Adaptive Stabilizing Controller Design Performing Under Stability of Sliding Type

S. Stanchev

Key Wors: *Design Method; Direct adaptive stabilizer; Sliding stability; Robustness; Inverted pendulum benchmark.*

Abstract. *This work studies an application of redesign of classical MRAC controller for nonlinear plant allowing description in canonical nonlinear form. Since MRAC scheme employs a linearized model of the plant in a state space form, focused not only on robustness, but first on stability analysis of the system. The simplicity of this redesign having in mind repeating of the main steps of considered classical design with all advantages of the method is compensated by introducing another stability type which is more reliable and mathematically simpler. The manner of using of sliding stability removes its main disadvantage – the chattering phenomenon. Chattering disappears because in this redesign the use of variable structure is avoided, since, the Lyapuov derivative of classical design is modified for guaranteeing sliding stability.*

Introduction

This paper considers a method for designing adaptive stabilizing controllers or stabilizers for plants allowing description in nonlinear canonical form in continuous time (CT) region on the basis of classical adaptive design by direct method using reference, or target model, known as Model Reference Adaptive Controller (MRAC) design. The most characteristic feature of this design and the following redesign is to ensure or to rebuild the stability type so that the real time performance of the derived adaptive stabilizing system be under sliding stability. Obviously, this is the main difference between the proposed design and the well known classical one with its many variants, see e.g., [2,9, 23,24,25,26,28,30] in CT, and [22] in discrete-time (DT) regions, respectively. One of the main purposes of this redesign is the possibility to study the robustness properties of such a system regarding uncertainties like neglected dynamics, external disturbances, etc., governed by sliding stability from another point of view, having in mind the invariant properties of the sliding mode.

Note that all known designs of MRAC systems aim at guaranteeing asymptotic or exponential stability, mainly, using linear or linear approximations of plant models, see e.g., [2, 9] and papers sited therein. Such systems function in real time but their general application, although for linear systems, is a rare case in practice, since its application is a serious problem. Some attempts for applying MRAC designs for nonlinear systems are mainly in the linearized or working region of the system.

Recent nonlinear theory, see e.g. [3,4,6,30], uses for such kind of designs the method known as back-stepping technique [6]. The main advantage of this approach is the high degree of the robustness of the derived system.

In the paper it is shown also that although this design uses linear approximation of the plant model it successively stabilizes the system demonstrating good performance. This is shown by simulations on the classical example of a planar inverted pendulum – a device described by nonlinear and unstable model belonging to the considered nonlinear canonical model. In this way, differentiating this redesign from the cited back-stepping technique is made where nonlinearities are considered with maximum precision. However in practice, the problem would be somewhat different, needing more efforts to guarantee satisfactory robustness.

This class of objects, allowing description by nonlinear canonical structure, is a subject of study in our previous papers, [10]. In [13,14] hybrid schemes are synthesized in CT and DT regions, respectively, for single dimensional nonlinear canonical form. In [15] multidimensional nonlinear canonical model is studied in CT settings and in [12,13] hybrid case is considered in the sense of CT plant nature and DT controller with additional switching conditions.

In the cited papers, however, the sliding mode is introduced only partially on a certain region of changes (critical for some systems, e.g., for unstable plants) of the output variable and the asymptotic stability is saved on the remainder of the output variable range in the sense of classical design by direct method [16]. The results of the simulations show also the perfect performance of the obtained adaptive system.

The organization of the paper is as follows. In the next section 2 the statement of the problem is given and the purpose of the proposed study is formulated. In subsection 3a the needed preliminaries of classical MRAC design are given and the closed loop system in its linearized form is formed. In the next subsection 3b the adaptive law is derived and the sliding stability of the system is proven. Next subsection 3c deals with the convergence of signals and system error. Subsection 3d considers to some extent the robustness of the adaptive system scheme and in section 4 an example is modeled demonstrating how the controller works. At the end of the paper some conclusions are made about the system performance and robustness and some directions for further studies are drawn.

2. Statement of the Problem

The subject of this study is a nonlinear plant generally given by the scalar nonlinear normal form or Brunovski form in continuous time settings

$$\begin{aligned} &\dot{z}_1 = z_2 \\ &\quad\vdots \\ (1)\quad &\dot{z}_n = f(t) + g(t)u \\ &y = h(z) \end{aligned}$$

In (1), $z(t) \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}$ is the output variable, and $u \in \mathbb{R}$ is the input variable. The nonlinear functions $f(t) \in \mathbb{R}$, $g(t) \in \mathbb{R}$, $h(t) \in \mathbb{R}$ are generally assumed smooth and their derivatives of the necessary order exist. It is also assumed that $f(0) \equiv 0$, $g(0) \neq 0$, $h(0) \equiv 0$, i.e. the standard controllability and stabilizing conditions are satisfied. Without loss of generality, for brevity and clearance the arguments are designated as subscripts or dropped out (if this does not lead to obscurity or incorrectness).

