Print ISSN:1312 – 2622; Online ISSN: 2367-5357
DOI:10.2478/itc-2013-0005

Predictive Control of a Laboratory Time Delay Process Experiment

S. Enev

Key Words: *Model predictive control; time delay process; experimental results.*

Abstract. *The paper presents the design and implementation of a Model Predictive Control (MPC) scheme of a laboratory heat-exchange process with a significant time delay in the input-output path. The optimization problem formulation is given and an MPC control algorithm is designed, achieving integral properties. Details, related to the practical implementation of the control law are discussed and the first experimental results are presented.*

1. Introduction

The presence of a time delay in the process to be controlled makes the design problem more difficult and reduces the achievable performance within the classical feedback loop structure. The Model Predictive Control [1] is inherently an appropriate option when the control of such processes is considered, since its fundamental mechanism of control elaboration relies on prediction of the process behavior, which in principle "counteracts" the delay. Moreover, the time delays are easily incorporated and handled in discrete-time descriptions, a fact that corresponds well to MPC, being inherently discrete-time.

Nowadays linear MPC, in its many variants, may be called a mature industrial technology in the process control field with well-developed theoretical basis and methodology that enable the control system characterization with respect to stability and performance. Following the ever increasing computational power of modern computers, the application areas of MPC grow in number and are beginning to bridge the milli- and micro-second sampling period range [2] (where a good overview of the current and future perspectives, challenges and research directions can also be found). These advances practically support the idea to transfer MPC algorithms towards and into the lower level control loop of the plant control system hierarchy and to take advantage of their capabilities.

In this paper a linear MPC scheme is implemented which controls a laboratory heat-exchange process with a significant time delay in the input-output path. Based on a discrete-time model of the process, a formulation of the optimization problem with respect to the control input changes is developed and an output feedback MPC algorithm achieving integral action, is designed. Some experimental results are presented, along with details concerning the implementation of the control algorithm.

2. Process Description and Mathematical Model Structure

The laboratory heat-exchanger is depicted in *figure 1*. The heated fluid, being water, passes through an electrical flow heater with a constant flow rate and then enters a long coiled pipeline. The water temperature at the pipeline exit point is the controlled variable. The transportation process in the pipeline introduces a significant time delay in the system input-output path.

The control input is the duty cycle - μ in the PWM scheme used to modulate the power to the electric heater, i.e., the fraction of the maximal heating power available. The main disturbance is the temperature of the inflow stream - θ_{in}. Within the existing installation, it is observed that after a certain period of operation, the thermal capacity of the structure begins to affect the outlet temperature. Besides, with the pipeline not being well insulated, the outlet temperature is affected by the heat transfer towards/from the environment when there is a significant deviation from the

