = 4\frac{L(L + T)}{4L + \pi T^2 + 4T}.$$

The advantages of the phase characteristic linear approximation are the following:

a. Simple optimal solution can be found with no additional approximation even in cases the nonlinear approach gives sub-optimal one.

b. The optimal parameter $T_{1_{opt}}^{lin}$ has no restricted values during the optimal solution search.

Example. We can study graphically on *figure 2* the interdependence between the gain margin and phase margin, their cross-dependence and the value of the tunable parameter T_1 for simple three-parameter model (1) with the values $K_0 = 1$, $T = 10$ s, $L = 2$ s.

The dash line on the zero value of the phase margin can be called „line of instability". Here the phase margin is a function of the total open-loop CS gain. From *figure 2* one can find the system instability ultimate gain in order to choose the gain margin.

The phase characteristic with linear approximation comes close to the optimal solution for $T_1 = 0.5$. Here the phase margin is almost linear, it displays weak sensitivity during the gain K goes up with no dramatically drops for higher values of T_1. This tuning behavior proves the expectation that the CS saves good robustness in terms of sensitivity with respect to the increasing system gain and perturbations.

The proposed approach provides the optimal tuning of the PID controller parameters T_i and T_d as a function of T_1 in order the maximum CS gain margin to be achieved. Something more, analytical expressions of this gain margin can be obtained as well.

2.4. K_p Tuning for Desired Phase Margin

In this section K_p tuning will be presented for acceptable both CS signals speed performance and robust control. For that purpose an expression of the frequency ω_1 at which $|W_{olcs}(j\omega)| = 1$ has to be obtained:

$$\omega_1 = \frac{\sqrt{-T_1^2 K^2 + T^2 + 2TT_1 + T_1^2}\, K}{-T_1^2 K^2 + T^2 + 2TT_1 + T_1^2}.$$

The phase margin for this frequency is obtained from (8), replacing this critical frequency ω_1:

$$(17) \quad P_{olcs}(T_1) = -\frac{\pi}{2} + arctg\left(\frac{-T_1^2 K^2 + T^2 + 2TT_1 + T_1^2}{T_1\sqrt{-T_1^2 K^2 + T^2 + 2TT_1 + T_1^2}\,K}\right) - \frac{\sqrt{-T_1^2 K^2 + T^2 + 2TT_1 + T_1^2}\,K}{-T_1^2 K^2 + T^2 + 2TT_1 + T_1^2}L.$$

The approximation of (17) is made assuming the phase margin is a straight line – for $T_1 = 0$ in case of P controller and plant model - integrator plus time delay, when $\varphi_{olcs}(\omega) = -L\omega$. This condition is simple and conservative, because the phase margin increases for $T_1 > 0$. The result is

$$(18) \quad P_{olcs}^{approx} = -\frac{\pi}{2} + \frac{\pi K_p}{2G_{olcs}(T_1)}$$

where $G_{olcs}(T_1)$ is the pre-calculated gain margin (12) or (15)

but with the plant gain only. The expression (18) is used to derive a practical formula for choice of the proportional PID coefficient as a function of the desired phase margin P_{olcs}^{des}

$$(19) \quad K_p = \frac{90° - P_{olcs}^{des}}{90°} G_{olcs}(T_1) = k G_{olcs}(T_1),$$

with graphical representation of coefficient k shown on *figure 3*.

According to *figure 3* some standard phase margins are given in *table 1*.

Table 1. K_p for desired phase margin

Desired phase margin P_{olcs}^{des} (deg)	Necessary K_p
- 45	$K_p = G_{olcsn}(T_1)/2$
- 60	$K_p = G_{olcsn}(T_1)/3$
- 75	$K_p = G_{olcsn}(T_1)/6$

2.5. Tuning Algorithm

The following steps are used to tune the PID controller (2):

Step 1. The time-constant T_1 can be chosen as the smaller value of (12) and (15).