The purpose of this study is, for the plant given by (1), a CT stabilizing controller to be designed using as a basis the classical methodology of the respective MRAC designs by direct method for a linear plant described in state space. However, the new moment in the proposed technique is that this redesign is modified with the main purpose to exchange the stability type of the classical design, that is, the asymptotic stability by sliding stability type. The stability of the system is proven next. Its robustness properties are analyzed, having in mind the invariance properties of the employed stability type. Finally, the proposed redesign is applied on a classical unstable and nonlinear example, the planar inverted pendulum, in order for the obtained results to be confirmed by simulations.

3. Design of an Adaptive Stabilizing Controller

a) Description of the Closed Loop System

Starting with the proposed redesign in this section some basic steps of classical adaptive designs by using reference or target model, [2,9] are reinitiated.

In linear, CT framework, the scalar nonlinear model (1) is analogical to the model (2) below, i.e., the linear canonical state-space form

$$(2)\quad \begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= c^T x(t) \end{aligned}.$$

The meaning of the symbols in (2) are as follows: $x(t) \in \mathbb{R}^n$ is the state vector with the same dimension as in (1), $y(t) \in \mathbb{R}$ is the output variable, $u(t) \in \mathbb{R}$ is the input variable. The symbol „$(.)^T$", is used for specifying a vector or matrix transpose. In linear model (2), the respective dimensions of the state matrix A, input vector b, and measurement vector $c^T = [1 \quad 0 \quad \cdots \quad 0]$, ($d = 0$) are equal to that of (1). Only the input vector is trivially modified, i.e. $b^T = [0 \quad 0 \quad \cdots \quad b_n]$.

The chosen stabilizing adaptive (bilinear) structure is the frequently used in the literature for such kind of designs, i.e.

$$(3)\quad u(t) = D^T(t)x(t).$$

In (3), $D(t) \in \mathbb{R}^n$ is the adaptive law and is one of the purposes of this (re-) design. Let separate the adaptive law as follows, $D(t) = D_n + D_a(t)$, where D_n is an appropriately chosen constant part, and $D_a(t)$ is the variable part subject of the proposed redesign. Clearly, the constant part of (3) is determined by appropriately chosen initial conditions, $D_n = -K_c$. As mentioned in the simulation part below, the choice of the initial conditions can be provided by pole assignment method, optimal control methods and many others. In the numerical part of this study it is also shown that using the pole assignment method has some advantages as a possibility for fast correction of initial data.

Further, some of the necessary steps of classical designs are reinstated in order to complete the mathematical basis of the considered problem. Substituting controller equation (3) into the linear plant equation (2), the so called plant-controller equation (4) is obtained

$$(4)\quad \dot{x}(t) = A_n x(t) + \Phi(t)x(t)$$

where in (4), $A_n = A - bD_n^T$ is the nominal and $\Phi(t) = bD_a^T(t)$ is the adaptive part of the system, respectively. Defining further the system error $e(t) \triangleq x^e(t) - x(t)$, the nominal or target model with the same structure and dimensions as (1) and (2), and the closed-loop error of the system, respectively are

$$(5)\quad \dot{x}^e(t) = A_e x^e(t),$$

and

$$(6)\quad \dot{e}(t) = \dot{x}^e(t) - \dot{x}(t) = A_e e(t) - \Phi(t)x(t).$$

Clearly, the purpose of design of a target model depends on the common criteria, mainly stability of the system and all other necessary requisites as convergence of all signals, robustness features, the quality of system performance, etc. In order for the system requirements to be met, let us subordinate the nominal model by the following

Assumption: $A_e \equiv A_n$,

i.e., the nominal and target models are equivalent. This Assumption implies, however, that the closed loop error equation (6) can be written as follows

$$(6a)\quad \dot{e}(t) = -\dot{x}(t) = A_n e(t) - \Phi(t)x(t).$$

It should be noted that, because of the above assumption, the closed loop error can be defined without taking into account the target model (5), i.e., $e(t) \triangleq A - x(t)$.

b) Deriving of the Adaptive Law and Analyzing Stability of the System

From the classical results of adaptive controller design by means of the direct method [2,9] Lyapunov function candidate is the standard quadratic form

$$(7)\quad V(t) = e^T(t)Pe(t) + D^T(t)\Gamma^{-1}D(t).$$

As is known, the matrices P and Γ are positive definite,

Γ is taken diagonal, and $e(t)$ is the system error, (6), (6a). The reason to include only the adaptive law $D(t)$ to be designed in the second term in (7) and not $\widetilde{D}(t) \triangleq D(t) - D^*$, as in [2,9], with D^* a real valued vector to which the adaptive law converge, is the chosen stability type, as will be shown further in this section. The Lyapunov derivative below is slightly different in comparison with the classical one because of the introduced sliding term $\dot{V}_{SI}(t)$ [1,17,18,27,29], added to and subtracted from (7) which construction will be clarified below

(8) $$\dot{V}(t) = -e^T(t)Qe(t) + 2D^T(t)\Gamma^{-1}\dot{D}(t) - 2(\Phi(t)x(t))^T Pe(t) \pm 2\dot{V}_{SI}(t).$$

In (8) Q is a positive definite matrix, a solution of Lyapunov equation. After some algebraic manipulations on the term $(\Phi(t)x(t))^T Pe(t) = (bD_a^T(t)x(t))^T Pe(t)$ in (9), since, $D^T(t)x(t) = (D^T(t)x(t))^T = u(t)$, and $f_p(e(t)) = b^T Pe(t) = b_n(p_{1n}e_1(t) + p_{2n}e_2(t) ... + p_{nn}e_n(t))$, we have $(\Phi(t)x(t))^T Pe(t) = (D^T(t)x(t))(b^T Pe(t)) = D^T(t) f_p(e(t))x(t)$.

