Embedded Robust Control of Self-balancing Two-wheeled Robot

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Key Words: *Robust control; embedded systems; two-wheeled robots; μ-synthesis; MATLAB.*

Abstract. *This paper presents the design and experimentation of a two degree-of-freedom robust controller for a self-balancing two-wheeled LEGO Mindstorms NXT robot. A 12-th order discrete-time controller is designed by using the techniques of μ-synthesis. The closed-loop control system achieves robust stability and robust performance in the presence of two uncertain friction coefficients. The experimental results show that the robot preserves stability in the vertical plan for deviations greater than 16є.*

1. Introduction

In the recent years, there is a growing interest in the research and education implementation of miniature robots, build on the basis of LEGO Mindstorms NXT developer kit (see for instance [1]-[3]). The control of such robots is carried out by 32-bit ATMEL ARM 7 (AT91SAM7S256) microcontroller with 48MHz speed. This microcontroller works under the operational system nxtOSEK and has 64KB RAM, which makes it suitable for implementation of sufficiently complex control laws. The LEGO Mindstorms NXT kit is used in [4] to build the self-balancing two-wheeled robot NXTway-GS, which implements a linear quadratic regulator for robot digital control (stabilization of vertical body position and achieving a reference position in the horizontal plane). The software product Embedded Coder Robot NXT [5] is used to implement additional tasks related to the robot control (system initialization, avoiding obstacles and battery voltage checking).

The robot balancing is achieved by rotating the wheels in the appropriate direction. The computation of control actions to both DC drive motors is realized in single precision on the basis of signals from the MEMS gyroscopic sensor which measures the angular rate (and, after integration, the pitch angle) of the robot body in the vertical plane and signals from rotary encoders which measure the wheels rotation angles. The control of the DC motors is accomplished by Pulse Width Modulated (PWM) signals. To avoid obstacles the robot is equipped with ultrasonic sensor.

The general view of the NXTway-GS robot in vertical stabilization mode is shown in *figure 1*.

In this paper we present the design of a robust controller for NXTway-GS robot with the aim to implement in maximum degree the available software for real-time control presented in [4]. Since the robust controllers are of higher order the basic problem is to check the possibility to implement such controllers on the available microcontroller working with sampling frequency f_s = 250 Hz in the stabilization loop. The results obtained show that the microcontroller under consideration implements without difficulties robust discrete controller of 12[th] order that allows to improve the closed-loop system performance. Results from the simulation of the closed-loop system as well as experimental results obtained during the real implementation of the controller designed are given.

2. Uncertain Model of the Two-wheeled Robot

The nonlinear differential equations describing the robot motion are derived in details in [4] by using the Lagrange method. These equations are linearized analytically around the balance point (equilibrium position) obtaining as a result state space description of 4[th] order for the vertical plane motion and 2[nd] order description for the rotation around vertical axis. In the first case, the average angle θ of left and right wheels rotation and the body pitch angle ψ, as well as their derivatives (the corresponding angular rates), are used as state vectors components. In the second case, state variables are the body yaw angle φ and its derivative. Introducing the state and control action vectors

(1) $x_1 = [\theta, \psi, \dot{\theta}, \dot{\psi}]^T,\ x_2 = [\phi, \dot{\phi}]^T,\ u = [u_l, u_r]^T,$