Step 2. Parameters T_i and T_d are calculated from (4) and (5).

Step 3. K_p is taken from (19) with k from *figure 3* for desired phase margin.

3. A Novel Method for 2DOF PID Controller Tuning

3.1. Preliminarily Theoretical Notes

One possible way to get over the conflicting CS requirements for speed performance and robustness is to use two-degree of freedom (2DOF) controller:

$$(20) \quad W_{PID\_2DOF}(s) = K_p\left(b + \frac{1}{T_i s} + cT_d s\right)r(s) - K_p\left(1 + \frac{1}{T_i s} + T_d s\right)y(s)$$

where b is the weighting coefficient of the reference signal in the 2DOF PID controller proportional part and c is the weighting coefficient of the reference signal in the 2DOF PID controller derivative part [1].

The following statements explain the reason for this assumption:

Statement 2. *The weighting coefficients b and c in (20) do not influence the CS stability.*

Proof. Let the transfer function of the closed-loop system $W_{close}(s)$ be defined by

$$W_{close}(s) = \frac{y(s)}{r(s)}.$$

Respectively the expressions of y and u are presented in the form:

$$y(s) = u(s)W_{plant}(s),$$

$$u(s) = K_p\left(b + \frac{1}{T_i s} + cT_d s\right)r(s) - K_p\left(1 + \frac{1}{T_i s} + T_d s\right)y(s)$$

After few obvious substitutions

$$y(s) = K_p\left(b + \frac{1}{T_i s} + cT_d s\right)W_{plant}(s)r(s) - K_p\left(1 + \frac{1}{T_i s} + T_d s\right)W_{plant}(s)y(s)$$

and

$$\left[1 + K_p\left(1 + \frac{1}{T_i s} + T_d s\right)W_{plant}(s)\right]y(s) = K_p\left(b + \frac{1}{T_i s} + cT_d s\right)W_{plant}(s)r(s).$$

the following expression is formed

$$(21) \quad W_{close}(s) = \frac{y(s)}{r(s)} = \frac{K_p\left(b + \frac{1}{T_i s} + cT_d s\right)W_{plant}(s)}{\left[1 + K_p\left(1 + \frac{1}{T_i s} + T_d s\right)W_{plant}(s)\right]}.$$

The **Statement 2** is true because the characteristic polynomial in (21) does not depend on the weighting coefficients b and c in (20).

According to this statement the following remark is done:

Remark 1. *The coefficients b and c in PID controller (20) can have any real values but not just the standard $b, c \in [0,1]$.*

Obviously the very proper values of the weighting coefficients for the reference signal in P- and D- part of (2) are result from the traditional engineering concepts and presumptions in order to avoid high overshoot in step response or non-minimal phase dynamics, rather than any mathematical considerations.

Statement 3. *The coefficients b and c in (20) do not take part in the sensitivity function, but only in the complement sensitivity function.*

Proof. According to the traditional definitions the sensitivity function $S(s)$ is

$$S(s) = \frac{e(s)}{r(s)} = \frac{1}{\left[1 + K_p\left(1 + \frac{1}{T_i s} + T_d s\right)W_{plant}(s)\right]},$$

where $e(s) = r(s) - y(s)$ is the CS error, and the complementary sensitivity $T(s)$ is

$$T(s) = W_{close}(s) = \frac{K_p\left(b + \frac{1}{T_i s} + cT_d s\right)W_{plant}(s)}{\left[1 + K_p\left(1 + \frac{1}{T_i s} + T_d s\right)W_{plant}(s)\right]}.$$

So, the proof of the statement stands for reason.

