Lyapunov Stability and Robustness of Fuzzy Process Control System with Parallel Distributed Compensation

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Key Words: *Linear matrix inequalities; Lyapunov stability; MATLAB™ real time temperature control; parallel distributed compensation PI process controller design; robust performance; time delay.*

Abstract. *The Parallel Distributed Compensation (PDC) established itself as a general systematic approach for fuzzy control systems design from stability criteria, using Lyapunov direct method to define analytically and the Linear Matrix Inequalities (LMIs) technique to solve numerically. Its main applications target is the mechanical systems. To spread it to process control some problems need engineering solutions. The aim of the investigation is to adapt the PDC-LMI approach for processes with time delay and uncertainty and PI fuzzy controllers in order to improve system performance and facilitate the PDC-LMI industrial application. The main results are: 1) development of identification based T-S modeling methodology employing Ziegler-Nichols approximation and PI local controllers; 2) derivation of Lyapunov fuzzy system stability conditions and the corresponding LMIs; 3) development of PDC PI controller design method for plants with time delay, ensuring system stability and robust performance; 4) its implementation for temperature control. Measurable improvements achieved by the PDC control in comparison with an ordinary PI control are the decreased by 30% overshoot and settling time.*

1. Problem Area

In recent investigations fuzzy control systems (FCS) are developed on the basis of dynamic fuzzy Takagi-Sugeno (T-S) plant and controller models [1] with common premises in the fuzzy rules. This approach known as Parallel Distributed Compensation (PDC) became popular, as it is systematic with emphasis on system stability and robustness, which can be studied analytically and combined with performance restrictions. It employs Lyapunov direct method [2,3] which does not require explicit plant mathematical model and well complements with advanced numerical techniques from convex programming [1,4,5] based on Linear Matrix Inequalities (LMIs) [1,4,5]. Stability and robustness are essential for practical feasibility of the FCS and also difficult to ensure because of the immanent nonlinear nature of both plant and controller.

The PDC-LMI builds a general and systematic approach [1,4,5] since: 1) any smoothly nonlinear dynamic system with restricted number of interruption points can be approximated by dynamic fuzzy T-S models [1] as well as any smoothly nonlinear controller can be approximated by PDC; 2) many control problems (stability analysis, optimal and robust design under restrictions on signals and system performance) can be formulated and solved in a unified way as LMIs both for linear and nonlinear, MIMO or SISO, discrete or continuous systems for control of plants with or without time delays and model uncertainties.

The same approach is applied for the design of state observers, output feedback or dynamic controllers. It integrates the best of the classic practice of fuzzy systems and the powerful means for design especially of linear systems of the modern control theory thus building a general methodology for control systems analysis and design. In this synergism the fuzzy logic offers both simple tools for decomposition of the modeling and design problem into a number of local linear easily solved tasks, and also a mechanism for aggregation of the local solutions in one common model or control. However, the PDC-LMI technique is developed for a specific fuzzy controller structure and T-S dynamic model of the plant.

This approach has been successfully applied mainly to mechanical systems with known nonlinear model. It is constantly being improved including diverse requirements to the performance of the FCS, signal restrictions, and reflecting more of the plant peculiarities such as time delay, model uncertainty, disturbances and noise, multivariable character, etc., characteristic especially for complex processes.

2. Theoretical Preliminaries

2.1. Parallel Distributed Compensation Principle

The Parallel Distributed Compensation determines a controller structure, based on the T-S model of the dynamic fuzzy system (DFS) – the plant, sharing with it common premises, i.e. each rule of the controller compensates a rule of the plant. The consequents in the fuzzy rules of the plant and the controller describe linear relationship between corresponding inputs and outputs. The PDC principle has been introduced as simplifying the Lyapunov requirement for global stability of the fuzzy closed loop system [1-3] and hence easing its solution.

It is assumed that the nonlinear plant can be described by a number of linear plant models usually in the state space, obtained either by linearization of a known nonlinear plant model in several operation points or as a result of experiments and identification often involving training of Sugeno type artificial neural networks (ANN) [6]. The linear plant models are supposed to be observable and controllable. They constitute the consequents in the fuzzy rules of the T-S plant model. A linear controller mainly a state feedback controller is designed for each linear plant, employing the well-developed linear systems design techniques to ensure local linear system stability and robustness. These local linear controllers are consequents in the fuzzy rules of the T-S controller. The global nonlinear fuzzy T-S controller is obtained as fuzzy blending between local linear controllers. The fuzzy relations (rules) of the plant and the controller in the *i*-th linearization sub-domain of the plant are respectively:

(1) $\mathbf{R_i}$: **IF** $z_1(t)$ is M_{i1} **AND...AND** $z_p(t)$ is M_{ip}

THEN. $\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + D_i d(t) \\ y(t) = C_i x(t) \end{cases}$;

(2) $\mathbf{R_i}$: **IF** $z_1(t)$ is M_{i1} **AND...AND** $z_p(t)$ is M_{ip}
THEN $u(t) = -\boldsymbol{F}_i x(t)$, $i=1\ldots r$,

where M_{ij} are linguistic values, defined as membership function (MF) of fuzzy sets; $x(t)\in\mathbf{R}^n$ is the state vector; $u(t)\in\mathbf{R}^m$ is the input control vector; $d(t)\in\mathbf{R}^m$ is the vector of the input disturbances; $y(t)\in\mathbf{R}^q$ is the output vector; $\boldsymbol{F}_i$ is the controller gain matrix; $\boldsymbol{A}_i\in\mathbf{R}^{nxn}$, $\boldsymbol{B}_i\in\mathbf{R}^{nxm}$ and $\boldsymbol{C}_i\in\mathbf{R}^{qxn}$; $z(t)=[z_k(t)]$, $k=1...p$ is the vector of the premise variables, which can be measured or estimated as a function of other measurable variables – state space variables, disturbances, time, plant performance measures, etc.

In absence of disturbances for given $[x(t)\ u(t)]$ the plant output after a Center-of-Gravity (**CoG**) defuzzification is obtained as:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(z(t))\{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^{r} w_i(z(t))} = \sum_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u(t)\},$$

(3)
$$y(t) = \frac{\sum_{i=1}^{r} w_i(z(t)) C_i x(t)}{\sum_{i=1}^{r} w_i(z(t))} = \sum_{i=1}^{r} h_i(z(t)) C_i x(t)$$

where $w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)), \left| \begin{matrix} \sum_{i=1}^{r} w_i(z(t)) > 0 \\ w_i(z(t)) \geq 0 \end{matrix} \right.$ is the degree of fulfillment of the compound fuzzy condition in the premise, $\boldsymbol{M}_{ij}(z_j(t))$ is the degree of match of $z_j(t)$ to $\boldsymbol{M}_{ij}$,

$h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}, \left| \begin{matrix} \sum_{i=1}^{r} h_i(z(t)) = 1 \\ h_i(z(t)) \geq 0 \end{matrix} \right.$ is the strength of firing of the rule and is normalized, which implies that the MFs comprise an orthogonal system.