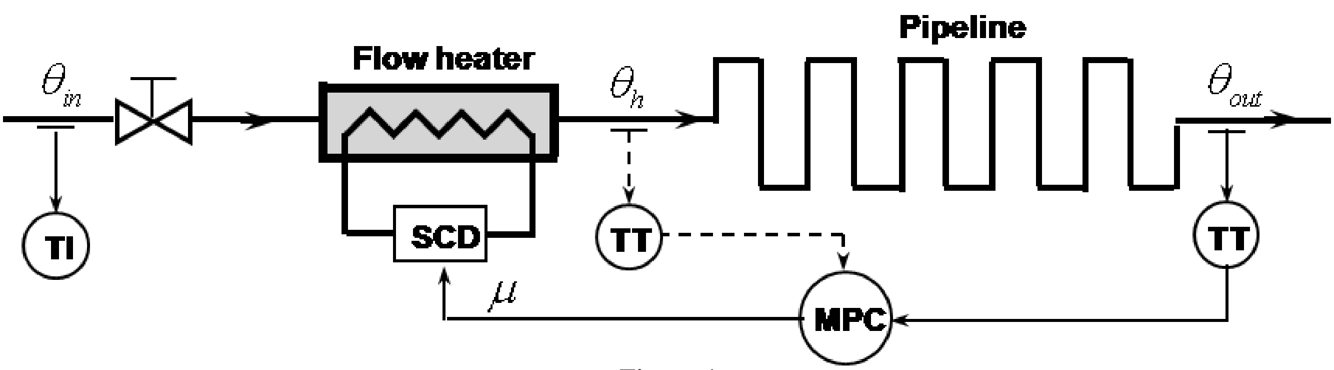

Figure 1

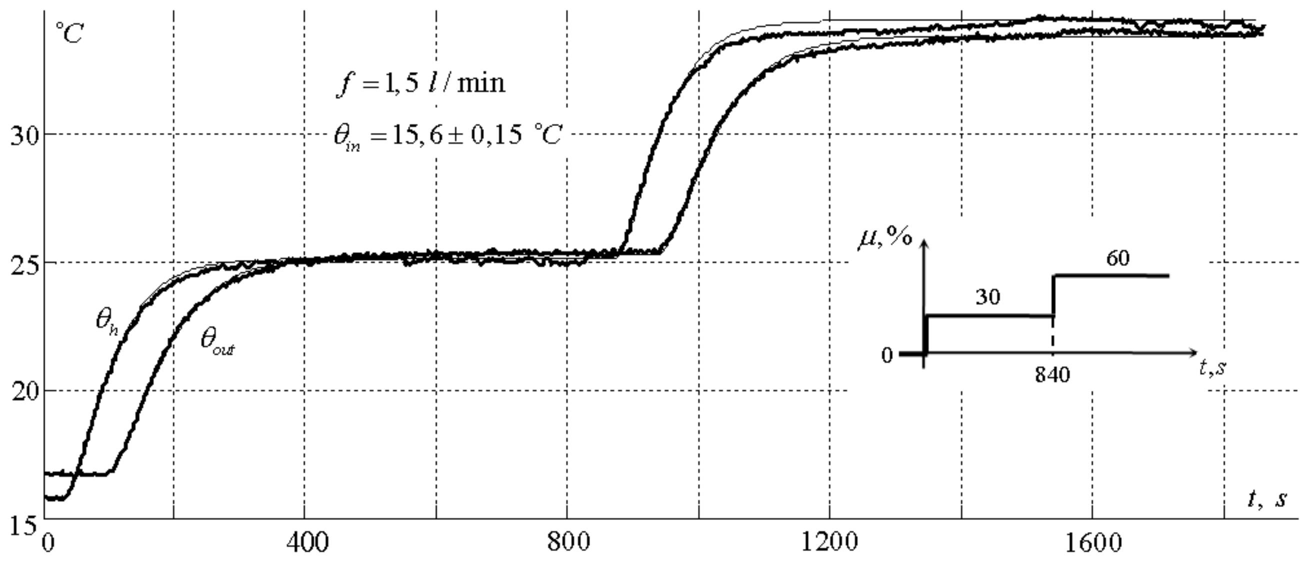

Figure 2

ambient temperature. Finally, the variations of the flow rate will significantly affect the performance of the control system, since the flow rate itself is a major factor determining the dynamic properties of the process. Though the constant considered is set by the manually operated valve at the piping input for modeling and throughout the experiments, such variations exist, due to the pressure variations at the installation input.

The outlet temperature is measured by a 4-20 mA temperature transmitter using a Pt100 resistor as a sensor and a temperature indicator is available for the inlet temperature. The PWM control is implemented with the help of a Solid State Relay (SSR).

A set of open loop experiments on the process were realized in order to acquire data for identification purposes. It was found that for a constant flow rate the process has a linear steady-state characteristic and quite similar transient curves throughout the entire operating domain. *Figure 2* shows the process responses to a series of two step changes of the control input, with the flow rate set at f = 1,5 l/min.

The parameters of the second-order plus time delay models were fitted to the actual responses, and the following parameters were obtained.

(1) $W(p) = \frac{\theta_{out(h)}(p)}{\mu(p)} = \frac{k_\mu}{(T_1 p+1)(T_2 p+1)} e^{-\tau p}$ – transfer function model

with

(1.1) $k_\mu = 0{,}285^\circ$ C / %, T_1 = 60 s, T_2 = 40 s, τ = 95 s for the output temperature θ_{out};

(1.2.) $k_\mu = 0{,}31^\circ$ C / %, T_1 = 40 s, T_2 = 40 s, τ = 30 s for θ_h – the temperature at the heater exit point.

The responses of the models obtained are shown on the same graph with the processes (*figure 2*). The first-principle modeling considerations suggest that the entry point of the disturbance in the system is at the input, so the same transfer functions can be considered, but with a steady-state gain equal to one (k_d = 1).

3. MPC Convex QP Problem Formulation with Respect to the Control Input Changes

In this paragraph the formulation of the optimization problem is described. The underlying process model used is of the following discrete-time description:

$$\text{(2)} \quad \begin{aligned} \bar{\boldsymbol{x}}_{(k+1)} &= \bar{\boldsymbol{A}}\bar{\boldsymbol{x}}_{(k)} + \bar{\boldsymbol{b}}u_{(k)} + \bar{\boldsymbol{b}}_d d_{(k)} \\ y_{(k)} &= \bar{\boldsymbol{C}}\bar{\boldsymbol{x}}_{(k)} \end{aligned},$$

where $\bar{x}_{(k)} \in R^n$, $y_{(k)} \in R$, $u_{(k)} \in R$, $d_{(k)} \in R$, $\bar{A} \in R^{n\times n}$, $\bar{b} \in R^n$, $\bar{b}_d \in R^n$, $\bar{C}^T \in R^n$, $s_{(k)}$ denotes the value of the signal $s(t)$at the sampling instant $t = kT_0$, and T_0 - the sampling period. The input structure is chosen according to the process model.

A basic requirement and design specification towards a control system is the ability to achieve offset-free control for constant reference in the presence of unknown constant disturbances and/or model mismatch, providing stability. Two basic approaches exist to achieve an integral action in MPC. The first approach introduces an estimated disturbance (possibly a vector) in the description, in which the steady-state values (input and state) are assumed dependent [3,4]. The second approach used here [1], is based on the following reformulation of the process model, in which the dynamics are described in terms of the changes as follows:

$$\begin{aligned} \Delta\bar{\boldsymbol{x}}_{(k+1)} &= \bar{\boldsymbol{A}}\Delta\bar{\boldsymbol{x}}_{(k)} + \bar{\boldsymbol{b}}\Delta u_{(k)} + \bar{\boldsymbol{b}}_d \Delta d_{(k)} \\ y_{(k+1)} &= y_{(k)} + \bar{\boldsymbol{C}}\bar{\boldsymbol{A}}\Delta\bar{\boldsymbol{x}}_{(k)} + \bar{\boldsymbol{C}}\bar{\boldsymbol{b}}\Delta u_{(k)} + \bar{\boldsymbol{C}}\bar{\boldsymbol{b}}_d \Delta d_{(k)} \end{aligned}$$