3.2. Task Formulation and its Theoretical Solution

The authors plan to use the formulated statements in section 3.1 for *tuning the b and c coefficients independently both from the other coefficients of the PID controller (20) and the model parameters in (1)* in the following way:

First, the algorithm from section 3.1 is to be applied for

tuning the basic PID parameters K_p, T_i and T_d. At this stage the sensitivity function is implicitly limited. Second, the coefficients b and c as functions both of the already tuned PID parameters (20) and the parameters of the plant model (1), are to be tuned in order to maximize the CS speed performance for overshoot under 20%. This task is a subject of formulation and solution. The authors approach to the task goes through the idea *that the desired quality of the PID control can be assured by CS (21) with minimum inertia of the reference signal transforming into output one. To realize this goal (theoretically zero overshoot) it is necessary the poles and the zeros of (21) to be cancelled.* In practice the solution is a result of approximation both in the model (mainly because of the time delay) and in mathematical expressions (mainly because of simplification), so some overshoot of the CS output for the step reference can be observed.

Let replace $W_{plant}(s)$ in (21) with $W_m(s)$ (1) plus first order Pade approximation of the time delay in (1), and then apply the PID controller (3), (4) and (5). The result will be the following expression:

$$(22) \quad W_{close}(s) = \frac{K_o K_p \left(c T T_1 s^2 + b(T + T_1)s + 1 \right)\left(1 - \frac{L}{2}s \right)}{(Ts+1)\left[\frac{1}{2}\left(\frac{T+T_1}{K_o K_p} - T_1 \right) L s^2 + \left(\frac{T+T_1}{K_o K_p} - \frac{L}{2} + T_1 \right)s + 1 \right]}$$

The CS with standard PID ($c = b = 1$) has the transfer function:

$$(23) \quad W_{close}(s) = \frac{K_o K_p (T_1 s + 1)(Ts+1)\left(1 - \frac{L}{2}s \right)}{(Ts+1)\left[\frac{1}{2}\left(\frac{T+T_1}{K_o K_p} - T_1 \right) L s^2 + \left(\frac{T+T_1}{K_o K_p} - \frac{L}{2} + T_1 \right)s + 1 \right]}.$$

Its PID parameters are tuned applying the algorithm from section 2. But a faster CS (22) can be designed with no waste of its robustness or gain margin by tuning the additional parameters b and c of the control low (20) in the following way: the zeros in (22) have to compensate the zeros in (23) modified by the increased with $\chi \geq 1$ value of the time-constant T_1, i.e. the multiplier $(\chi T_1 s + 1)$ takes place in (23) instead of $(T_1 s + 1)$.

Then after $(T_1 s + 1)(Ts + 1) = TT_1 s^2 + (T + T_1)s + 1$ an equation is got

$$\left(c T T_1 s^2 + b(T + T_1)s + 1 \right) = \left(T \chi T_1 s^2 + (T + \chi T_1)s + 1 \right)$$

so

$$(24) \quad c = \chi; \quad b = \frac{(T + \chi T_1)}{(T + T_1)}.$$

Taking into account that an CS output signal overshooting due to the increasing value of the PID D-part or/and a large control due to the value of the PID P-part may occur, so the choice of χ has to be balanced.

4. Comparative Study

4.1. Minimum-phase Plant with Time-delay

The plant is described by the transfer function:

$$W(s) = \frac{2s + 1}{(10s + 1)(7s + 1)(3s + 1)} e^{-4s}.$$

The tree-parameter plant model (1) is identified using the original software system PIDBul:

$K_0 = 1$, $T = 12.8499$ s, $L = 9.4425$ s.

Step 1. In conformity with the algorithm in section 2.4 and using the control signal constraints [0,5] we chose the bigger time-constant $T_1 = 4.3681$ in order to use the more aggressive PID tuning based on a model with too big time delay. The corresponding PID parameters are

$K_p = 0.85$, $T_i = 17.2180$, $T_d = 3.2599$.

Step 2. After the choice of various b and c values (*table 2*) the CS signal behaviour are shown on *figure 4*.