For the fuzzy controller similarly it is obtained

(4)
$$u(t) = -\frac{\sum_{i=1}^{r} w_i(z(t)) F_i x(t)}{\sum_{i=1}^{r} w_i(z(t))} = -\sum_{i=1}^{r} h_i(z(t)) F_i x(t)$$

Thus for the closed lop system is computed:

(5) $\dot{x}(t)$ (or for discrete system $x(t+1)$ =

$$= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \{A_i - B_i F_j\} x(t)$$

The fuzzy closed loop system is asymptotically quadratically stable in global iff there exists a common for all local linear sub-systems positive definite matrix $\mathbf{P}$, that satisfies the Lyapunov stability inequality $A_{ij}^{T}\mathbf{P}A_{ij} - \mathbf{P} < 0$ for $\forall i, j$ ($A_{ij} = A_i - B_i . F_j$) [1-3]. Once $\mathbf{P}$ found the controller F_j is determined from the same condition. The controller design is iterative and not unique.

2.2. Linear Matrix Inequalities Technique

Linear Matrix Inequalities is a numerical computation technique for solving optimization problems of mathematical programming under convex restrictions – the LMIs, using the recently developed interior-point method [1,4,5]. The LMIs are generally of the type $G(x) = G_0 + \sum_{i=1}^{m} x_i G_i > 0$, where $\mathbf{G}(x)$ is a positive definite matrix, $\mathbf{G}_i$ are symmetric matrices and x is the unknown variable or matrix.

The solution of the Lyapunov problem is simplified by applying the LMIs techniques for computation of $\mathbf{P}$ (x=$\mathbf{P}$). After introducing $\mathbf{X}=\mathbf{P}^{-1}>0$ and $\mathbf{M}_j = F_j\mathbf{X}$ the Lyapunov inequality condition is first turned into the convex LMIs $\mathbf{X} - \{A_i\mathbf{X} - B_i\mathbf{M}_j\}^{T}\mathbf{X}^{-1}\{A_i\mathbf{X} - B_i\mathbf{M}_j\} > 0$ [1], which by the help of the Schur complement become

$$\begin{bmatrix} \mathbf{X} & (A_i\mathbf{X} - B_i\mathbf{M}_j)^{T} \\ (A_i\mathbf{X} - B_i\mathbf{M}_j) & \mathbf{X} \end{bmatrix} > 0, \ i, j = 1\ldots r.$$

The quadratic system stability is guaranteed by the existence of $\mathbf{X}>0$ and $\mathbf{M}_j$, that fulfill the LMIs. The controller is determined straightforward by $F_j = \mathbf{M}_j\mathbf{X}^{-1}$. The performance requirements are often included in the optimization problem as additional convex restrictions. So, from the set of fuzzy controllers (F_j: $\mathbf{X}>0$, $\mathbf{M}_j$) that ensure system stability, are selected those, which: 1) minimise α, thus restricting the damping of the system, defined as $\dot{V}[x(t)] \leq -2\alpha V(x(t)]$, where $V[x(t)]$ is the Lyapunov function; 2) maximise γ, thus suppressing the disturbances $d(t)$ - $\sup_{\|d(t)\|_2 \neq 0} \frac{\|y(t)\|_2}{\|d(t)\|_2} \leq \gamma$; 3) restrict the plant input ($\|u(t)\|_2 \leq \mu$) or output signals; 4) minimise integral squared error criterion with restriction on the control; 5) any combinations among 1)...4).

As the system performance is bounded with the stability requirements, the fuzzy controller design is an iterative and slow process. The requirements for stability and performance can be independently fulfilled without iterations from two different fuzzy rules sub-sets to design a state feedback controller for mechanical plants [7].

2.3. State-of-the-Art of Parallel Distributed Compensation Process Controller Design

There are a great number of derived LMIs for different cases of systems [1]. Their solution is the basis for the design of the fuzzy controller. The systems differ by the type of: 1) plant (with time delay(s), with diverse model uncertainties, etc.); 2) PDC based fuzzy controller (state feedback with or without an observer, output feedback, dynamic, etc.); 3) additional design requirements for desired performance, disturbance rejection, signals restriction, etc. 4) stability criteria used to simplify the controller design and reduce the conservatism of the stability conditions, which are sufficient [8].

In this investigation the design of PDC based process fuzzy controllers is considered out of the closed loop system stability and robustness requirements. This requires accounting for such characteristics of the plant as time delay and model uncertainties.

The general T-S model for plants with time delay in the state variables τ_1 and at the input τ_2 is described by fuzzy rules of the type:

(6) **R_i: IF** $z_1(t)$ is M_{i1} **AND...AND** $z_p(t)$ is M_{ip} **THEN**

$$\dot{x}(t) = A_{i0}x(t) + A_{id}x(t-\tau_1) + B_{i0}u(t) + B_{id}u(t-\tau_2), i=1\ldots r.$$

In case of disturbances $d(t)$ the fuzzy rules consequent is changed to:

(7) $\dot{x}(t) = A_{i0}x(t) + A_{id}x(t-\tau_1) + B_{i0}u(t) + B_{id}u(t-\tau_2) + D_i d(t)$, $i=1\ldots r$.

Independent of the time delay stability conditions or Lyapunov-Krasovski stability conditions for restricted variable time delay and corresponding LMIs for these cases are derived in [9,10] for PDC state feedback controller and observer design. The approach is tested on a constantly stirred tank reactor with irreversible exothermic reaction and recycling flow.

In case of plant model uncertainty the T-S plant model rules consequent is of the type [1]:

(8) $\dot{x}(t) = (A_i + D_{ai}\Delta_{ai}(t)E_{ai})x(t) + (B_i + +D_{bi}\Delta_{bi}(t)E_{bi})u(t)$ $i=1\ldots r$,

where $\|\Delta_{ai}(t)\| \le 1/\gamma_{ai}, \Delta_{ai}(t) = \Delta_{ai}(t)^{\mathrm{T}}$, $\|\Delta_{bi}(t)\| \le 1/\gamma_{bi}, \Delta_{bi}(t) = \Delta_{bi}(t)^{\mathrm{T}}$ are the restricted uncertainties.