where u_l, u_r are the control actions to the left and right wheel,

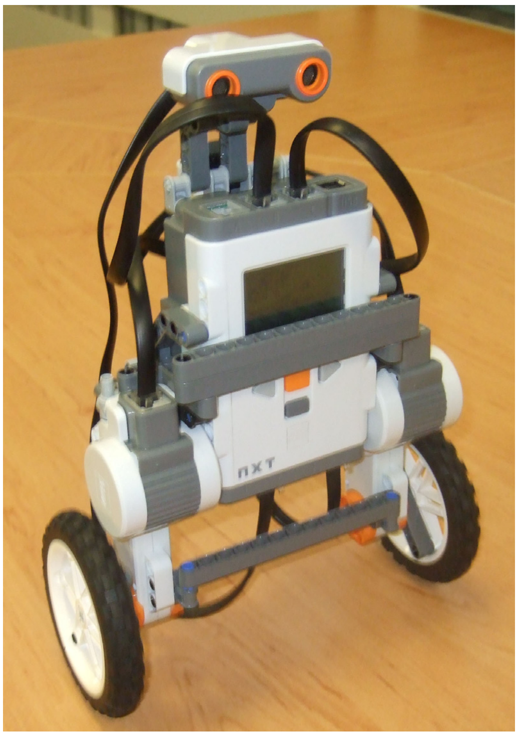

Figure 1. General view of the two-wheeled robot

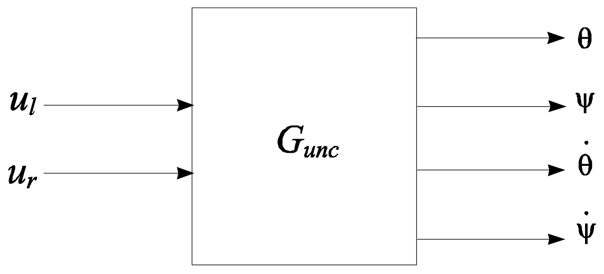

Figure 2. Uncertain model of the two-wheeled robot

respectively, the equations of the linearized system are obtained in the form

(2) $\dot{x}_1 = A_1 x_1 + B_1 u$

(3) $\dot{x}_2 = A_2 x_2 + B_2 u,$

where the expressions for the elements of matrices A_1 and B_1, as well as the values of the corresponding parameters, are given in [4]. It should be noted that the robot model is not determined with sufficient accuracy but for the aim of robust control design this fact is not so essential. In the given case as uncertain parameters we consider the friction coefficient f_m between the robot body and DC motor and the friction coefficient f_n between the robot wheels and motion surface. In particular, we assume that the coefficient f_m is known with 20% uncertainty and the coefficient f_n - with 100% uncertainty. The analysis performed later on shows that the coefficient f_m has more significant influence on the system dynamics.

As a results, for the subsystem (2) one obtains an uncertain plant G_{unc} with 2 inputs and 4 outputs (*figure 2*).

The magnitude responses of the nominal and uncertain plant (for random parameter values in the assumed range) are shown in *figure 3*.

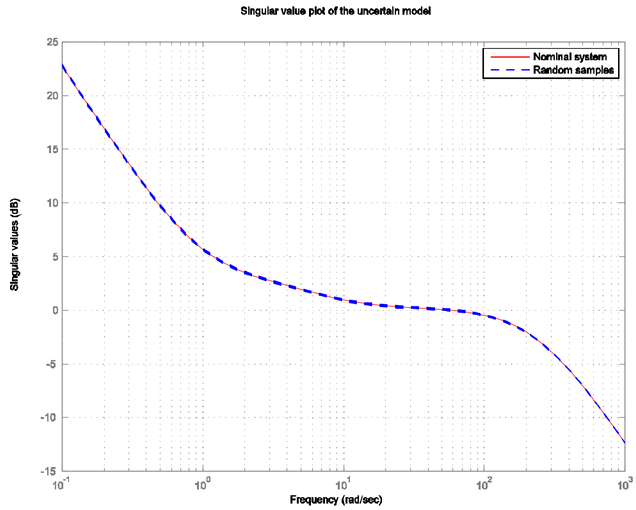

Figure 3. Magnitude responses of the nominal and uncertain plant

3. Design of Robust Controller

The design of a two degree-of-freedom discrete controller for the subsystem (2) is realized by using the μ-synthesis, which usually ensures best performance of the closed-loop system [6,7]. The design is intended for 250 Hz sampling frequency, which corresponds to the frequency of the signal at the output of gyro sensor. However, the following problem appears during the design. The experiments with different standard design configurations show that they cannot ensure good tracking of the reference angle θ, and hence the desired position in the motion plane. That is why apart from the constraints on the sensitivity function we add also a constraint on the integral of the error in wheels rotation angle. This allows to ensure sufficient accuracy in tracking the angle θ. The integral component is added also in the system feedback. The subsystem (3) is controlled by PID regulator as shown in [4] and is not considered in the present design.

The closed-loop structure with the sensitivity function requirement on the output variables as well as the requirement on the control actions is shown in *figure 4*. The transfer function matrix W_p reflects the requirements to the system performance,