Replacing $\dot{V}_{Cl}(t) = -e^T(t)Qe(t)$ and $\dot{V}_{SI}(t) \triangleq k_s \|D(t)\|^2 e^T(t)Q_{SI}\dot{e}(t)$, with Q_{SI}, a positive-definite matrix similarly constructed as the matrix Q, for Lyapunov derivative (9) is obtained

(9) $$\begin{aligned}\dot{V}(t) &= \dot{V}_{Cl}(t) + 2\dot{V}_{SI}(t) \\ &+ 2D^T(t)(\Gamma^{-1}\dot{D}(t) - f_p(e(t))x(t) - k_s D(t)e^T(t)Q_{SI}\dot{e}(t))\end{aligned}.$$

Applying, further, Krasovskii [5], La Salle's [8] invariance principle,

(9a) $\lim_{t\to\infty}(\dot{D}(t) - \Gamma(f_p(e(t))x + k_s D(t)e^T(t)Q_{SI}\dot{e}(t)) = 0$,

finally, for the adaptive law and Lyapunov derivative, respectively, is obtained

(10) $$\dot{D}(t) = \Gamma(f_p(e(t))x + k_s D(t)e^T(t)Q_{SI}\dot{e}(t)),$$

(11) $$\dot{V}(t) = \dot{V}_{Cl} + \dot{V}_{SI}(t) = -e^T(t)Qe(t) + 2k_s\|D(t)\|^2 e^T(t)Q_{SI}\dot{e}(t).$$

Note that the sign of the real constant k_s must be changed during the real-time control process, e.g. by a condition as defined below

(12) $$k_s = \begin{cases} k_s^*, & if \ \dot{V}_{SI} < 0 \\ -k_s^*, & if \ V_{SI} \geq 0 \end{cases}.$$

The choice of (12) is discussed below and will also be considered in the numerical part of this study.

From (7), (11), the control law (3), the adaptive law (10), and the sign condition (12), by applying Lyapunov stability theorem [2, p. 109] and further, Krasovskii [5], La Salle invariance principle [8], it follows that the closed loop system (6), (6a), will perform under sliding stability, since the second term in (11), as is well known, is a sufficient condition for the existence of sliding mode. Note however, that in order for the second term to be dominating, the influence of the first quadratic term in (11) is sufficiently decreased. Such a behavior can be guaranteed if the matrix Q is taken approximately zero, $Q \approx 0$ or small enough, computing for this purpose the matrix P to obtain $f_p(e(t))$ and the adaptive law (10), so for Lyapunov derivative (11) to obtain

(11a) $$\dot{V}(t) = \dot{V}_{SI}(t) = 2k_s\|D(t)\|^2 e^T(t)Q_{SI}\dot{e}(t).$$

In this case the controller will perform purely under sliding stability if the sign of k_s, (12) is changing appropriately for keeping the pseudo-quadratic term $e^T(t)Q_{SI}\dot{e}(t)$ and then, (11a) negative-definite.

This finishes the proof of stability of the system, (1), (2), (3), (5), (6) or (6a), (10). (11), and (12). □

c) Boundedness and Convergence of Signals

On the basis of the proven sliding stability of the obtained adaptive scheme in the previous section, i.e., (1), (2), (3), (6), or (6a), (10), (11), and (12), all signals are bounded and convergent, as proven below.

Proof. The Lyapunov function (7) is positive definite and radially unbounded, since its derivative (11a) together with (12) is negative definite. From the Lyapunov stability theorem it follows that the system is asymptotically stable and (7) is a decreasing function with respect to time (w.r.t.). This fact implies boundedness of system error (6), (6a), and adaptive law, (10), i.e. $e(t), D(t) \in \mathcal{L}_\infty$. From (6) and (6a) follows also that the error rate is bounded, $\dot{e}(t) \in \mathcal{L}_\infty$, implying boundedness of the state vector $x(t) \in \mathcal{L}_\infty$, since the signals of the target (stable) model are bounded. The boundedness of the system error in quadratic sense can be proven as follows. Integrating both sides of (11) and substituting for $\dot{e}(t)$ from (6) give $\int_0^t \dot{V}(\tau)d\tau = -2k_s\int_0^t |D(t)|^2 e'(t)Q_{SI}(A_n e(\tau) - \Phi(x(\tau)x(\tau))d\tau$.