The plant output is computed as:

(9) $\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\{(A_i + D_{ai}\Delta_{ai}(t)E_{ai})x(t) + (B_i + D_{bi}\Delta_{bi}(t)E_{bi})u(t)\}$.

Substituting the controller (4) in (9) yields the output of the closed loop system. For it the corresponding robust stability conditions are derived, which are solved by the help of LMIs as a maximization of the norms of the uncertainty blocks under the convex LMIs restrictions.

The T-S model for plants with time delay τ in the state variables and model uncertainties Δ**A**, Δ**B** and Δ**C** is presented with the fuzzy rules consequents $\dot{x}(t) = [A_{i0} + \Delta A_{i0}(t)]x(t) + [A_{id} + \Delta A_{id}(t)]x(t-\tau) + [B_{i0} + \Delta B_{i0}(t)]u(t)$. For PDC state feedback controller a generalized Lyapunov-Krasovski stability criterion and a model transformation with free matrices simplify the corresponding LMIs but increase the computations [11]. Similar is the approach in [12] for plants with unknown constant time delay in the control.

Less conservative robust stability conditions are derived in [13,14] for a nonlinear plant with model uncertainties and a PDC controller using a new stability criteria and defining robustness domains in the parameter space. The stability conditions do not require existence of a common Lyapunov function for the local linear sub-systems in the T-S model. The same task is solved in [15] in order to design a PDC state feedback controller for a plant with unknown constant or variable upper bounded time delays in the input and in the state variables. The T-S plant model rules consequents are $\dot{x}(t) = [A_{i0} + \Delta A_{i0}(t)]x(t) + [A_{id} + \Delta A_{id}(t)]x(t-\tau_1(t)) + [B_i + \Delta B_i(t)]u(t-\tau_2(t))$, where the uncertainty is presented in a structured fractal form with restricted norm thus allowing rational uncertainties. The closed loop T-S model is first transformed into a descriptor model to escape the free weighted matrices problem. By applying Lyapunov-Krasovski approach that utilizes all the information about the time delay, the conservatism of the robust stability conditions is reduced.

2.4. Aim and Tasks of the Investigation

Though appealing the PDC-LMI industrial application in process control encounters some basic problems.

1. Difficulties in T-S modeling of the plant, based on identification when a dynamic nonlinear mathematical model is not available, due to restrictions on experimentation and plant complexity.

2. Complex and not unique derivation of stability conditions and their transformation into LMIs, high computation effort and knowledge, required for the LMIs solution, despite the techniques for simplification and conservatism reduction – proper MFs [7], alternative stability criteria [9,10,11,15], fuzzy descriptor models instead of T-S models [1,15], etc.

3. Increased number of LMIs and unknown matrices to be computed with the increase of the requirements set on the system (desired performance, robustness, restriction of signals, time delays considerations, etc.), of the plant order and the number of linearisation sub-domains, which complicates or makes impossible the solution.

4. Difficulties for experts to assign plant model uncertainties, time delays and performance restrictions due to the high level of abstractness that deprives the T-S model of linguistic meaning.

5. PDC controller's complexity – unsuitable for industrial completion and implementation, where tuning facilities, simple and transparent structure and design are preferred, due to its high order as the order of the fuzzy system is high (including an observer – another dynamic T-S model in case of fuzzy state feedback controller, or a plant model augmentation in case of dynamic feedback controller), high controller's gains computed, causing saturation of controller's signals, inapplicability of design results to fuzzy controllers for similar plants.

The aim of the recent investigation is to adapt the PDC-LMI approach for smoothly nonlinear complex processes with time delay and model uncertainty and PI fuzzy controllers in order to improve system performance and facilitate the PDC-LMI industrial application. The main tasks are:

1. Develop a system T-S dynamic modelling methodology, using state space presentations of Ziegler-Nichols linear plant models and dynamic PI controllers in the consequents.

2. Derive Lyapunov global fuzzy closed loop system stability conditions and the corresponding LMIs.

3. Develop a PDC PI controller design method for plants with time delay on the basis of local robust performance and global Lyapunov stability criteria.

4. Implement the PDC PI controller design method for real time control of the water temperature in a tank and assess the improvements by comparison of system performance with the performance of a conventionally designed classic PI control system.

3. T-S Modeling of Industrial Processes

The T-S model of an industrial process is derived in a similar way to [16], using identification. Step responses $y_l(t)$, $l=1\ldots g$, to plant input changes $u_l(t)=A_l.1(t)$ in different operation points are experimentally recorded and after their approximation Ziegler-Nichols models are estimated. Adjacent step responses that correspond to models with close parameters are grouped to determine r sub-domains of linearisation. For each i-th sub-domain an average Ziegler-Nichols model with transfer function of the type $P_i^o(s)=K_i^o.e^{-\tau_i^o s}.(T_i^o s+1)^{-1}$ is accepted as a nominal local linear plant. Due to the smooth plant nonlinearity the plant model gain K, time constant T and time delay τ vary with the operation point. So, K_i, T_i and τ_ι are different in each linear sub-domain. The sub-domains can be recognized by the plant output $y(t)$. When under closed loop control, the plant output follows the reference y_r and smoothly passes through all sub-domains from the current to the final. So, the multiplicative uncertainty model for each sub-domain can be defined as $l_i(s)=\Delta P_i(s)/P_i^o(s)$, where the additive uncertainty $\Delta P_i(s)=P_i^o(s)-P_w(s)$ is determined on the basis of the „worst" perturbed plant $P_w(s)$ for all sub-domains – collective virtual plant with the greatest gain $K_w=K_{imax}$ and time delay $\tau_w=\tau_{imax}$, and the smallest time constant $T_w=T_{imin}$, $i=1\ldots r$, with worst effect on system stability. The local linear controllers for the linear plants in the sub-domains are dynamic PI controllers with transfer functions $C_i(s)=K_{pi}.[1+1/(T_{ii}s)]$, where the controller's tuning parameters are the gain K_{pi} and the integral action time T_{ii}.

The local plant and controller differential equation are respectively

(10) $T_i^o\dot{y}_i(t)+y_i(t)=K_i^o.u_i(t-\tau_i^o)$;

(11) $u_i(t)=K_{pi}e_i(t)+(K_{pi}/T_{ii})\int e_i(t)dt$,

where $e_i(t)=y_r-y_i(t)$ is the error in the local closed loop system for constant reference y_r.