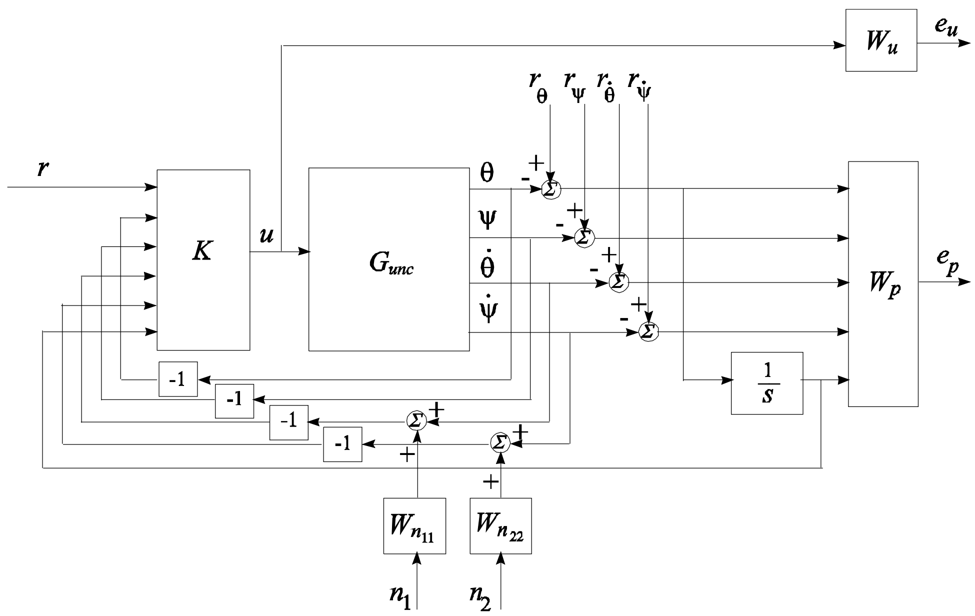

Figure 4. Block-diagram of the closed-loop system with performance requirements

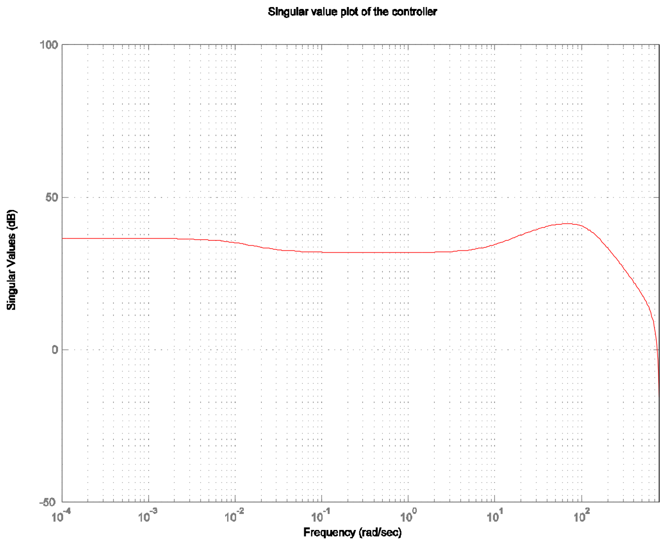

Figure 5. Singular value plot of the controller

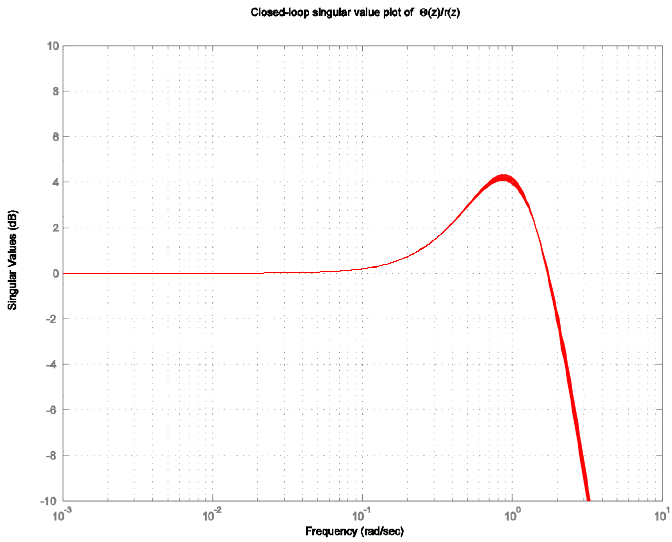

Figure 6. Magnitude response of the uncertain closed-loop system

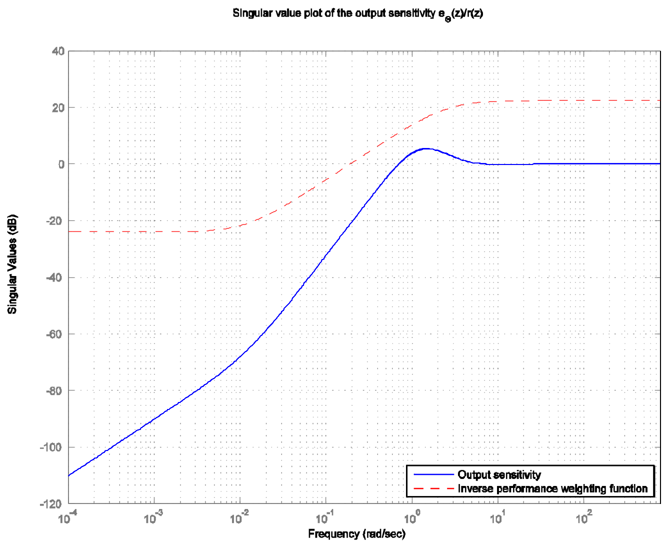