The boundedness of adaptive law can be expressed as $|D(t)|^2 \leq \Lambda_D$. Using, further, an estimation of the matrix Q_{SI} by its maximum eigenvalue $\lambda_{\min}[Q_{SI}]$ and for the normal matrix A_n, (6), by $\lambda_{\min}[A_n]$, we have

$$k_e\int_0^t (e'(\tau)e(\tau) - e'(\tau)\Phi(x)x(\tau))d \leq -1/2\int_0^t \dot{V}(\tau)d\tau,$$

where $k_e = k_s\Lambda_D\lambda_{\min}[Q_{SI}]\lambda_{\min}[A_n]$. After some algebra for the integral quadratic norm of the system error we obtain

$$\int_0^t |e(\tau)|^2 d\tau + \int_0^t e(\tau)'\Phi(x(\tau))x(\tau)d\tau \leq \frac{1}{2k_e}\Lambda_V$$

where $\frac{1}{2}\Lambda_V = \frac{1}{2}(V_0 - V_t) = -1/2\int_0^t \dot{V}(\tau)d\tau$. Since the right hand side of the above expression is bounded, it follows that the integral quadratic error is bounded, $e(t) \in L_2$. By applying, further, the Corollary of Barbalat's lemma, see e.g. [2, 9], mainly if its true that $e(t), \dot{e}(t) \in L_\infty$, and $e(t) \in L_2$, it follows that the system error (6) or (6a) will limit to zero, $\lim_{t \to \infty} e(t) = 0$.

With the above was proven that all signals in the closed loop system defined by (6), (6a) with or without using a target model (5), the plant-controller equation (4), the control law (3),the adaptive law (10), switching condition (12), with the Lyapunov derivative (11), are bounded and convergent, and the system error converges to zero. □

Notes

(i) Obviously, the main assumption, i.e. the uniform continuity when applying Barbalat's Lemma, of the system error and the adaptive law is also needed. The vector of system error cannot be zero if all of its elements are not zero; the case when all elements of system error are zero simultaneously is practically impossible. The above is valid and for the adaptive law vector.

(ii) Note also that for the adaptive law we only have proved that it is convergent but there is no convergence to some true values. This is one of the difficult questions in the adaptive control, especially in the direct adaptive control as is the case here, causing unexpected uncertainties in practice. This fact decreases the 'transparency' of direct control in comparison with the indirect or explicit variants. The application of the first one makes it more appealing. Obviously, the mentioned problem needs special attention from theoretical and practical points of view.

In the next part, the robustness properties of the obtained closed loop system, in terms of stabilization of nonlinear plant given by (1) is considered.

d) Robustness of the System

Clearly, as a result of this redesign, the Lyapunov derivative (11), (12) and the adaptive law (10) are different from that of classical design. Regarding the robustness properties of the derived adaptive system, the following conclusions are made.

As mentioned at the beginning, in the closed loop of the system the nonlinear model (1) is included instead of linear model (2). The numerical study below is based on the same idea. In the case under consideration, the system error will be

(13) $e(t) = e_m(t) + \Delta_e(t)$,

where, $e_m(t)$ designates the system error formulated via real measurements, i.e. scanning the system output variable with a certain sampling time period to have $e_m(t) = x^e(t) - z(t)$, where $z(t)$ is the state vector of the nonlinear model (1). The second term in (13), $\Delta_e(t)$ is the difference between the real, measured error and the ideal one of (6), (6a).

From (11) и (12) it is clear that, independently of the error (6) or (6a), the Lyapunov derivative is negative definite. If the second term in (11) is dominating – if the right hand side of Lyapunov equation $A_n^T P + PA_n = -\mu I$, with right side $I[n\,x\,n]$ the identity matrix and a real valued parameter μ – implies that the system performance will be, indeed, under sliding stability. The limit condition (9a), obtained after applying the invariance principle, is valid and is leading to the adaptive law (10). As is seen from (10) the expressions of the adaptive law are correct numerically, i.e. with assumption (9a) a real-time solution of the adaptive law (10) exists and is unique. These conclusions are confirmed numerically in the next section.

Since the existence condition of sliding mode [1,17,18,27,29] is fulfilled immediately at the beginning of the control process (11), this fact means that for an appearance of sliding mode reaching phase is not necessary. In order that the attractive condition of the sliding mode is guaranteed, analogically to classic designs by using a variable structure, similar technique can be used in this case.

Eventual fluctuations, i.e. 'chattering phenomenon', in such a design is impossible or if appears, their amplitude reduces extremely fast. When the system error (6) is decreasing and passes through zero, it changes the sign. However, assume that its derivative preserves the same sign. Because of condition (12), it follows that the Lyapunov derivative will be negative definite again and sliding stability will exist. In the opposite case, when the system error is increasing, in passing through zero the error derivative will be the same, positive again and the above conclusion is valid as well. It is important to note that the respective terms of the adaptive law (13) will also change their sign, but as the system error is very close to zero, the adaptive law cannot cause high frequency fluctuations of the control law. Obviously, this feature of the above redesign combines the quality of adaptation, mainly its performance with small signals, and the main advantage of sliding mode - its exclusive robustness. In other words, in the designed controller/stabilizer the main disadvantage of sliding mode is eliminated.