To escape the integrating in (11), both sides of (10) and (11) are differentiated to yield:

(12) $T_i^o\ddot{y}_i(t)+\dot{y}_i(t)=K_i^o.\dot{u}_i(t-\tau_i^o)$;

(13) $\dot{u}_i(t)=K_{pi}\dot{e}_i(t)+(K_{pi}/T_{ii})e_i(t)=$

$=-K_{pi}\dot{y}_i(t)+(K_{pi}/T_{ii})[y_r-y_i(t)]$.

This is equivalent to augmentation of the local plant with the integrator of the controller and the transformation of the local position PI controller into an incremental PI .

The state space representation of the local plant and controller is respectively:

(14) $$\left|\begin{array}{l}\dot{x}_i(t)=A_{io}x_i(t)+B_{id}\dot{u}_i(t-\tau_i^o)\\ y_i(t)=C_ix_i(t)\end{array}\right.;$$

(15) $\dot{u}_i(t)=-F_ix_i(t)+G_ix_r$,

where $x_i(t)=\begin{bmatrix}x_{i1}(t)=y(t)\\ x_{i2}(t)=\dot{x}_{i1}(t)\end{bmatrix}, A_{io}=\begin{bmatrix}0 & 1\\ 0 & -1/T_i^o\end{bmatrix}, B_{id}=\begin{bmatrix}0\\ K_i^o/T_i^o\end{bmatrix}, C_i=[1\ \ 0]$

and $x_r=\begin{bmatrix}x_{r1}=y_r\\ x_{r2}=0\end{bmatrix}$; $F_i=[K_{pi}/T_{ii}^o\ \ K_{pi}]$, $G_i=[K_{pi}/T_{ii}^o\ \ 0]$.

The local closed loop state space model is obtained after substituting the delayed control rate from (15) $\dot{u}_i(t-\tau_i^o)=-F_ix_i(t-\tau_i^o)+G_ix_r$ into (14):

(16) $$\left|\begin{array}{l}x_i(t)=A_{io}x_i(t)+B_{id}[-F_ix_i(t-\tau_i^o)+G_ix_r]\\ y_i(t)=C_ix_i(t)\end{array}\right..$$

Thus the fuzzy rules in the T-S models of the plant and the controller for $i=1\ldots r$ are finally determined:

(17) **R$_i$: IF** $y(t)$ is M_{i1} **AND** $e(t)$ is M_{i2} **AND** $\dot{e}(t)$ is M_{i3}

THEN. $$\left|\begin{array}{l}\dot{x}_i(t)=A_{io}x_i(t)+B_{id}\dot{u}_i(t-\tau_i^o)\\ y_i(t)=C_ix_i(t)\end{array}\right.$$

(18) **R$_i$: IF** $y(t)$ is M_{i1} **AND** $e(t)$ is M_{i2} **AND** $\dot{e}(t)$ is M_{i3}

THEN $\dot{u}_i(t)=-F_ix_i(t)+G_ix_r$

or $\dot{u}_i(t)=K_{pi}\dot{e}(t)+(K_{pi}/T_{ii})e(t)$.

The PDC fuzzy controller has three inputs $y(t)$, $e(t)$ and $\dot{e}(t)$ - $z^T=[y\ e\ \dot{e}]$ (or $y(t)$, y_r and $\dot{y}(t)$), and one output – the control rate $\dot{u}(t)$, computed as in (4), which after integration is passed onto the plant input. The inputs are normalized to have the same range and to be dimensionless. The fuzzy blending between outputs of several linear controllers cannot be normalized. Therefore a gain as a tuning parameter is used before the integrator.

The solution of the rules (17)-(18), using **CoG** defuzzication, is computed in a similar way to (5) to yield [1]:

(19) $\dot{x}(t)=\sum_{i=1}^{r}h_i^2(z)\{A_{io}x(t)-B_{id}F_ix(t-\tau_\iota^o)\}+$

$+2\sum_{i=1}^{r}\sum_{j>i}^{r}h_i(z)h_j(z).0.5\{(A_{io}+A_{jo})x(t)-(B_{id}F_j-B_{id}F)x(t-\tau_\iota^o)\}$

4. Design of Dynamic Fuzzy PI Controller

The design of the PI T-S controller involves two separate stages. First the parameters of the local linear PI controllers are tuned to ensure local system stability and robustness. The complex nonlinear plant uncertainty is scattered to smaller local plants uncertainties, which can be objectively determined from measured quantities such as the worst and the nominal plant model parameters, included in $l_i(s)$, and estimated in the frequency domain, which is good when time delay is present. Besides, the smaller plants uncertainties facilitate the achievement of good local system robustness. Next the global system stability is checked.

4.1. Tuning of Local PI Controllers from Robust Performance Criterion

A good criterion for tuning a PI controller for a linear plant, described by a Ziegler-Nichols model with time delay, could be the system robust stability or the robust performance [17]. They are defined in the frequency domain, so are suitable for stable plants with time delay.

given perturbations but also keep its transient response close to the nominal. Therefore robust performance as more general is selected for a tuning criterion.

An algorithm and a MATLAB™ program are developed for given nominal and perturbed plant models parameters to compute the PI controller parameters (K_p, T_i), which satisfy (21). First, the significant frequency range is estimated on the basis of the main frequency of the plant ω_o. Next, the ranges for the tuning parameters are calculated from restrictions set on the settling time and the overshoot accounting for the fact that the tradeoff for good robustness of the system is its slow transient response. Then for incremental values of the tuning parameters the Nyquist stability of the nominal system is checked and the fulfillment of (21). From all couples (K_p, T_i) that satisfy (21) the best is selected from the criterion for disturbance suppression (K_p/T_i)=max.

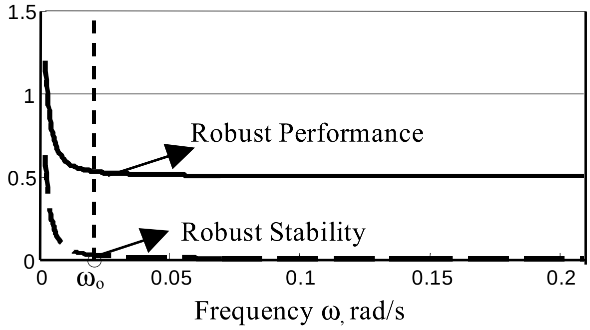

Figure 1. Illustration of fulfillment of robust criteria

The MATLAB™ program is used to tune all local PI controllers independently.