where $\Delta\bar{x}_{(k)} = \bar{x}_{(k)} - \bar{x}_{(k-1)}$, $\Delta u_{(k)} = u_{(k)} - u_{(k-1)}$, $\Delta d_{(k)} = d_{(k)} - d_{(k-1)}$. The model is expressed in the following form:

$$\text{(3)} \quad \begin{aligned} \boldsymbol{x}_{(k+1)} &= \boldsymbol{A}\boldsymbol{x}_{(k)} + \boldsymbol{b}\Delta u_{(k)} + \boldsymbol{b}_d \Delta d_{(k)} \\ y_{(k)} &= \boldsymbol{C}\boldsymbol{x}_{(k)} \end{aligned},$$

where

$$\boldsymbol{x}_{(k)} \equiv \left[\Delta\bar{\boldsymbol{x}}^T_{(k)} \;\vdots\; y_{(k)} \right]^T \in \mathbf{R}^{n+1},\ \boldsymbol{A} \equiv \begin{bmatrix} \bar{\boldsymbol{A}} & \boldsymbol{0}_{[n]} \\ \overline{\boldsymbol{CA}} & 1 \end{bmatrix} \in \mathbf{R}^{n+1 \times n+1},\ \boldsymbol{b} \equiv \begin{bmatrix} \bar{\boldsymbol{b}} \\ \overline{\boldsymbol{Cb}} \end{bmatrix} \in \mathbf{R}^{n+1},\ \boldsymbol{b}_d \equiv \begin{bmatrix} \bar{\boldsymbol{b}}_d \\ \overline{\boldsymbol{Cb}}_d \end{bmatrix} \in \mathbf{R}^{n+1},$$

$$\boldsymbol{C} \equiv \left[\boldsymbol{0}_{[1\times n]} \;\vdots\; 1 \right],\ \boldsymbol{C}^T \in \mathbf{R}^{n+1}.$$

The following so-called “prediction horizon vectors” are defined:

$\boldsymbol{y}_{(k)} \equiv [y_{(k+1|k)}, y_{(k+2|k)}, \dots, y_{(k+N|k)}]^T \in \mathbf{R}^N$ – a prediction horizon output vector;

$\boldsymbol{y}_{R,(k)} \equiv [y_{R,(k+1|k)}, y_{R,(k+2|k)}, \dots, y_{R,(k+N|k)}]^T \in \mathbf{R}^N$ – a prediction horizon reference signal vector;

$\Delta\boldsymbol{u}_{(k)} = [\Delta u_{(k|k)}, \Delta u_{(k+1|k)}, \dots, \Delta u_{(k+N-1|k)}]^T \in \mathbf{R}^N$ – a prediction horizon input change vector, with $\Delta u_{(k+i|k)} = u_{(k+i|k)} - u_{(k+i-1|k)}$;

$\Delta\boldsymbol{d}_{(k)} = [\Delta d_{(k)}, \Delta d_{(k+1|k)}, \dots, \Delta d_{(k+N-1|k)}]^T \in \mathbf{R}^N$ – a prediction horizon disturbance change vector, where N denotes the prediction horizon, $s_{(k+i|k)}$ being the predicted future value of the signal $s(t)$ at the sampling instant $t = kT_0$.

Remark. $\Delta u_{(k|k)}$ is the control input change to be applied to the plant at $t = kT_0$ (zero-order hold is assumed at the plant input, so that the control is kept constant during each sampling interval), determined as a result of the optimization procedure, performed at $t = kT_0$.

The objective function is expressed in a prediction horizon vector form as:

$$(4)\quad J_{(k)} = \frac{1}{2}(\boldsymbol{y}_{R,(k)} - \boldsymbol{y}_{(k)})^T \boldsymbol{Q}(\boldsymbol{y}_{R,(k)} - \boldsymbol{y}_{(k)}) + \frac{1}{2}\Delta\boldsymbol{u}^T_{(k)}\boldsymbol{P}\Delta\boldsymbol{u}_{(k)},$$

where

$\boldsymbol{Q} = \boldsymbol{diag}\ (q_i) \in \mathbf{R}^{N\times N}$, $i = 1...N$

and

$\boldsymbol{P} = \boldsymbol{diag}\ (p_i) \in \mathbf{R}^{N\times N}$, $i = 1..N$ – weight coefficients.

We have the following:

$$(5)\quad y_{(k)} = M\Delta u_{(k)} + Gx_{(k)} + M_d\Delta d_{(k)},$$

where

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{Cb} & 0 & 0 & \cdots & 0 \\ \boldsymbol{CAb} & \boldsymbol{Cb} & 0 & \cdots & 0 \\ \boldsymbol{CA}^2\boldsymbol{b} & \boldsymbol{CAb} & \boldsymbol{Cb} & \ddots & \vdots \\ \vdots & & & \ddots & 0 \\ \boldsymbol{CA}^{N-1}\boldsymbol{b} & \boldsymbol{CA}^{N-2}\boldsymbol{b} & \cdots & & \boldsymbol{Cb} \end{bmatrix},$$

$$\boldsymbol{M}_d = \begin{bmatrix} \boldsymbol{Cb}_d & 0 & 0 & \cdots & 0 \\ \boldsymbol{CAb}_d & \boldsymbol{Cb}_d & 0 & \cdots & 0 \\ \boldsymbol{CA}^2\boldsymbol{b}_d & \boldsymbol{CAb}_d & \boldsymbol{Cb}_d & \ddots & \vdots \\ \vdots & & & \ddots & 0 \\ \boldsymbol{CA}^{N-1}\boldsymbol{b}_d & \boldsymbol{CA}^{N-2}\boldsymbol{b}_d & \cdots & & \boldsymbol{Cb}_d \end{bmatrix},$$