Figure 7. Output sensitivity function of the closed-loop system

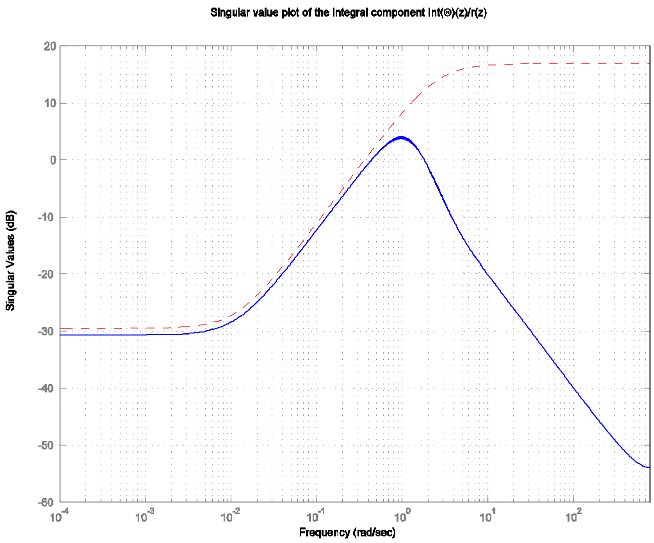

Figure 8. Magnitude response of the integral component (the frequency domain constraints on this component are shown with dashed line)

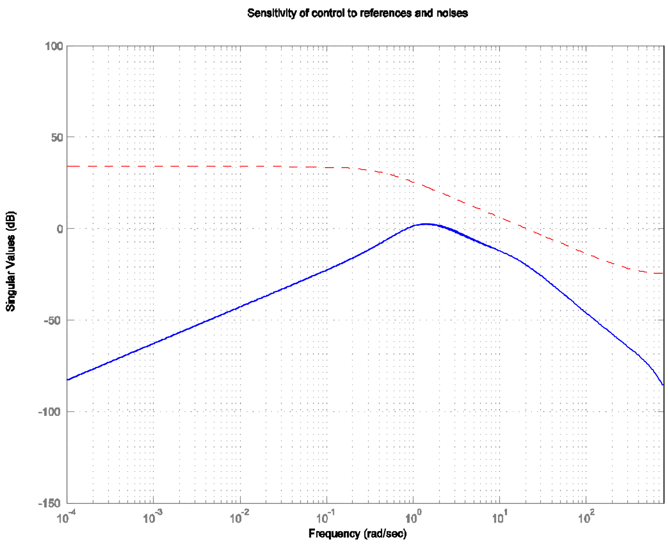

Figure 9. Sensitivity of control action to references and noises (the control constraints in the frequency domain are shown by dashed line)