Sample graphs of the left-hand functions in (20) and (21) and their location with respect to 1 (one) for a robust system are shown in *figure1*.

The linear system is robustly stable if for all significant frequencies ω:

(20) $\left|\Phi^o(j\omega).l(j\omega)\right| < 1,\ \forall\omega$

or $\sup_{\omega}\left|\Phi^o(j\omega).l(j\omega)\right| < 1,\ \forall\omega$,

where $|\Phi^o(j\omega)|$ is the magnitude of the frequency response of the closed loop system for nominal plant, obtained for $s=j\omega$ from the closed loop system transfer function with respect to reference $\Phi^o(s) = y^o(s)/y_r(s) = P^o(s).C(s)[1 + P^o(s).C(s)]^{-1}$.

The robust performance is defined as a bounded $\boldsymbol{H}_\infty$-norm of the magnitude of the system error e - $\sup_{\omega}\left|e(j\omega)\right| < 1$ for all significant frequencies ω. The linear system has a robust performance if:

(21) $\left|S^o(j\omega).W_f(j\omega)\right| + \left|\Phi^o(j\omega).l(j\omega)\right| < 1,\ \ \forall\omega \geq 0$,

where $|S^o(j\omega)|$ is computed for nominal plant from the system sensitivity function $S(s) = e(s)/[-f(s)] = [1 + P(s).C(s)]^{-1}$ and $|W_f(j\omega)|$ is the magnitude frequency response of the shaping filter for the disturbance at the plant output (usually $\left|W_f(j\omega)\right|$=0,3...0,9 [17]).

As (21) includes (20) it sets stronger requirements for robustness – the system should not only preserve stability for the

4.2. Lyapunov Fuzzy Closed Loop System Stability Conditions

The design of the PDC PI controller is successful if the global fuzzy closed loop system with the already tuned local linear PI controllers is stable. The following proposition states a delay-independent sufficient stability condition for the system (19).

Proposition. The closed loop system (19) is quadratically stable if there exist matrices $\boldsymbol{P} > 0$, and $\boldsymbol{Q} > 0$ such that the following matrix inequalities are satisfied for i, j=1…r, j>i:

(22) $PA_{io} + A_{io}^T P + PB_{id}F_i Q^{-1} F_i^T B_{id}^T P + Q < 0$

(23) $P.0.5(A_{io} + A_{jo}) + [0.5(A_{io} + A_{jo})]^T +$
$+ 0.5(B_{id}F_j Q^{-1} F_j^T B_{id}^T + B_{jd}F_i Q^{-1} F_i^T B_{jd}^T) + Q \leq 0.$

The proof is given in Appendix.

Inequalities (22) and (23) are turned into LMIs, applying Schur compliment, and adding also the inequalities for positive definite $\boldsymbol{P}$ and $\boldsymbol{Q}$, the final LMIs become:

(24) $$\begin{vmatrix} \begin{bmatrix} PA_{io}+A_{io}^{T}P+Q & PB_{id}F \\ F_i^T B_{id}^{T} P & -Q \end{bmatrix} < 0 \\ P.0.5(A_{io}+A_{jo})+[0.5(A_{io}+A_{jo})]^T P+0.5(B_{id}F_jQ^{-1}F_j^{T}B_{id}^{T}+B_{jd}F_iQ^{-1}F_i^TB_{jd}^{T})+Q\le 0 \\ -P<0 \\ -Q<0 \end{vmatrix}$$

The LMIs (24) are solved with respect to the unknown matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ in MATLAB™ [4,5]. If solution exists the system is stable, if – not it may be stable or unstable as the proposition defines sufficient conditions.

3) the electrical heater and the heating of salty water itself, which also introduce inertia and time delay – for the short duration of pulses and pauses of PWM the heater keeps emitting an average heat and needs a long time to switch onto another average heat that corresponds to a different duty ratio, besides the dynamic asymmetry – the plant time constant and delay in heating and cooling differ for the same operation point. The noise filter adds to the inertia – good suppression of noise is related with slowing of measurement. The operation range of the plant is for temperatures [18, 70] °C with references from [25, 50] °C. The main disturbances are the ambient temperature, which varies in the range [15, 35] °C, the draught, the presence of people in the room and the initial concentrations of salts. The generalized plant progressive step responses for heating, presented in *figure 3*, can be approximated with Ziegler-Nichols models with variable gain, time constant and time delay in the different operation points. Similar but with different model parameters are the progressive step responses for cooling. The plant is with self-regulation. The natural circulation and the close temperature of the cooling air to the temperature of heating make the cooling not enough effective thus the self-regulation of the plant is slow.

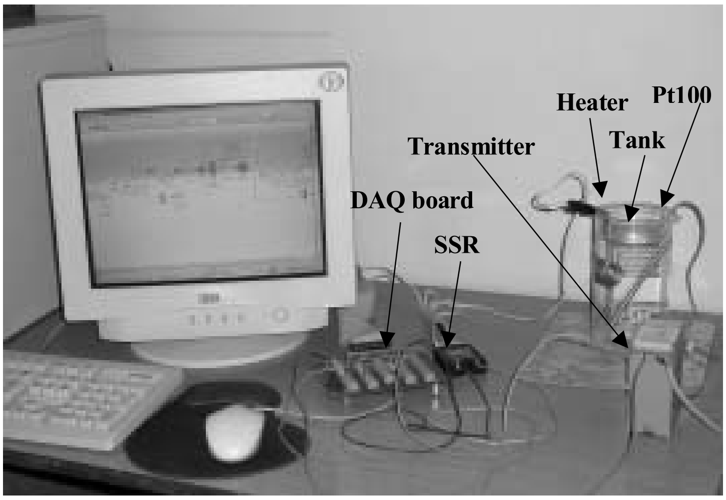

Figure 2. Laboratory tank water heating system

5. Case Study

5.1. T-S Modeling

The step responses of salty water temperature θ in a laboratory tank are experimentally studied in MATLAB™ real time [18] in [19] using the experimental setup, shown in *figure 2*. The generalized plant considered includes a Simulink completion of a Pulse-Width Modulator (PWM), a Solid State Relay (SSR), an electrical heater, the heating process with convectional cooling, measurement devices – a resistance thermometer Platinum 100 (Pt100) as a temperature sensor connected by a three wired scheme to a transmitter with normalized output voltage in the range [0, 10] V, a Simulink voltage-to-temperature converter and exponential noise filter, which output comprises the plant output.