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{CA} \\ \boldsymbol{CA}^2 \\ \boldsymbol{CA}^3 \\ \vdots \\ \boldsymbol{CA}^N \end{bmatrix} \quad \boldsymbol{M},\ \boldsymbol{M}_d \in \mathbf{R}^{N\times N},\ \boldsymbol{G} \in \mathbf{R}^{N\times n}.$$

Remark. If no apriori information is available, a constant disturbance is usually assumed during the prediction horizon, that is, $d_{(k+i|k)} = d_{(k)}$, $i = 1..N-1$, which in turn leads to $\Delta d_{(k+i|k)} = 0$, $i = 1..N–1$. Thus all elements of the vector $\Delta\boldsymbol{d}_{(k)}$, except for, probably, the first one, are equal to zero (if unmeasured disturbance is considered, the last term in (5) may not be considered).

The objective function is now rewritten as

$$(6)\quad \begin{aligned} J_{(k)} &= \frac{1}{2}\left(\boldsymbol{y}_{R,(k)} - \boldsymbol{M}\Delta\boldsymbol{u}_{(k)} - \boldsymbol{f}_{(k)}\right)^T \boldsymbol{Q}\left(\boldsymbol{y}_{R,(k)} - \boldsymbol{M}\Delta\boldsymbol{u}_{(k)} - \boldsymbol{f}_{(k)}\right) + \frac{1}{2}\Delta\boldsymbol{u}^T_{(k)}\boldsymbol{P}\Delta\boldsymbol{u}_{(k)} = \\ &= \frac{1}{2}\Delta\boldsymbol{u}^T_{(k)}\left(\boldsymbol{M}^T\boldsymbol{QM} + \boldsymbol{P}\right)\Delta\boldsymbol{u}_{(k)} + \left(\boldsymbol{f}_{(k)} - \boldsymbol{y}_{R,(k)}\right)^T \boldsymbol{QM}\Delta\boldsymbol{u}_{(k)} + \frac{1}{2}(\boldsymbol{y}_{R,(k)} - \boldsymbol{f}_{(k)})^T\boldsymbol{Q}(\boldsymbol{y}_{R,(k)} - \boldsymbol{f}_{(k)}), \end{aligned}$$

where $\boldsymbol{f}_{(k)} = \boldsymbol{Gx}_{(k)} + \boldsymbol{M}_d\Delta\boldsymbol{d}_{(k)}$. The last term can be removed from the expression since it is no function of the optimization variables.

Let L denotes the control horizon. Then the following is obtained:

$$\Delta\boldsymbol{u}_{(k)} = \begin{bmatrix} \Delta\boldsymbol{u}_{L,(k)} \\ \boldsymbol{0}_{[N-L]} \end{bmatrix},\ \Delta\boldsymbol{u}_{L,(k)} = [\Delta u_{(k|k)}, \Delta u_{(k|k+1)}, \dots, \Delta u_{(k|k+L-1)}]^T \in \boldsymbol{R}^L$$

and the objective function can be rewritten as

$$(7)\quad J_{(k)} = \frac{1}{2}\Delta\boldsymbol{u}^T_{L,(k)}\boldsymbol{\Lambda}^T_L\left(\boldsymbol{M}^T\boldsymbol{QM} + \boldsymbol{P}\right)\boldsymbol{\Lambda}_L\Delta\boldsymbol{u}_{L,(k)} + (\boldsymbol{f}_{(k)} - \boldsymbol{y}_{R,(k)})^T\boldsymbol{QM\Lambda}_L\Delta\boldsymbol{u}_{L,(k)},$$

where $\boldsymbol{\Lambda}_L = \begin{bmatrix} \boldsymbol{I}_{[L\times L]} \\ \boldsymbol{0}_{[N-L\times L]} \end{bmatrix}$, $\boldsymbol{I}_{[L\times L]}$ – the identity matrix with the respective dimensions.

The Hessian and the gradient of the objective function are

$$(8)\quad \nabla J_{(k)} = \boldsymbol{\Lambda}^T_L\ (\boldsymbol{M}^T\boldsymbol{QM} + \boldsymbol{P})\boldsymbol{\Lambda}_L\Delta\boldsymbol{u}_{L,(k)} + (\boldsymbol{f}_{(k)} - \boldsymbol{y}_{R,(k)})^T\boldsymbol{QM\Lambda}_L,$$

$$\nabla^2 J_{(k)} = \boldsymbol{\Lambda}^T_L\ (\boldsymbol{M}^T\boldsymbol{QM} + \boldsymbol{P})\boldsymbol{\Lambda}_L.$$

It is easily shown that the Hessian – $\boldsymbol{\Lambda}^T_L\ (\boldsymbol{M}^T\boldsymbol{QM} + \boldsymbol{P})\boldsymbol{\Lambda}_L$ is a symmetric positive definite matrix. Thus, $J_{(k)}$ is a convex function of $\Delta\boldsymbol{u}_{L,(k)}$ on $\boldsymbol{R}^L$.

Expressing the objective function in terms of the control input changes has also the following advantages:

- Problem size reduction – the optimization problem is on R^L, $L = 3 – 6(7)$ typically.
- Equality constraints appearing when $L < N$ (which is practically always the case) are eliminated, that is, they are incorporated in the objective function (an additional useful feature as a result of the problem size reduction).