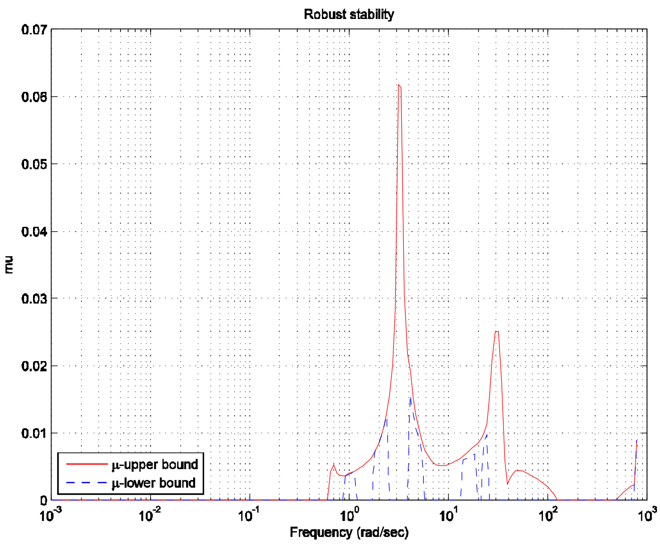

Figure 10. Robust stability of the closed-loop system

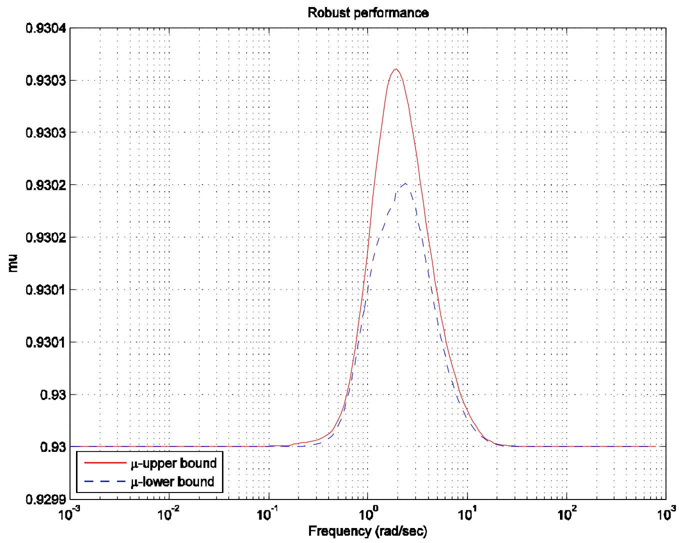

Figure 11. Robust performance of the closed-loop system

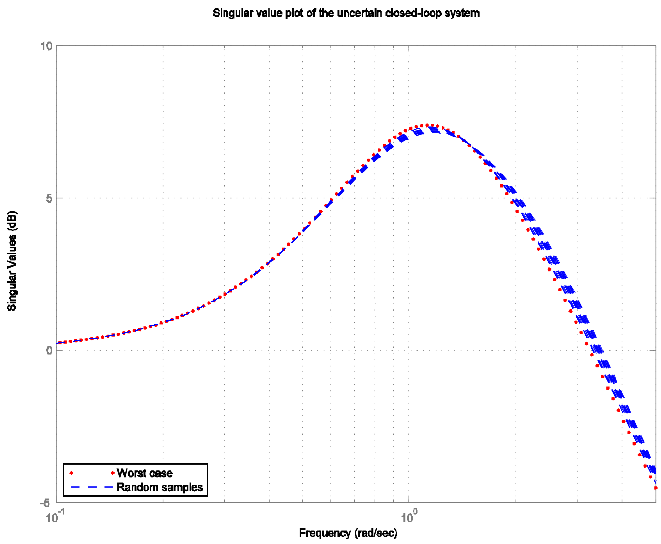

Figure 12. Worst-case magnitude response of the closed-loop system

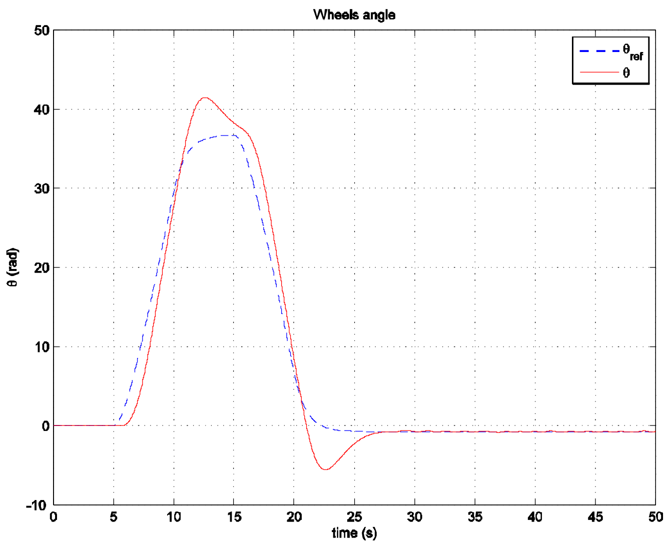

Figure 13. Transient response of the closed-loop system

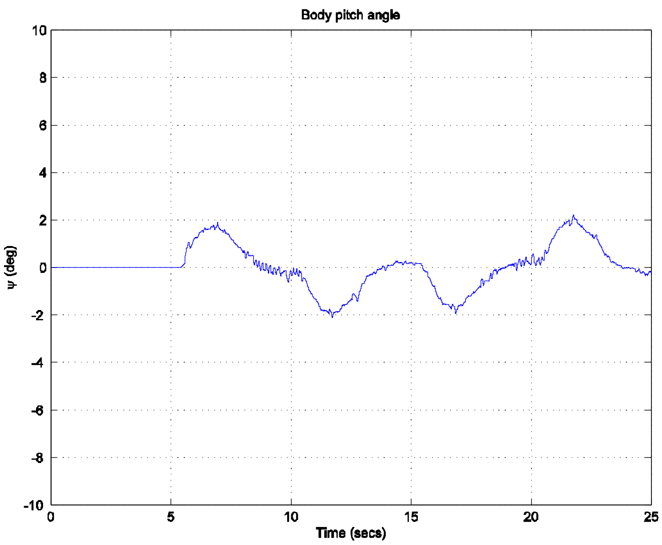

Figure 14. Transient response of the closed-loop system - body pitch angle

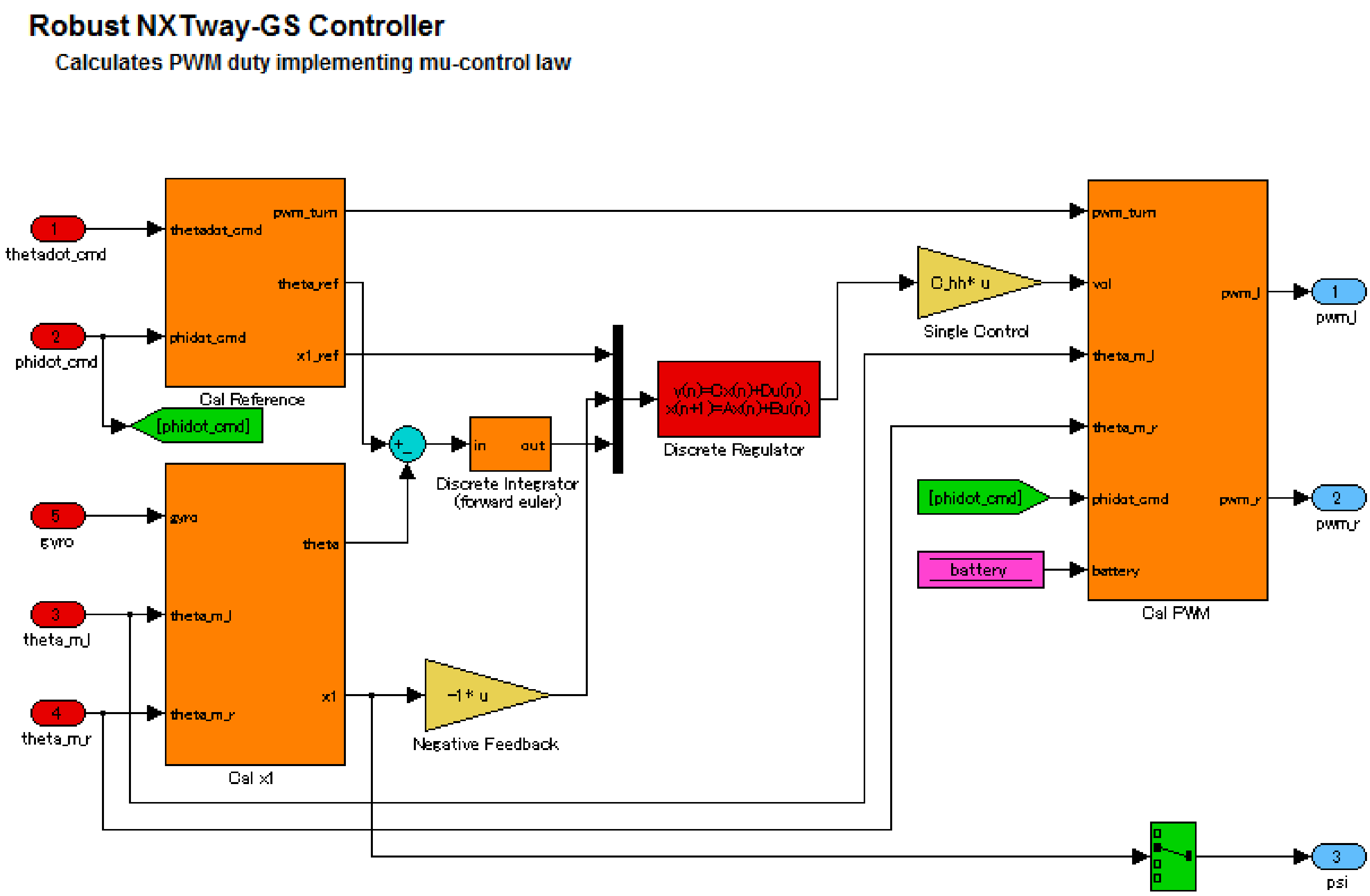

Figure 15. Controller implementation

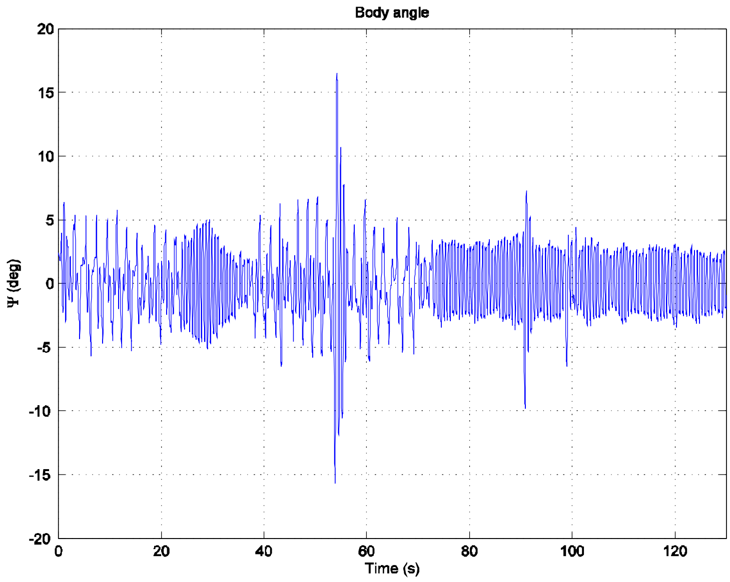

Figure 16. Transient responses for deviations from balance point

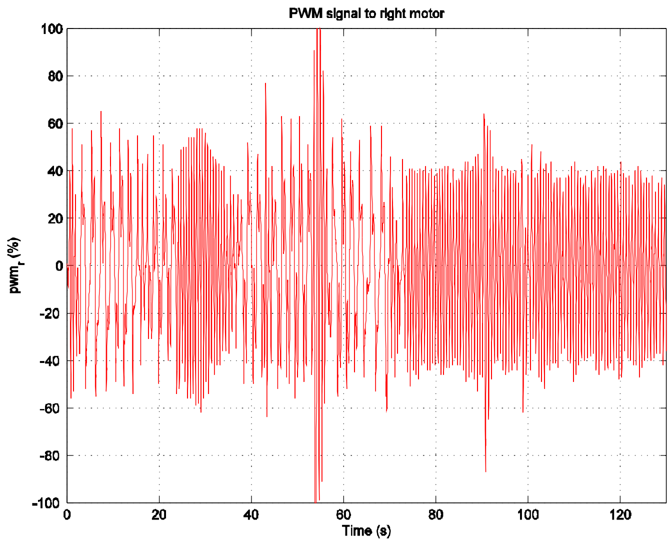

Figure 17. Control action to the right wheel motor