Our previous studies in hybrid settings, [11,12], show that because of the high invariance properties of the sliding stability, the scanning period and the discrete time of the controller/ stabilizer, respectively, can be chosen much bigger than the known cases found in the references. This fact allows performance of the system with initial conditions comparatively 'bigger' than these in the stabilizing or the linear region, which are usually used in almost all studies. Changing the stability type of the system does not allow the determined initial robustness, e.g. by pole assignment technique to be violated by an adaptation in a wrong direction - a situation observed in many cases of classical designs. Future analytical and numerical investigations will address these problems.

Let us mention at the end of this section that in our previous studies the sliding stability is introduced only partially at a certain interval of the output variable changes considering the additional Lyapunov function [10,11,12,13,16].

4. Numerical Study

In this part the derived adaptive stabilizer is tested on the well-known classical example – the planar inverted pendulum [7]. This example have been considered in our previous studies in SISO [13,12], and [15,11,14] in MIMO cases, respectively. CT case is considered in [13,15], and in hybrid settings in [12,11,14]. This example represents a nonlinear and unstable the plant [10] and is typical enough for this class of plants. One of the methods applied for controller design for this plant, used by many researchers, is the back-stepping method [6], see also [19]. Another known approach uses passivity theorem and is based on the energy characteristics of the system, [20].

In the proposed experiment it is assumed that real-time measurements are 'close' to the data from the nonlinear model (1), and disturbances of any kind are not taken into account. For clarity of the application of the method, the dynamics of the actuator device in the presented example below is neglected, i.e. the considered simplified pendulum model is as follows

$$\dot{\theta}_1 = \theta_2$$
$$\dot{\theta}_2 = f(t) + g(t)u\ ,$$

with nonlinear functions given as

$$f(t) = -a_1 \sin(\theta_1) - a_2\theta_2$$
$$g(t) = b_1 \cos(\theta_1) \qquad .$$

The parameter values of the respective linear model valid for the region of the upper equilibrium point are $a_1=-19.5,\ a_2=0.0,\ b_2=2.1951$. The linear model (2) in this case is

$$A=\begin{bmatrix} 0 & 1 \\ 19.5 & 0 \end{bmatrix},\quad b=\begin{bmatrix} 0 \\ 2.1951 \end{bmatrix}.$$

The eigenvalues of the state matrix are $\mathrm{eig(A)}=[4.6405, -4.6405]$, i.e. one stable and one unstable pole. The experimentally chosen poles of the normal and target model (5) are $\mathrm{eig(An)}=[-4.3, -5.7]$, see (4), and the initial conditions of the adaptive law (10) are $D'(0) = \begin{bmatrix} D_1(0) & D_2(0) \end{bmatrix} = \begin{bmatrix} 20.9757 & 4.5556 \end{bmatrix}$.

For the adaptive stabilizer (3), the state vector is $x(t) \in R^2$ and, respectively, the adaptive law (10) is with the same dimension as it is in the initial conditions above. The function $f_p(e(t))$ in this case is $f_p(e(t)) = b_2(p_{1,2}e_1(t) + p_{2,2}e_2(t))$, and the coefficients are the respective entries of the matrix P, $p_{1,2} = 0.0102$, $p_{2,2} = 0.0260$ – the solution of the Lyapunov equation with an appropriately chosen matrix $Q = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$. The matrix Q_{SI} determining the sliding mode, also experimentally chosen, is $Q_{SI} = \begin{bmatrix} -2.0 & 0 \\ 0 & -8.0 \end{bmatrix}$. The chosen real constant and the adaptive gain are $k_s^* = 0.01$, (12) and $\gamma = 0.01$, (10), respectively.

The numerical simulations are carried out in two parts. The first part is with initial conditions much different from the respective of the linear region $\theta = \begin{bmatrix} 65.5^0 & 0 \end{bmatrix}$. The results from the simulations are depicted in *figures 1, 2, 3 and 4.* The second part is with initial conditions closer to the stabilizing region, $\theta = \begin{bmatrix} 34.5^0 & 0 \end{bmatrix}$ and the results are shown in *figures 5, 6, 7 and 8.* It should be noted that the stabilizer works in the whole region without any necessary adjustments of its parameters - a result, to the knowledge of the author, not reached in any previous studies. The proposed numerical experiment confirms the obtained results, mainly the exclusive robustness properties of the system.

Regarding the robustness part of this paper from *figures 4 and 8* it is seen that at time moments $t = 0.25;\ 0.71$, the Lypunov derivative (11) has changed its sign because of the switching condition (12). It is clear that the system performance remains smooth as in the previous parts of these figures. A practical realization of this stabilizer, as is the case on the device of the Institute for Measurement and Control of University of Hanover [11,12], as in many other apparatus, may need a hybrid design. This is a better variant having in mind the forming of all the needed differences from the real-time discrete measurements and thus there is no necessity of a complicated observer design.