Ultimately, this leads to an unconstrained optimization problem.

Inequality constraints are introduced in the optimization problem to account for the limited power availability and the physically possible in the experiment process control inputs. The control input must satisfy the following constraints

(9) $0 \leq u_{(k)} \leq u_{\max}, \forall k.$

They are written in a standard form in terms of the control input changes as

$$\begin{aligned} \varphi_{i,1,(k)} &\equiv -\left(u_{(k-1)} + \sum_{j=k}^{k+i-1} \Delta u_{(j)}\right) \leq 0,\ i = 1..L \\ \varphi_{i,2,(k)} &\equiv u_{(k-1)} + \sum_{j=k}^{k+i-1} \Delta u_{(j)} - u_{\max} \leq 0,\ i = 1..L \end{aligned} \quad (10)$$

We have

$$\varphi_{i,(k)} = \varphi_{i,1,(k)} \varphi_{i,2,(k)} = \left(u_{(k-1)} + \sum_{j=k}^{k+i-1} \Delta u_{(j)}\right)\left(u_{\max} - u_{(k-1)} - \sum_{j=k}^{k+i-1} \Delta u_{(j)}\right),\ i = 1..L.$$

The sets $\boldsymbol{S}_{\Delta,(k)}^{L} = \{\Delta \boldsymbol{u}_{L,(k)} \mid \boldsymbol{0}_{[N]} \leq \boldsymbol{u}_{(k)} \leq \boldsymbol{1}_{[N]} u_{\max}\}$ ($u_{(k)}$ being defined in a similar way to $\Delta \boldsymbol{u}_{L,(k)}$ with respect to the control inputs rather than to the control input changes) are convex, since it can be shown that they are obtained through convexity preserving transformations from the sets $\boldsymbol{S}_{u,(k)} = \{\boldsymbol{u}_{(k)} \mid \boldsymbol{0}_{[N]} \leq \boldsymbol{u}_{(k)} \leq \boldsymbol{1}_{[N]}\ \boldsymbol{u}_{max}\}$ which are obviously convex. Thus, the overall problem is a convex optimization problem.

In order to incorporate the inequality constraints into the objective function, a logarithmic barrier function [5] is constructed as $\phi_{(k)} = \sum_{i=1}^{L} \phi_{i,(k)}$,

where $\phi_{i,(k)} = -\ln(-\varphi_{i,1,(k)}) - \ln(-\varphi_{i,2,(k)}) = -\ln(\varphi_{i,(k)})$.

It is easily shown that $\varphi_{i,(k)}$ are concave and positive on $\boldsymbol{S}_{u,(k)}^{-} = \{\boldsymbol{u}_{(k)} \in \boldsymbol{S}_{u,(k)} \mid \boldsymbol{u}_{(k)} \neq \boldsymbol{h} u_{max}\}$, from which it follows that the functions $\phi_{i,(k)}$ are convex on $\boldsymbol{S}_{u,(k)}^{-}$ and $\mathbf{S}_{\Delta,(k)}^{L-}$ ($\mathbf{S}_{\Delta,(k)}^{L-}$ defined in the same way, but on $\mathbf{S}_{u,(k)}^{L}$). Hence, $\phi_{(k)}$ is convex.

We have

$$\nabla \phi_{i,(k)} = -\frac{1}{\varphi_{i,(k)}} \nabla \varphi_{i,(k)};$$

$$\nabla^2 \phi_{i,(k)} = \frac{1}{\varphi_{i,(k)}^2} \nabla \varphi_{i,(k)} \nabla \varphi_{i,(k)}^T - \frac{1}{\varphi_{i,(k)}} \nabla^2 \varphi_{i,(k)};$$

$$\nabla \phi_{(k)} = \sum_{i=1}^{L} \nabla \phi_{i,(k)}; \quad \nabla^2 \phi_{(k)} = \sum_{i=1}^{L} \nabla^2 \phi_{i,(k)};$$

$$\nabla \varphi_{i,(k)} = \Big[\underbrace{\psi_{i,(k)}, \psi_{i,(k)}, .., \psi_{i,(k)}}_{i \text{ elements}}, \boldsymbol{0}_{[1\times L-i]}\Big]^T \in \mathbf{R}^L,\ \psi_{i,(k)} = -2\sum_{j=k}^{k+i-1} \Delta u_{(j)} + u_{\max} - 2u_{(k-1)};$$

$$\nabla^2 \varphi_{i,(k)} = \left[\begin{array}{c|c} \boldsymbol{S}_2 & \boldsymbol{0}_{[i\times L-i]} \\ \hline \boldsymbol{0}_{[L-i\times i]} & \boldsymbol{0}_{[L-i\times L-i]} \end{array}\right] \in \mathbf{R}^{L\times L},\ \boldsymbol{S}_2 = -2.\boldsymbol{1}_{[i\times i]}.$$

The gradient and the Hessian operators are defined as follows:

$$\nabla f \equiv \left(\frac{df}{d\boldsymbol{x}}\right)^T = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, .., \frac{\partial f}{\partial x_n}\right], f \in \mathbf{R},\ \boldsymbol{x} \in \mathbf{R}^n,\ \nabla f \in \mathbf{R}^n$$

and

$$\nabla^2 f \equiv \left(\frac{d}{d\boldsymbol{x}} \nabla f\right) = \left[\left(\frac{d}{d\boldsymbol{x}} \frac{\partial f}{\partial x_1}\right)^T, \left(\frac{d}{d\boldsymbol{x}} \frac{\partial f}{\partial x_2}\right)^T, .., \left(\frac{d}{d\boldsymbol{x}} \frac{\partial f}{\partial x_n}\right)^T\right]^T, \nabla^2 f \in \mathbf{R}^{n\times n}.$$

The output and state constraints, expressed using the underlying prediction model, can be introduced in the same way to the problem.