the matrix W_u reflects the requirements to the control actions and the transfer functions W_{11} and W_{22} reflects the influents of noises in the gyro and encoder measurements. In the given case the following weighting functions are determined

$$W_p = diag(W_{p11}, W_{p22}, W_{p33}, W_{p44}, W_{p55})$$

$$W_{p11} = 0.95\frac{0.4s+1}{5s+0.06} \qquad W_{p22} = 0.93$$

(4) $$W_{p33} = 0.15 \qquad W_{p44} = 0.22\frac{1.1s+1}{1.0s+1}$$

$$W_{p55} = 1.8\frac{0.4s+1}{5s+0.06}$$

$$W_u = diag(W_{u11}, W_{u22})$$

(5) $$W_{u11} = 0.02\frac{1.2s+1}{0.0024s+1} \qquad W_{u22} = 0.02\frac{1.2s+1}{0.0024s+1}$$

$$W_n = diag(W_{n11}, W_{n22})$$

(6) $$W_{n11} = 0.1\frac{1.0s+2}{0.001s+1} \qquad W_{n22} = 0.1\frac{1.0s+2}{0.001s+1}$$

The design of two degree-of-freedom controller for the subsystem (2) is realized by the function *dksyn* from Robust Control Toolbox of MATLAB. As a result after 3 iterations one obtains a 12th order controller, which ensures a minimum value of the structured singular value μ equal to 0.930. This shows that the closed-loop system achieves robust performance in respect to the variation of both uncertain coefficients.

The singular value plot (the magnitude response) of the controller obtained is shown in *figure 5*.

The maximum controller gain is 41db, which ensures acceptable control actions. The experiments show that larger controller gains lead to actuators saturation and instability of the real system.

4. Closed-loop System Properties

After determining the controller, it is possible to compute several frequency responses and time responses of the closed-loop system that give profound information about its properties.