Regarding the adaptive law, as observed from the figures 3 and 6, it performs fast enough and in a comparatively wider range around the stabilizing region. In this range a pole assignment controller, which works satisfactory enough in the stabilizing region outside it, however, performs slower and when the initial conditions are increased the system loses stability. This is valid and for the classical adaptive controller as briefly derived in the introductory part, i.e. without using a sliding term in (10). Its performance is violated on the nonlinear range where a readjustment of the adaptive gain is needed - a fact, decreasing the effect of adaptation to a great extent. A main advantage of the latter remains mainly in the linear part, i.e. for initial values of the angle about 5 deg. In the simulations not shown here, the adaptive controller, however, performs better than a pole assignment controller.

Remark. A part of the numerical experiments, not included in this paper, show that the sliding stability can exist and without changing the sign of the sliding term; this observation could be proven also analytically. Clearly, this fact depends on the fulfillment of the condition for existence of sliding mode.

Conclusion

One of the purposes of this paper is a rebuild of the asymptotic stability type of the classical MRAC design to sliding mode stability of the system. The invariant features of sliding stability in comparison with the classical one, clearly, lead to more desirable robustness behavior of the system as is also demonstrated numerically in the previous section using for this purpose nonlinear and unstable plant model.

Introducing sliding stability, in addition, leads to a 'faster

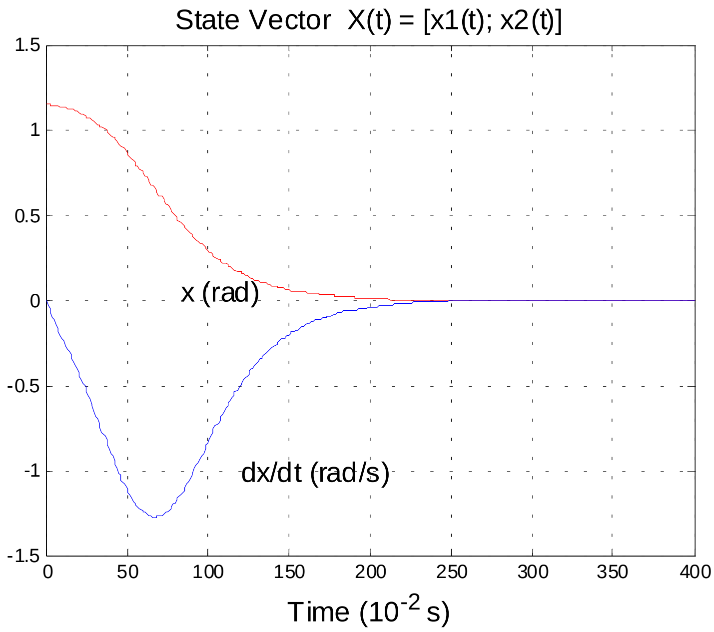

Figure 1

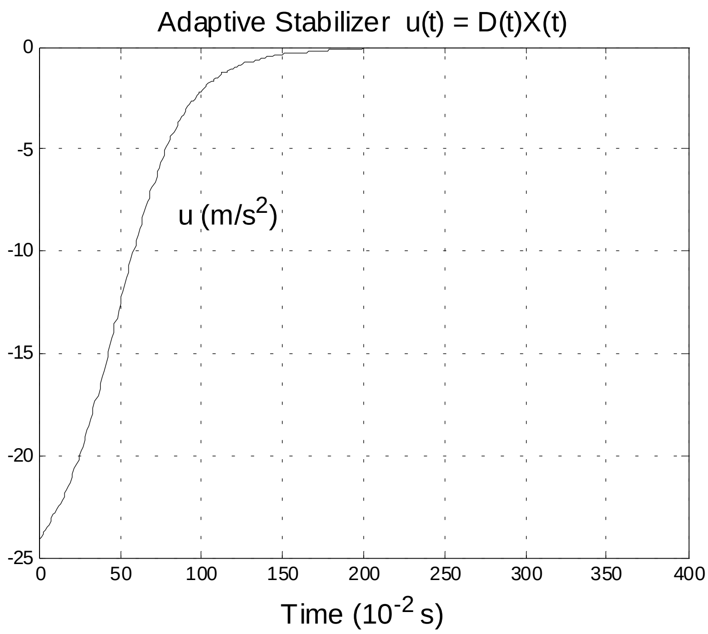

Figure 2

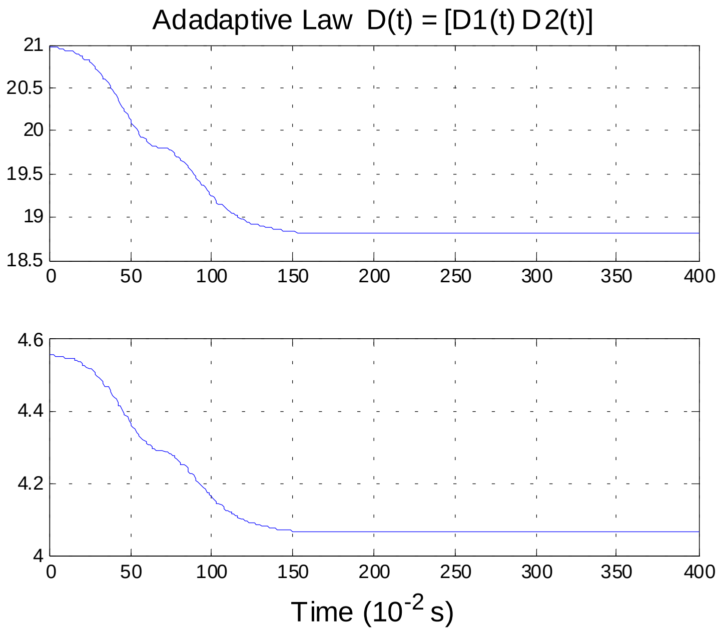

Figure 3

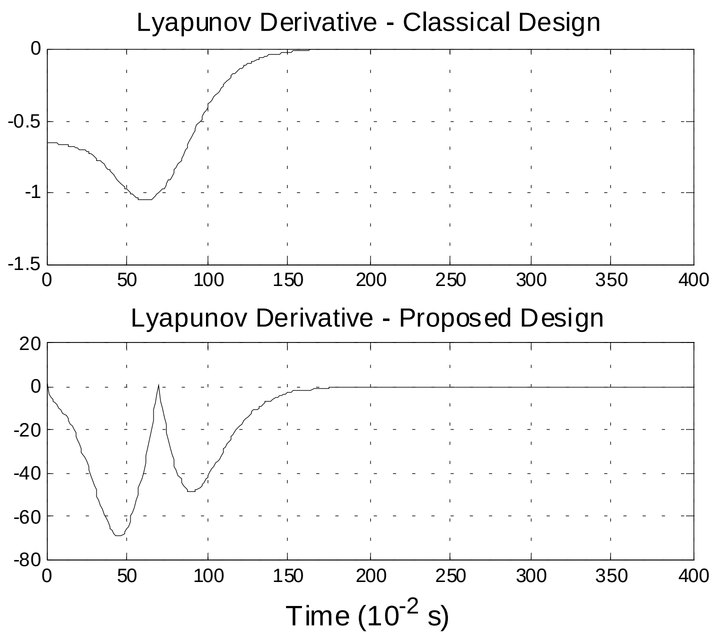

Figure 4

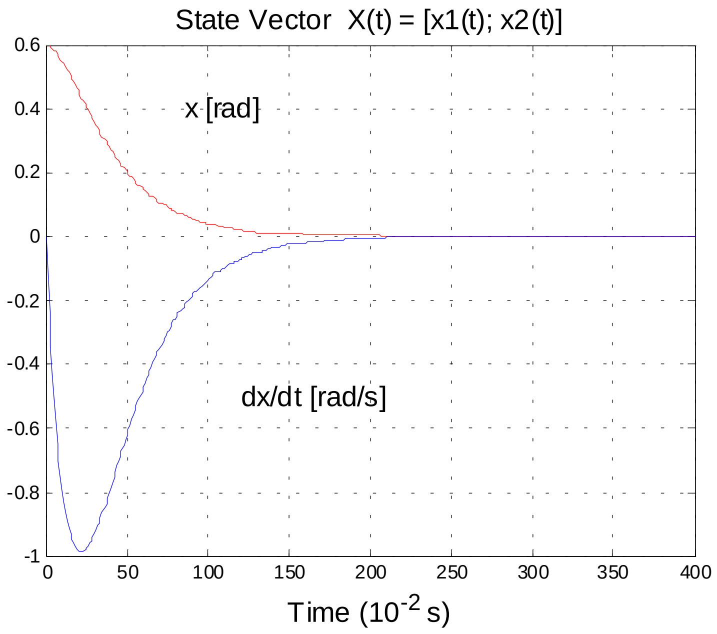

Figure 5

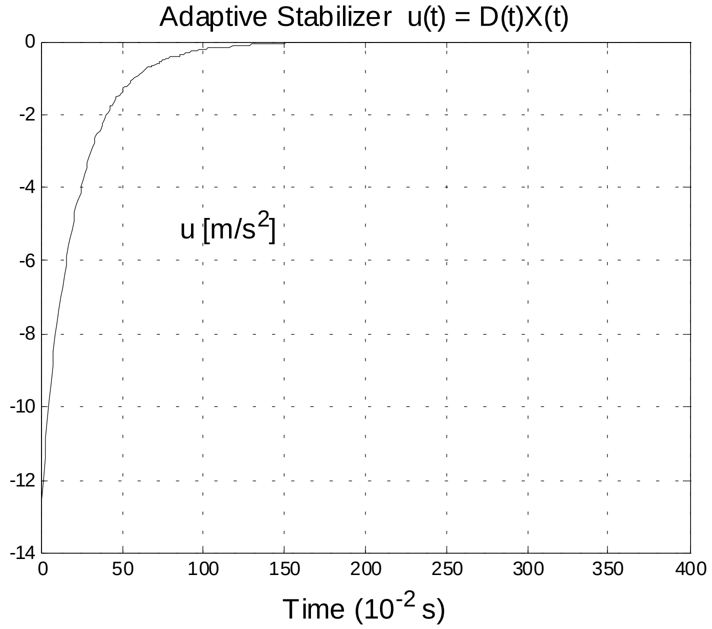

Figure 6

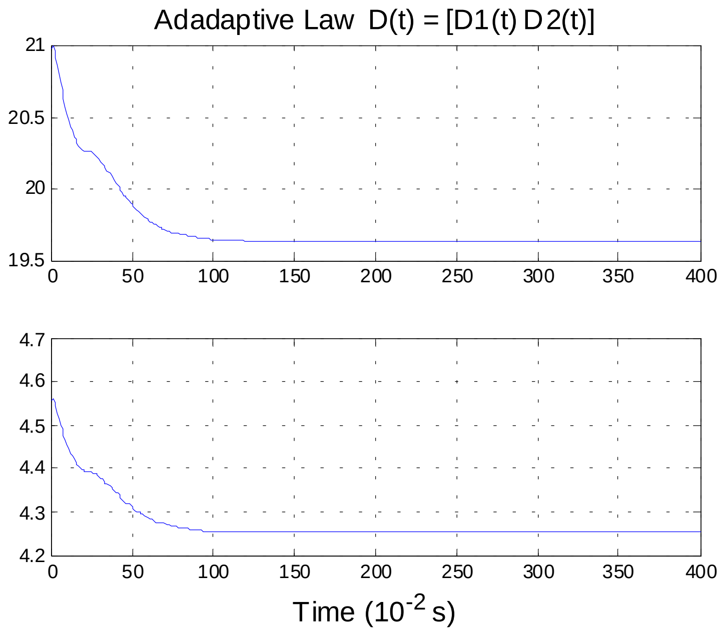

Figure 7

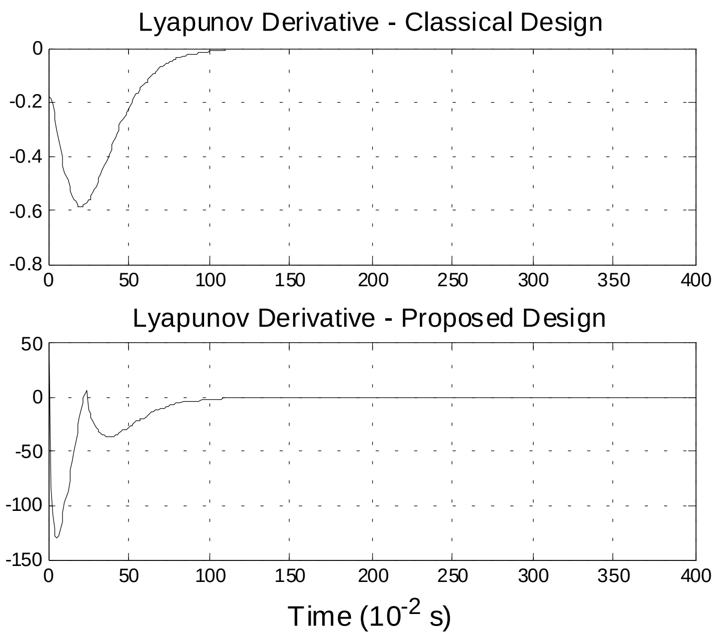

Figure 8

adaptation' despite the presence of terms in the model comparable with earth gravity. The obtained system has enough 'instruments' for an influence on the quality characteristics of the transient process.

One of the important features of the stabilizer and of the method, respectively, is the fact that since the adaptation performs under sliding stability, i.e. following an implicit sliding trajectory, the adaptation cannot get worse. It is already clear that this fact is as a result of the improved robustness features ensured by this redesign. Numerical experiments have shown that such a system performance is impossible to be met by a classical adaptive stabilizer.

Concluding this paper, it should be noted that this design, having as a main purpose the modifying of the stability type based on the conventional 'linear' adaptive theory, i.e. using linear approximation of the plant model, can be applied successfully for controlling this class of plants.

Future studies concerning application of the method for other classes of plants will give answer to the issue of the advantages and applicability of the proposed method.

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Stefan Stanchev *received his D-r Eng. and Ph.D. (C.Sc.) degrees both from the Department of Automation Technique, Slovak Technical University, Bratislava, Slovakia.*
Currently, Mr. Stanchev is Senior Research Fellow at the Department of Adaptive and Robust Control of the Institute of System Investigations and Robotics, Bulgarian Academy of Sciences.
His research interests are in the field of system analysis, stability theory, robustness studies, mathematical theory of automatic control, adaptive control, continuous-time (CT) and discrete-time (DT) control systems.

Contacts:
Bulgarian Academy of Sciences,
Institute of System Investigations and Robotics,
Acad. G. Bonchev St., Bl. 2, P Box 79,
1113 Sofia, Bulgaria, EU
E-mail: stefanstanchev@abv.bg, WEB http://www.bas.bg