An approximate problem is formulated, in which a barrier term is added to the objective function, accounting for the inequality constraints:

(11) $\min\ \tilde{J}_{(k)} = J_{(k)} + r\phi_{(k)}$.

We also have

$\nabla \tilde{J}_{(k)} = \nabla J_{(k)} + r \nabla \phi_{(k)}$;

$\nabla^2 \tilde{J}_{(k)} = \nabla^2 J_{(k)} + r \nabla 2\phi_{(k)}$.

As $r \to 0$ the approximate problem goes to an inequality constrained problem. As it can be seen, the approximate problem is an unconstrained convex optimization problem.

The MPC is introduced solving the above formulated unconstrained optimization problem at each sampling instant, yielding the optimal sequence $\Delta \boldsymbol{u}_{L,(k)}^{opt}$. From the optimal sequence, the first element is applied as a change in the control input to the process. As seen from the formulation, the term $f_{(k)}$ is updated at each sampling instant, which requires state and disturbance information.

4. Extension of the Basic Algorithm and Implementation

Since information about all states is required as an initial condition for the optimization procedure, the overall control law is augmented by a Luenberger type of an observer, based on model (2)

(12) $\hat{\boldsymbol{x}}_{(k+1)} = \boldsymbol{A}\hat{\boldsymbol{x}}_{(k)} + \boldsymbol{b}\Delta u_{(k)} - \boldsymbol{L}(\tilde{\boldsymbol{C}}\hat{\boldsymbol{x}}_{(k)} - \xi_{(k)})$,

where $\xi_{(k)}$ is the vector of measured variables $\xi_{(k)} \equiv [\theta_{out,(k)}, \Delta\theta_{h,(k)}]^T \equiv [y_{(k)}, \Delta\theta_{h,(k)}]^T$, with $\Delta\theta_{h,(k)} = \theta_{h,(k)} - \theta_{h,(k-1)}$. The output matrix of the observer is augmented by a second row and defined as $\tilde{\boldsymbol{C}} \equiv \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{C}_{\theta} \end{bmatrix} \in \mathbf{R}^{2\times n+1}$. The observer gain matrix $\boldsymbol{L} \in \mathbf{R}^{n+1\times 2}$ is chosen so that $\boldsymbol{A} - \boldsymbol{LC}$ is Hurwitz (a closed-loop observer is required, since as a consequence of the problem formulation, $\boldsymbol{A}$ has a pole equal to 1).

Then the optimization problem solved at each sampling instant is constructed using the estimated values $\hat{\boldsymbol{x}}_{(k)}$, that is, the term $\boldsymbol{f}_{(k)}$ in the objective function (7) and its gradient (8) are calculated with the estimates. No information of the disturbance has been used in the control law.

It should be noted that if the reference signal for $t = (k + 1)T_0$ is known at the sampling instant $t = (k - 1)T_0$, then it is possible to determine the control input (solve the optimization problem) to be applied in the interval $kT_0 \leq t < (k+1)T_0$, in the interval $(k - 1)T_0 \leq t < kT_0$, since all the required information is available at $t = (k - 1)T_0$. This possibility is not exploited in the current implementation.

The optimization problem (10) is solved by implementing Newton's method and backtracking the line search [5] with a fixed value of r (though not advised in [5], satisfying results were found during the preliminary tests). The value of r was set to 10^{-8} and Newton's algorithm sub-optimality tolerance, estimated by Newton's decrement, was set to 10^{-7}.

The overall MPC algorithm was implemented in *Matlab*, using the capabilities of the *Data Acquisition Toolbox* to interface the USB DAQ device *NI 6009*, used to realize the physical interface to the process.

The discrete-time prediction model of the process is obtained by discretizing, assuming zero-order holds at the inputs, phase-variable canonical form representation of the delay-free part (the rational part) of (1). Then the time delay is accounted in the model by introducing $n_d = t / T_0$ additional states at the system output, such as $x_{j,(k+1)} = x_{j+1,(k)}$, $j = 1 \,..\, n_d - 1$ and $x_{n_d,(k+1)} = y^{df}_{(k)}$, with $y^{df}_{(k)}$ – the output of the delay-free part of the process model.

Thus, the following state space matrices were obtained and inserted in (2) (we have $n = n_d + 2$ according to the definitions in Section 3):

$$\overline{\boldsymbol{A}} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \cdots & \cdots & \ddots & 0 & \vdots \\ 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & \cdots & 0 & a_{11} & a_{12} \\ 0 & 0 & \cdots & 0 & a_{21} & a_{22} \end{bmatrix} \in \mathbf{R}^{(n_d+2)\times(n_d+2)},$$

$$\left[\overline{\boldsymbol{b}} \;\middle|\; \overline{\boldsymbol{b}}_d\right] = \left[\begin{array}{c|c} 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ \hline b_{11} & b_{12} \\ b_{21} & b_{22} \end{array}\right] \in \mathbf{R}^{(n_d+2)\times 2}, \quad \overline{\boldsymbol{C}}^T = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in \mathbf{R}^{(n_d+2)},$$