Table 2. CS signal behaviour

b	c	Settling Time (2%),s	Rise Time, s	σ, %	$\max_t \|u(t)\|$
1	1	64.1	29.9	0	1.32
1.25	2	55.7	22.4	0	2.37
1.51	3	24.3	13.8	1	3.45
1.76	4	45.2	9.24	11	4.52

For the same plant dynamics the well-known Astrom-Hagglund (AH) tau-method [1] for maximum sensitivity M=1.4 gives the following PID parameters:

$K_p = 0.5460$, $T_i = 13.2469$, $T_d = 3.4431$, $b = 0.7134$.

It is important to mark that both methods ensure very close:

• normalised gain margin $G_m^n = G_m K K_p = 6.89$ as the maximum value of the CS gain which preserves its stability.

• maximum value of the sensitivity function $\max_\omega \|S(\omega)\| = 0.9999$ as a measure of CS robustness (if $\max_\omega \|S(\omega)\| \leq 1$)

The corresponding CS signals of both methods are given on *figure 5*.

Table 3. Comparative study indices

Method	Rise Time, s	Settling Time, (2%), s	σ, %	$\max_t \|u(t)\|$
proposed	13.9	24.7	0.321	3.45
AH	37.4	62.8	0.861	1.05

The full indices of the comparative study can be found in *table 3* in spite of the very similar corresponding sensitivity functions on *figure 6*.

4.2. Non-minimum Phase Plant

A linearized model of a hydro generator has the following description

$$(25) \qquad W(p) = \frac{k_t k_g (1 - T_1 p)}{(T_c p + 1)(T_2 p + 1)(T_g p + 1)}$$

with $k_t = 0.468$, $k_g = 1.6$, $T_c = 0.47\ s$, $T_\Gamma = 10.1\ s$, $T_1 = 5.58s$ and $T_2 = 5.17\ s$.

The result from 3-parametrical identification by authors' tuning system PIDBul is $K_0 = 0.748675$, $T_m = 11.69637s$, $\tau_m = 8.856241s$.

Step 1. The PID controller parameters for the proposed method are:

$K_p = 1.25$, $T_i = 16\ s$, $T_d = 3.1416\ s$

In case of comparison the tuned parameters for AH method (M=2) are:

$K_p = 1.4605$, $T_i = 12.4938\ s$, $T_d = 3.0953\ s$, $b = 0.2939$, $c = 0$.

Step 2. The highest possible values of b and c so that the CS with PID tuned by the proposed method has strictly better performance (except control signal maximum) comparing with the CS with Astrom-Hagglund PID tuned parameters are calculated:

$b = 1.3766$, $c = 2.4$.

The results are presented on *figure 6* and *figure 7* and generalized in *table 4*.

Table 4. Comparative study indices

Method	Rise Time, s	Settling Time, s	σ, %	$\max_t \|u(t)\|$
proposed	8.65	17.1	0	5.2
AH	18.8	55	3	1.52

Method	Gain Margin	Sensitivity function
proposed	3.8626	0.9999
AH	3.9397	1.1104

One can see from *table 4* that despite the higher gain margin, the CS with AH tuned parameters is not robust at all (the sensitivity function have value higher than 1). The CS with proposed tuned parameters is both faster and more robust for the constraints over the control signal.

5. Conclusion

In this paper the authors present an original research - an alternative method for 2DOF-PID controllers tuning for self-stabilizing plants. A two-step procedure is proposed: in the first step the basic PID parameters are tuned according to the maximum gain margin criteria, and in the second step additional weighting coefficients for the reference signal in P and D parts of the PID controller are tuned according to a maximum speed performance criteria. The tuning expressions are worked out by Symbolic Toolbox of MATLAB. Novel approximations of certain phase characteristics are made as a basis of author's new tuning method for maximum gain margin and desired phase margin. The test examples demonstrate the efficiency of the novel method comparing with the well-known quality method of Astrom-Hagglund: the control systems with tuned by the proposed method modified PID show a substantially faster set-point response with an admissible control signal magnitude. It is very important that the proposed method *does not make the CS robustness worse*. The choice of a 2DOF PID controller conforms the better CS performance (increase signals speed with preserved system robustness) if the coefficients b and c are tuned more suitable than by the „traditional" in [1] values.