Input to the plant is the generated in Simulink stepwise changing virtual voltage u to the PWM. The PWM output pulses control via a DAQ board and the SSR the average heat, emitted by the electrical heater. Sources of nonlinearity are: 1) the PWM, which is insensitive to voltage smaller than 0.25 V; 2) the SSR that is switching the nets supply to the heater for the time of the pulses of the PWM when the nets voltage crosses the zero, thus introducing also a time delay in a similar manner like the discretization of time with equal sample period of 1 s;

Then three linearisation sub-domains are distinguished. The average parameters of the corresponding Ziegler-Nichols models in each sub-domain, shown in *table 1*, determine the nominal models of the local linear stable plants, which are assumed completely observable and controllable. The worst-case plant parameters, which define the local plants multiplicative uncertainties, are also given in *table 1*.

5.2. PDC PI Controller Design

The PDC controller has three inputs – the current moment measured rate-of-error Δe_k, error e_k and temperature θ_k, one output – the control rate Δu_k and 18 rules of the form:

R$_i$: IF Δe_k is M_{i1} **AND** e_k is M_{i2} and θ_k is M_{i3} **THEN**

$$\Delta u_{ki} = k_{pi}\Delta e_k + \frac{k_{pi}\Delta t}{T_{ii}} e_k, \; i=1...18,$$

where the MFs of the terms M_{ij} are shown in *figure 4*. The MFs for the error e_k and the rate-of-error Δe_k are selected from general empiric rules – symmetric and orthogonal. The MFs for the temperature θ_k are obtained from the plant steady state characteristic, computed from the step responses in *figure 3* [19].

Each PI controller in the rules consequents is tuned with respect to the corresponding sub-domain linear plant using the robust performance criterion (21) and the developed MATLAB™ program. The local controllers' parameters are added in *table 1*.

The parameters of an ordinary PI controller (K_{po}, T_{io}) are also tuned from the requirement for minimal overshoot and settling time according to an empirical tuning method for the worst-case plant and included in *table 1*.

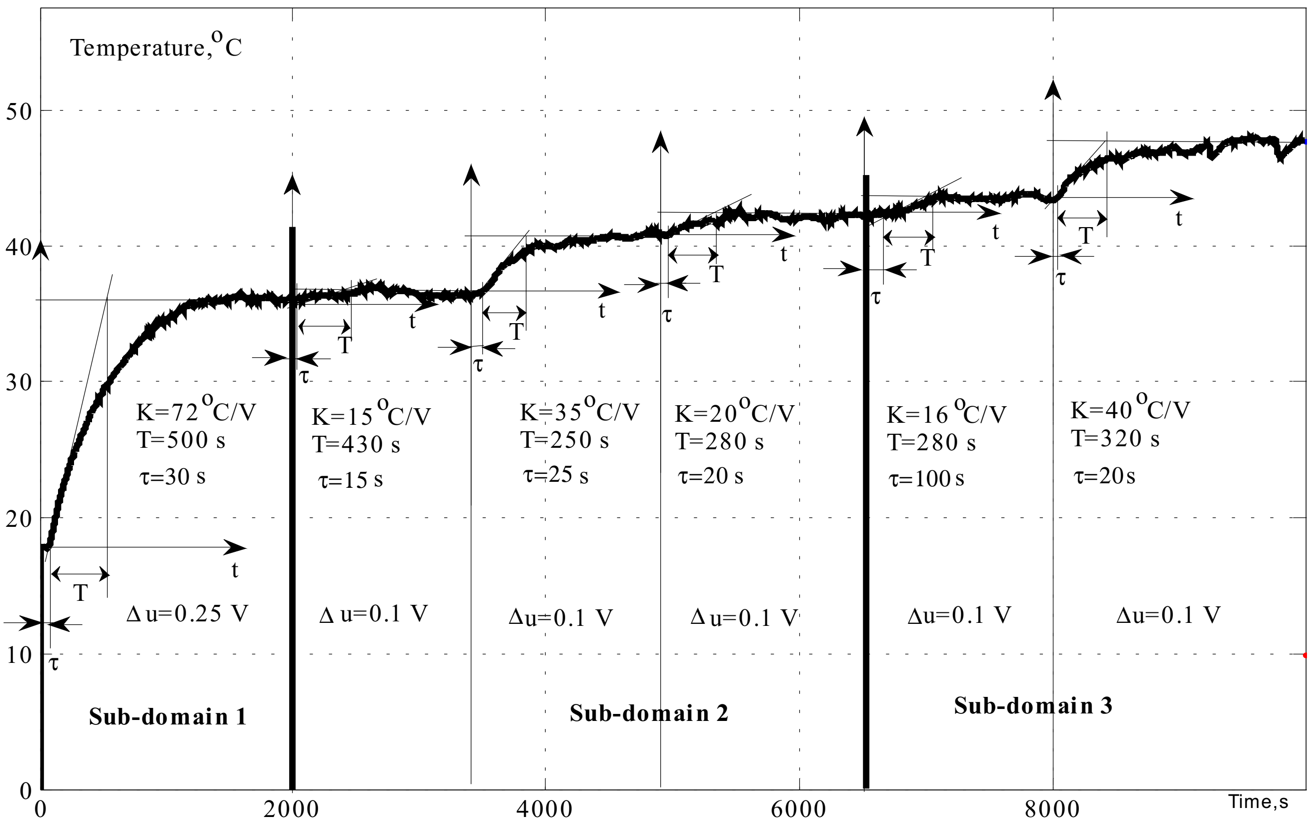

Figure 3. Plant step responses in different operation points

Table 1. Average plant model parameters and tuned local controller parameters

Plant model parameters	K_i^{av}, °C/V	T_i^{av}, s	τ_i^{av}, s	K_{pi}, V/°C	T_{ii}, s
Sub-domain 1	K_{max}=72	500	30	0.00252	200
Sub-domain 2	23	320	20	0,007	238
Sub-domain 3	28	T_{min}=300	τ_{max}=60	0,0018	68
Worst-case plant	72	300	60	$K_{po}=0.3T_{min}/(K_{max}\tau_{max})=$ 0.021	$T_{io}=0.6T_{min}$ = 180

In order to check the Lyapunov stability of the PDC PI control system eight LMIs are derived from (22)-(23) for i, j=1,2,3, $j>i$, to which also $\boldsymbol{P}>0$ and $\boldsymbol{Q}>0$ are added. They are numerically solved and the following positive definite matrices for $\boldsymbol{P}$ and $\boldsymbol{Q}$ are found –

$$P=\begin{bmatrix}1{,}95.10^{-2} & -7{,}32\\ 7{,}32 & 2{,}8.10^{3}\end{bmatrix},\quad Q=\begin{bmatrix}2{,}82.10^{-4} & -1{,}06.10^{-2}\\ -1{,}06.10^{-2} & 11{,}3\end{bmatrix}.$$

rate-of-error of 5 °C and temperature range of [18, 70], °C.