where

$$\boldsymbol{H} \equiv \begin{bmatrix} a_{11} & a_{21} \\ a_{21} & a_{22} \end{bmatrix}, \boldsymbol{H} = e^{T_0 \boldsymbol{F}}, \boldsymbol{F} = \begin{bmatrix} 0 & 1 \\ -1/T_1T_2 & -1/(T_1+T_2) \end{bmatrix}, \begin{bmatrix} b_{11} & b_{21} \\ b_{21} & b_{22} \end{bmatrix} = \left(\boldsymbol{H} - \boldsymbol{I}_{[2\times 2]}\right)\boldsymbol{F}^{-1}\begin{bmatrix} 0 & 0 \\ k_\mu / T_1T_2 & k_d / T_1T_2 \end{bmatrix}.$$

5. Some Experimental Results

For the experimental results discussed in this paragraph, the following values of the parameters were set: $N = 20$, $L = 4$, $\boldsymbol{Q} = diag(q_i)$, $q_i = i$, $i = 1 \,..N$, $\boldsymbol{P} = diag(p_i)$, $p_i = 0{,}001$, $i = 1 \,..N$.

The mean time, needed for solving the optimization problem on the computer configuration used, was less than 0.01 s.

A sampling period of 10 s was selected. The PWM period was set to 2 s, thus having 5 PWM periods per a sampling period. Several experiments were conducted with the implemented model predictive controller and some of the obtained transients are shown in *figure 3*.

For the experiment, shown in *figure 3a*, the process model was constructed using the rational part of (1) with parameters given in (1.1) (thus, using a transfer function relative to the output temperature). The number of the additional states was set to 9, i.e., $n_d = 9$ (an equivalent delay of 90 s). The second row in the observer matrix was defined as $\boldsymbol{C}_\theta \equiv \left[\boldsymbol{0}_{[1\times n_d - 3]} \;\vdots\; 1 \;\vdots\; \boldsymbol{0}_{[1\times 5]}\right]$, thus accounting for the 30 s delay, found in (1.2). The coefficients in the observer gain matrix were set by a trial and error manner, and the resulting closed-loop dynamics were determined by the following poles: 0,8, 0,7788 and 0,8465, the rest of the poles being equal to 0.

For the experiment, shown in *figure 3b*, the process model was constructed using the rational part of (1) with parameters given in (1.2). The number of the additional states was set to 10, i.e, $n_d = 10$ (an equivalent delay of 100 s). The same observer output matrix was used. The observer gains were set to attribute analogous closed-loop dynamics.

As seen on both graphs, a zero offset is indeed achieved and the control input drives the outlet temperature to its reference with satisfactory transient performance. Slow variations of the inlet temperature within the range 15.3- 15.7 °C were observed during the experiments. As expected, the induced effects are compensated by the control law and any deviations are hardly visible on the curves. Besides, the first transient in both series differs performance-wise from the following ones, which is probably due to the initial convergence of the observer estimates. Further experiments with more adequate initialization of the observer estimates and optimized observer closed-loop dynamics are planned.

6. Conclusion

An MPC scheme was implemented in order to control a laboratory heat-exchange process experiment with