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APPENDIX

Exact solution of the task $\quad T_1 = \arg\max G_{olcs}^{nonlin} = \arg\max \dfrac{T + T_1}{K\sqrt{T_1^2 + \dfrac{4L^2 T_1^2}{\left(\pi T_1 + \sqrt{\pi^2 T_1^2 - 4LT_1}\right)^2}}}$

(A.1) $\quad T_1 = \dfrac{1}{6} I_1 + \dfrac{2}{3} I_2 + \dfrac{1}{3} I_3$, where (A.2) $\quad I_1 = \dfrac{\sqrt[3]{Y_2}}{T\pi^2 \left(T + \tau + \pi^2 T\right)}$.

(A.3) $\quad I_2 = \dfrac{\left(30\pi^2 \tau T^3 + 12\pi^2 \tau^2 T^2 + 4\pi^4 \tau T^3 + 28\pi^2 T^4 + 13\pi^4 T^4 + \tau^4 + 32\tau T^3 + 16T^4 + 8\tau^3 T\right)}{T\pi^2 \left(T + \tau + \pi^2 T\right)\sqrt[3]{Y_2}} + ... +$

$$+ ... + \dfrac{\left(24\tau^2 T^2 + \pi^4 \tau^2 T^2 + 2\tau^3 \pi^2 T\right)}{T\pi^2 \left(T + \tau + \pi^2 T\right)\sqrt[3]{Y_2}}$$

(A.4) $\quad I_3 = \dfrac{\tau^2 + \pi^2 \tau T + 4T^2 + 4\tau T + 5\pi^2 T}{T\pi^2 \left(T + \tau + \pi^2 T\right)}$

(A.5). $\quad Y_1 = \dfrac{\left(27\pi^6 T^5 + 194\pi^4 T^5 + 102\pi^4 \tau T^4 + 24\pi^4 \tau^2 T^3 + 4\pi^4 \tau^3 T^2 + 435\pi^4 T^5 + 514\pi^2 \tau T^4 + 267\pi^2 \tau^2 T^3\right)}{T} + ... +$

$$+ ... + \dfrac{\left(72\pi^2 \tau^3 T^2 + 8\pi^2 \tau^4 T + 288T^5 + 624T^4 + 536\tau^2 T^3 + 228\tau^3 T^2 + 48\tau^4 T + 4\tau^5\right)}{T}$$

(A.6) $\quad Y_2 = 8\tau^6 + 12\sqrt{3Y_1}\pi^3 T^4 + 12\sqrt{3Y_1}\pi^5 T^4 + 756\pi^4 \tau^2 T^4 + 240\pi^2 \tau^4 T^2 + 312\pi^6 \tau T^5 + 1488\pi^4 \tau T^5 + 2784\pi^2 \tau T^5 + ... +$

$+ ... + 1032\pi^2 \tau^3 T^3 + 2352\pi^2 \tau^2 T^4 + 192\pi^4 \tau^3 T^3 + 48\pi^6 \tau^2 T^4 + 8\pi^6 \tau^3 T^3 + 24\pi^4 \tau^4 T^2 + 24\pi^2 \tau^5 T + 1536\tau T^5 + ... +$

$\cdot + ... + 1280\tau^3 T^3 + 480\tau^4 T^2 + 1920\tau^2 T^4 + 1212\pi^4 T^6 + 496\pi^6 T^6 + 1344\pi^2 T^6 + 108\pi^8 T^6 + 96\tau^5 T + 512T^6 + ... +$

$$+ ... + 12\sqrt{3Y_1}\pi^3 \tau T^3$$