5.3. Experimental Framework

Two types of real time control of the plant are carried out – with the robustly designed PDC controller and with the classically designed ordinary PI controller. The reference is changed stepwise three times by 5 °C. The temperature step responses are shown in *figure 5* for the PDC (top) and the PI (bottom)

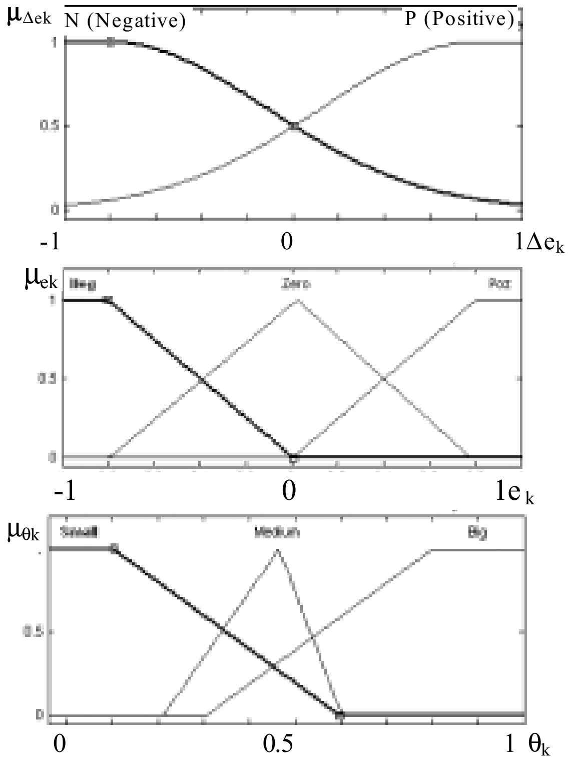

Figure 4. PDC controller membership functions for change-of-error, error and temperature

Table 2. Comparison of control systems performance indicators

	PDC controller			Ordinary PI		
Reference θ_r, °C	28	33	38	28	33	38
Overshoot $\times10^3$, %	30	30	45	60	50	45
Settling time $\times10^3$, s	0.8	0.8	0.8	1.5	1.0	0.9

This confirms that the global fuzzy closed loop system is quadratically stable.

The gain of the integrator for the defuzzified PDC controller output is tuned by simulation investigations to $K_{\Delta u}$ = 3 [19]. The initial insensitivity of the PWM of 0.25 V is compensated as an initial condition of the integrator. The normalization gains for the PDC inputs consider maximal absolute system error and control systems respectively. The systems overshoots and settling times in different operation points are estimated and given in *table 2* for easy comparison. The PDC system has on the average 30% reduced both overshoot and settling time, keeping these performance indicators almost the same in the three operation points – this is a measure for good robust performance. The behavior of the two systems in the first step responses is the most significant test for the controller how it tackles a potential problem of instability and high oscillations arising from the high plant gain in this operation point – reference of 28 °C. The last step response of the PDC system is a result of the application of external disturbance – pouring of 30% more cold water into the tank. The PDC controller compensated the impact of the disturbance in 1500 s. The PI control system needed for the same purpose 3500 s. Therefore its step response to disturbance is not shown in *figure 5*.

are derived for a PDC control system with dynamic local PI controllers and a plant with time delay, described by local Ziegler-Nichols models.

3. A simple and transparent design method for a process fuzzy controller is developed on the basis of robust performance and Lyapunov stability criteria by adapting the PDC-LMI approach for the case of dynamic local PI controllers and Ziegler-Nichols local linear plants.

4. A fuzzy PDC temperature control system is designed

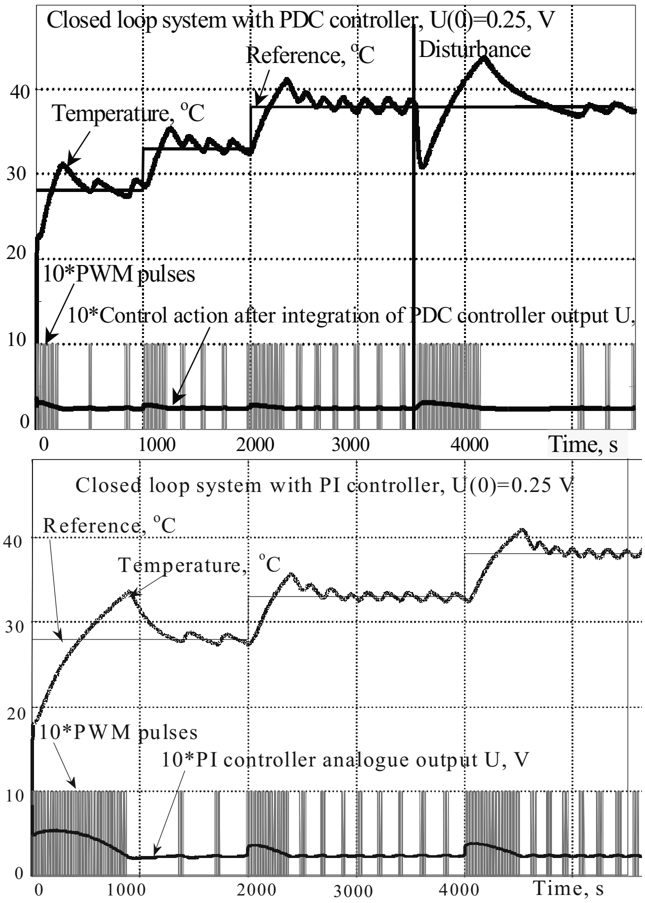

Figure 5. PDC and PI control systems step responses to reference change

6. Conclusions and Future Investigations

The main contributions of this investigation are the following:

1. A methodology for T-S modelling of complex processes with time delay and uncertainties, for which no mathematical dynamic nonlinear model is available, is suggested for the purpose of PDC design, based on the state space presentation of experimentally obtained Ziegler-Nichols models.