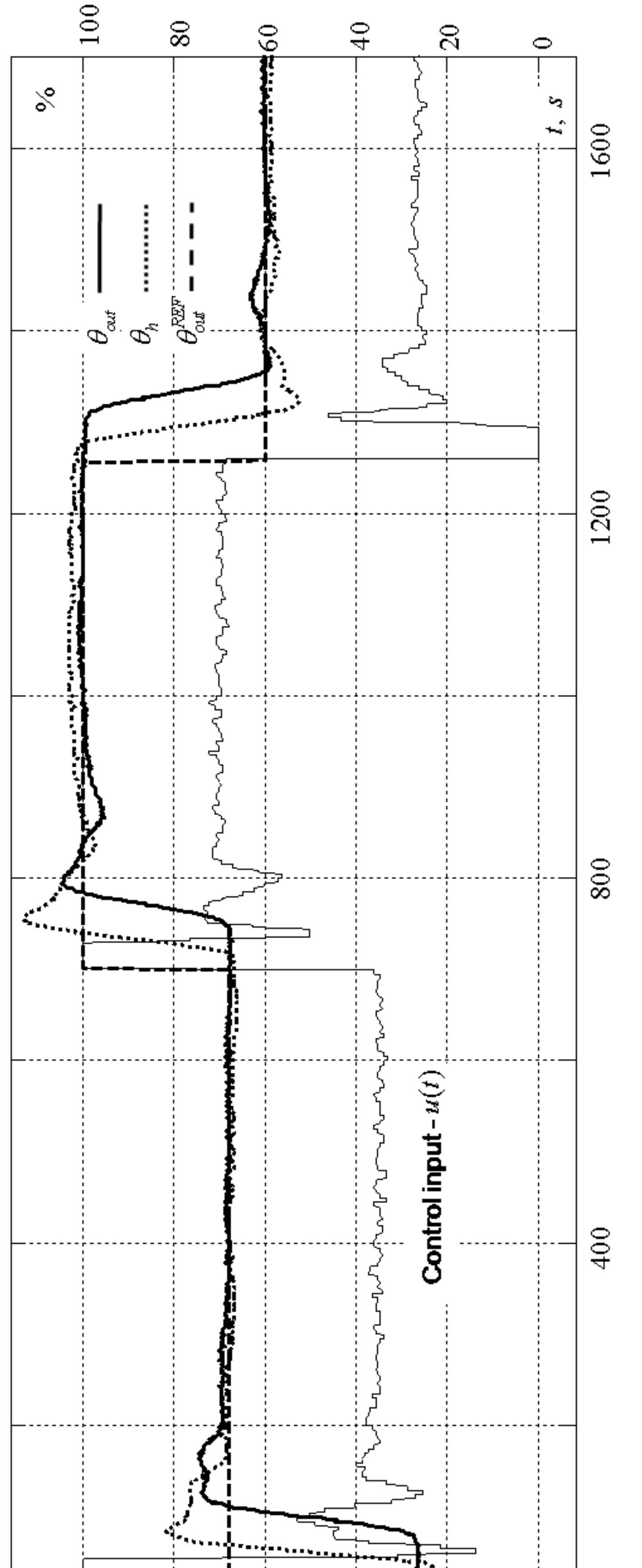

Figure 3. Transient responses (a)

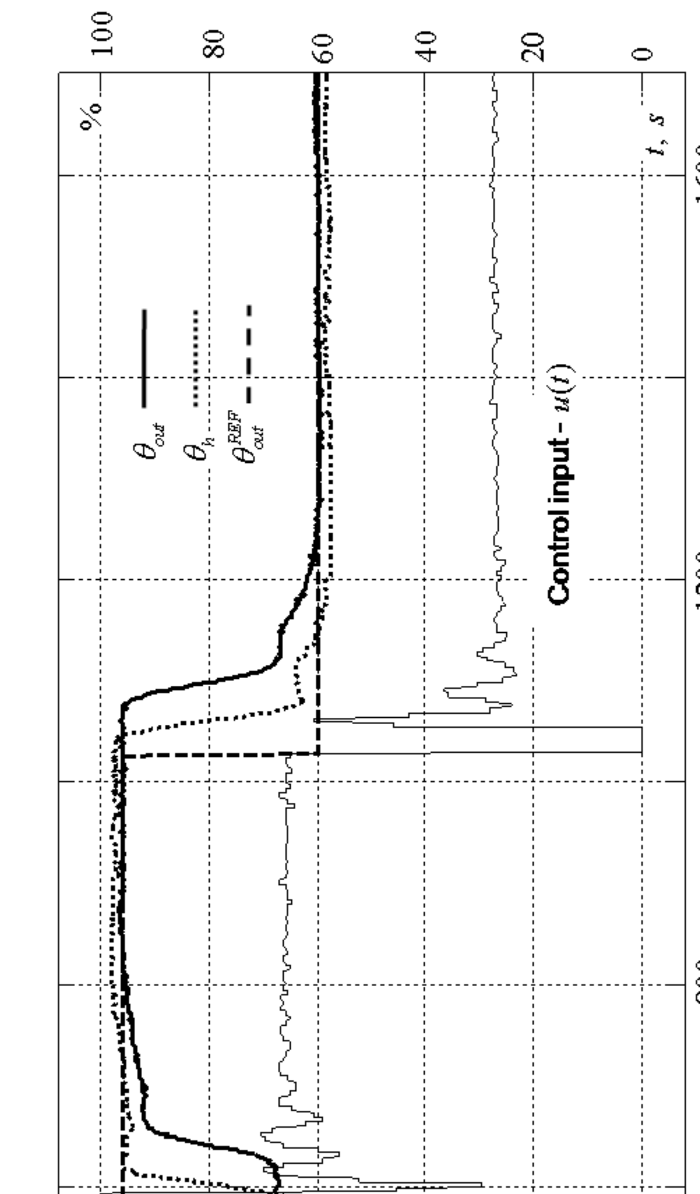

Figure 3. Transient responses (b)

a significant transportation time delay. The time delay is reflected in the prediction model by augmenting the state vector with the respective number of additional delayed states. Then a reformulation of the description in terms of the control and state changes is used to attribute an integral action to the overall control law which incorporates also a Luenberger observer, in order to estimate the non-measured state variables. First experiments with the controller proposed were conducted, which have confirmed the expected performance. Zero offset control was achieved without using disturbance information in the elaboration of the control signal.

References

1. Maciejowski, J. Predictive Control with Constraints. Prentice Hall, 2002.

2. Morari, M. Predicting the Future of Model Predictive Control. Systems and Process Control Centennial Session, 2008 Annual AIChE Mtg, 2008.

3. Maeder, U. Augmented Models in Estimation and Control. DSc Dissertation ETH Zurich, 2010, No. 19278.

4. Faanes, A., S. Skogestad. Offset-Free Tracking of Model Predictive Control with Model Mismatch: Experimental Results. *Ind. Eng. Chem. Res.*, 44 (11), 2005, 3966–3972, DOI:10.1021/ie049422y.

5. Boyd, St., L. Vandenberghe. Convex Optimization. Cambridge University Press, 2004.

Manuscript received on 2.12.2013

Stanislav Enev *(born 1980) received the M.Sc. degree in Electrical Engineering in 2004 and the Ph.D. degree in Industrial Automation in 2009 from the Technical University – Sofia, French Language Department of Electrical Engineering. Since March 2008 he is with the Department of Industrial Automation (FA), Technical University – Sofia as an Assistant Professor. His current research interests are in nonlinear drive and electromechanical systems control theory and applications, model-predictive and adaptive process control.*

Contacts:
Technical University – Sofia, Faculty of Automation
Department of Industrial Automation
e-mail: enev@tu-sofia.bg