In *figure 6* we show a family of frequency response characteristics of the closed-loop with input and output the desired and actual angle θ, respectively, for different random values of the uncertain parameters in the prescribed range. It is seen that the responses have acceptable picks, the frequency band width (at level -3db) being about 2.13 rad/s. For this band width it is possible to achieve good tracking of typical for the system under consideration signals.

The magnitude responses of the output sensitivity function of the closed-loop system along with the inverse performance function are shown in *figure 7*. It is seen that the disturbance suppression in the low frequency range is better than this required by the performance weighting function.

In *figure 8* we show the magnitude response of the loop from reference angle θ to the integral of the tracking error of this angle. Clearly, the prescribed frequency domain constraints on the integral error are fulfilled, the error suppression in the low frequency range (where the reference spectrum lies) being 30 db (i.e., more than 30 times). It is possible to achieve even better suppression of this error but in such a case the requirement for robust performance cannot be fulfilled (the value of μ increases).

In *figure 9* we show the magnitude response of the loop from references and noises to the control actions (motor controls). It is seen that this response is lying below the frequency response of the inverse control weighting filter, i.e., the prescribed constraints on the controls are fulfilled. These constraints are chosen so that to avoid saturation of the actuators which generate the PWM signals to the motors.

The robust stability analysis of the uncertain closed-loop system shows that the upper bound on the structured singular value does not exceed the value of 0.063. This means that the system may remain stable for much larger than the prescribed uncertainty in the corresponding parameters. This is not the case in respect to the robust performance. It is seen from the frequency response characteristic of the structured singular value, shown in *figure 11*, that its maximum value is almost equal to 0.9304. This shows that there exists an uncertainty which is only 1.075 times larger than the existing one, for which the closed-loop system loose robust performance.

In *figure 12* we show the worst case magnitude response of the closed-loop system obtained by using the function *wcgain* from Robust Control Toolbox. Due to the robust performance achieved, the worst case response is slightly different from the nominal one.

The simulation of the nonlinear closed-loop system in the time domain is performed by the program *nxtway_gs_vr.mdl* from [4], in which the controller, as implemented by the file *nxtway_gs_controller.mdl,* is modified in a way to use the μ-regulator designed.

In *figure 13* we show the actual reference angle θ, that is input to the controller as well as the transient response of the output angle θ. Multiplying these variables by the wheel radius $R = 0.04$ m it is possible to determine the accuracy of robot positioning in horizontal plane motion.

The body pitch angle, corresponding to the reference position given in *figure 13*, is shown in *figure 14*. Obviously, this angle is in the range of ±2 degrees.

The simulation of the closed-loop system for zero reference and initial deviation of robot body from the vertical plane shows that the robot successfully and quickly returns to the balance point. Since the simulation program *nxtway_gs_vr.mdl* implements the linearized plant model, it is not possible to establish the maximum value of the vertical deviation for which the robot preserves stability. This is done experimentally as described in the next section.

5. Experimental Results

The designed robust regulator of the two wheeled robot motion is experimented in practice using the available software for automatic generation of C control code and its loading in the digital robot controller, as presented in [4].

For this aim we use the modified block-diagram of the controller model in Simulink, shown in *figure 15*. The 12th order μ-regulator is represented by the state space block „Discrete Controller" and the integration of the tracking error is done by the block „Discrete Integrator". The generation of the control code is done by using the Simulink Coder. During the experiments one measures the control signals to both motors, the pitch angle ψ, the battery voltage and so on. The data transfer from robot to the personal computer is performed by using the bluetooth-protocol. The file containing the results is loaded to MATLAB which allows to visualize the experimental data. Since the angular rate dψ/dt measured by the gyroscope and its integral are contaminated by noises there is some bias in the computed value of ψ, which is removed by the MATLAB function *dtrend*.

In *figure 16* we show the results from one of the experiments performed. The robot is subject to two forced deviations from the vertical position at approximately $t = 54$ s and $t = 90$ s. For the bigger deviation of 16.5є at $t = 54$ s the regulator succeeds to stabilize the system and to return the robot to the balance point.

For the same experiment, the control signal to the right wheel is shown in *figure 17*. In stabilization mode the control is between 40% and 60% from the maximum one which guarantees that the saturation of the actuator will be avoided. Since the control at $t = 54$ s reaches 100% it is justified to accept that the deviation of 16.5□ is the maximum allowable one for which the robot is kept stable.

6. Conclusions

A robust 12th order discrete-time μ-controller is designed and implemented experimentally to stabilize the vertical position of two wheeled robot. The closed-loop control system achieves robust stability and robust performance in presence of uncertain friction coefficients and allows to keep the robot stable for deviations from vertical position up to 16.5є.

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