2. Lyapunov stability conditions and the corresponding LMIs

using the developed method and T-S modelling methodology and then studied in real time in different operation points. The comparison of the performance indicators of the designed PDC control system and of a conventionally designed ordinary PI control system proves 30% decrease of overshoot and settling time in the PDC system, preserving their values in the whole operation range of the experiments – an evidence for good performance robustness.

By this investigation engineering solutions are suggested for some basic problems of the PDC-LMI approach thus facilitating its industrial implementation for improvement of the control of complex nonlinear process with time delay.

The future work will be focused on the use of programmable logic controllers for completion of the designed PDC controller and real time experimentations with these industrial PDC controllers.

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Appendix

Proof of Proposition:

Consider the following Lyapunov function for the closed loop system, adapted from [1]:

$$V(x) = x(t)^T Px(t) + \int_{t-\tau}^{t} x(s)^T Qx(s)ds .$$

Taking the derivative of $V(x)$ along the closed loop system and considering that for any vectors x_1 and x_2 and matrix Y

$$x_1 Y x_2 + x_2^T Y^T x_1 \le x_1 Y R^{-1} Y^T x_1 + x_2^T R x_2 ,$$

where $\boldsymbol{R}$ is a positive definite matrix, it is obtained:

$$\dot{V}(x) = \sum_{i=1}^{r} h_i^2 x(t)^T \{PA_{io} + A_{io}^T P + PB_{id} F_i Q^{-1} F_i^T B_{id}^T P + Q\} x(t) + 2\sum_{i=1}^{r}\sum_{j>i}^{r} h_i h_j x(t)^T .$$
$$.\{P.0.5(A_{io} + A_{jo}) + [0.5(A_{io} + A_{jo})]^T P + 0.5(B_{id} F_j Q^{-1} F_j^T B_{id}^T + B_{jd} F_i Q^{-1} F_i^T B_{jd}^T) + Q\} x(t)$$

Since $\sum_{i=1}^{r} h_i > 0$, $h_i \ge 0$ and (22) and (23) are satisfied, $\dot{V}(x) < 0, \forall x \ne 0$ and the system (19) is stable.

References

1. Tanaka, K. and H. O. Wang. Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach. John Wiley & Sons, Inc., 2001, ISBN 0-471-32324-1.

2. Hellendoorn, J. Fuzzy Logic for Engineering Applications. Lecture Notes for MSc Programme in Systems and Control. Delft University of Technology, NL, 2008, <http://www.dcsc.tudelft.nl/~sc4150>.

3. Slotine, J. J. and W. Li. Applied Nonlinear Control. Englewood Cliffs, NJ: Prentice-Hall, 1991.

4. Yonchev A., M. Constantinov, P. Petkov. Linear Matrix Inequalities in Control Theory. S., Demetra, 2005, ISBN 954-9526-32-1 (in Bulgarian).

5. Balas, G., R. Chiang, A. Packard and M. Safonov. Robust Control Toolbox™3 User's Guide. The MathWorks, Inc., 2005.

6. Jang, J.-S.R., C.-T. Sun, and E. Mizutani. Neuro-Fuzzy and Soft Computing. A Computational Approach to Learning and Machine Intelligence. MATLAB Curriculum Series, Prentice-Hall, Inc., N.J., 1997, ISBN 0-13-261066-3.

7. Lam, H. K. and F. H. F. Leung. Fuzzy Controller with Stability and Performance Rules for Nonlinear Systems. – *J. Fuzzy Sets and Systems*, 158, 2007, No. 2, 147-163.

8. Cuesta, F. and A. Ollero. Fuzzy Control of Reactive Navigation with Stability Analysis Based on Conicity and Lyapunov Theory. – *J. Control Engineering Practice*, 12, 2004, No. 5, 625-638.

9. Gu, Y., H. O. Wang and K. Tanaka. Fuzzy Control of Nonlinear Time-Delay Systems: Stability and Design Issues. Proc. of the 2001 American Control Conf., Arlington, VA, June 25-27, 2001.

10. Cao, Y. Y. and P. M. Frank. Analysis and Synthesis of Nonlinear Time-Delay Systems via Fuzzy Control Approach. – *J. IEEE Trans. on Fuzzy Systems*, 2000, No. 8, 200-211.

11. Yoneyama, J. New Robust Stability Conditions and Design of Robust Stabilizing Controllers for Takagi-Sugeno Fuzzy Time-Delay Systems. – *J. IEEE Trans. on Fuzzy Systems*, 15, 2007, No. 5, 828-839.

12. Chen, B., X. Liu and S. Tong. Robust Fuzzy Control of Nonlinear Systems with Input Delay. – *J. Chaos, Solitons & Fractals*. 37, Aug. 2008, No. 3, 894-901.

13. Leung, F. H. F., H. K. Lam and P. K. S. Tam. Lyapunov Function Based Design of Robust Fuzzy Controllers for Uncertain Nonlinear Systems: Distinct Lyapunov Functions. Proc. IEEE World Congress on Computational Intelligence, FUZZ-IEEE, Anchorage, AK, May 4-9, 1998, 577-582.

14. Leung, F. H. F., H. K. Lam and P. K. S. Tam. Design of Fuzzy Controllers for Uncertain Nonlinear Systems Using Stability and Robustness Analyses. – *J. Systems & Control Letters*, 35, 19 Nov. 1998, No. 4, 237-243.

15. Li, L. and X. Liu. New Approach on Robust Stability for Uncertain

T-S Fuzzy Systems with State and Input Delays. –*J. Chaos, Solitons & Fractals*, Nov. 2007.

16. Petrov, M., A. Taneva, T. Puleva and E.Garipov. Parallel Distributed Fuzzy PID Control of Hydro Turbine Generator. Proc. 9th WSEAS Int. Conf. on Fuzzy Systems. Sofia, Bulgaria, 2-4 May, 2008, 73-78.

17. Morari, M. and B. Zafiriou. Robust Process Control. Prentice Hall, N.J., 1989.

18. MATLAB – Real-Time Workshop. User's Guide. The MathWorks, Inc., 1992.

19. Yordanova, S., B. Tabakova. Robust Fuzzy Parallel Distributed Compensation PI Control of Non-Linear Plant. Proc. 8th WSEAS Int. Conf. on Artificial Intelligence, Knowledge Engineering and Data Bases – AIKED'09, Cambridge, UK, 21-23 Feb. 2009, 128-133. ISBN 978-960-6766-56-5.

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